Expectations and the Structure of Share Prices

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This paper presents the results of an empirical study of year-end common-stock prices from 1961 through 1965. The ratios of market prices to earnings are related to such factors as earnings growth, dividend payout, and various proxy variables designed to measure the risk or quality of the returns stream.

Several previous empirical studies[1] have tried to explain share prices on the basis of such variables, but these investigations were forced to rely on published accounting data and untested hypotheses about the formation of expectations. V. Whitbeck and M. Kisor were able to increase the explanatory ability of their regression by substituting the estimates of security analysts of one firm for fabricated expectations variables based on simple extrapolations of past performance. Our study tries to determine whether the goodness of fit can be improved still further by substituting the estimates from several securities firms for the expectations of a single predictor and by using a wider variety of such expectational variables. The most important of the expectational variables employed are forecasts of short-term and long-term earnings growth, estimates of the "normal earning power" of each company, and estimates of the "instability" of the earnings stream. The data used are described in Section II.

It is found in Section III that an extermely close fit to the empirical structures of share prices is obtained with the use of such expectations data. These results are also contrasted with those obtained when only historic data are used. Section III then examines further the stability and predictive power of the model over time. Section IV discusses the usefulness of the model for security selection.

I. *Specification of a Valuation Model*

In the typical valuation model, the price of a share is taken to be the present value of the returns expected therefrom. In the simplest model, the price is the sum of the present values of a stream of dividends that is assumed to grow at a constant rate, g, over time. See, for example, J. B. Williams for one of the earliest statements of the problem and M. J. Gordon for a more recent treatment. Letting P stand for the (ex dividend) price of a share, D the (annual) dividend per share in the year just past, and r the appropriate rate of discount, we have

$$P = \sum_{i=1}^{\infty} D \frac{(1+g)^i}{(1+r)^i}, \tag{1}$$

provided $g<r$. Dividing both sides of (1) by earnings per share, E, and summing the

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[1] Cross-sectional empirical studies have been undertaken by F. D. Arditti, H. Benishay, R. S. Bower and D. H. Bower, G. R. Fisher, I. Friend and M. Puckett, M. J. Gordon, F. C. Jen, M. Kisor, Jr. and A. Feuerstein, Kisor and S. Levine, and R. Ortner.

progression we obtain an expression for the price-earnings multiple

$$(2) \qquad \frac{P}{E} = \frac{D}{E}\,\frac{(1+g)}{(r-g)}$$

The price-earnings ratio is seen to depend on the dividend payout ratio and the expected long-term growth rate of the dividend stream.

The specific model of security price-earnings ratios presented in equations (1) and (2) has several drawbacks. It is inapplicable in cases where no dividends are currently paid, it leads to an infinite value for the shares when $g>r$, and it requires projecting growth rates from now till Kingdom come.[2] Such difficulties have led several writers to formulate a finite-horizon model of share prices. See, for example, Charles Holt and Malkiel. P. F. Wendt presents a useful survey of a number of alternative models. The basic idea of the finite-horizon approach is that both dividends and earnings are assumed to grow at some rate g for N periods,[3] and then grow at a normal rate such as the growth rate for economy as a whole. This approach can be illustrated by the following very simple model.[4]

$$(3) \qquad \frac{P_0}{E_0} = \sum_{i=1}^{N} \frac{D_0}{E_0}\,\frac{(1+g)^i}{(1+r)^i} + (m_s)_0\,\frac{(1+g)^N}{(1+r)^N},$$

where $(m_s)_0$ is the average current price-earnings ratio for the market as a whole. The model in (3) appears to be highly nonlinear in the growth rate and payout ratio. Fortunately, however, a linear approximation to the true expression seems to work reasonably well for N as small as five, the period for which we have growth-rate estimates.[5]

The preceding model has abstracted entirely from the existence of risk. There are several possible ways in which risk can be represented in a valuation model. The theoretical justification for the alternatives rests on the assumptions employed.

A common way in which risk is introduced into empirical valuation models is to incorporate a term representing the (expected) variance of the future returns stream from each security. Such a procedure has been justified in two ways. First, it has been argued (e.g., see L. G. Peck) that the horizon, N, over which extraordinary growth can be forecast is itself a function of the variance or "dependability" of the returns stream. By this reasoning, investors would project extraordinary earnings growth over only a very limited horizon for companies where the anticipated variance of the earnings stream is large. Since it can easily be shown that $\partial(P/E)/\partial N>0$ for a growth stock according to the finite-horizon model (see Malkiel, pp. 1028–29), it follows that price-earnings multiples should be negatively related to the variance term.

[2] Moreover, since the growth rate estimates collected were specifically made for only the next five years, it would seem that this model is not consistent with the data.

[3] In some models, the growth rate is assumed to decline in stages to the final "mature" growth rate of the economy. In other models, the initial and terminal growth rates are estimated on the basis of such factors as the retention rate and the rate of return on equity.

[4] The rationale for this approach and the derivation of equation (3) is contained in Malkiel. It is assumed that after N periods, the price-earnings ratios for all stocks revert to the same average condition.

[5] The closeness of the proposed linear approximation was examined by fitting a regression of the form

$$(3') \qquad \frac{D_{j0}}{E_{j0}} \sum_{i=1}^{5} \frac{(1+g_j)^i}{(1+r)^i} + (m_s)_0\,\frac{(1+g_j)^5}{(1+r)^5} = A + Bg_j + C\frac{D_0}{E_0}$$

Values of the parameters $(m_s)_0$ and r were chosen to be consistent with experience during the 1961–65 period. The coefficient of determination, 0.97, was so high that it seemed safe to substitute the right-hand side of (3′) for the right-hand side of (3). It should be noted, however, that this argument assumed that the horizon N is the same for all companies.

A second justification for the inclusion of a variance term in the model rests on recent theoretical work by William Sharpe, John Lintner, and Jan Mossin, extending the Markowitz portfolio selection model. In these models the market establishes "prices" for the expected return and "risk" of each security, where risk consists of the sum of the variance of that security's return and its covariances with all other returns multiplied by the number of shares. If we assume that the returns from different securities are uncorrelated with each other, however, it turns out that the price of a security should simply be a linear function of the expected return and the variance associated with the security. This suggests not only that a variance term should be included in the model but also provides some justification for the linear specification employed in this study.

The second risk measure employed in this study, an index of the conformance between the returns of each individual security and that of a market index, rests on more realistic assumptions. In Sharpe's simplification of the Markowitz model, covariances are assumed to arise because all returns depend on one or a few common factors, such as a market or industry return. For example, the returns from each security, R_i, might first be related to the returns from some index of security prices

$$(4) \quad R_i = \alpha_i + \beta_i(\text{Return to Index}) + \mu_i$$

The total risk of an asset (i.e., the scatter of the R_i around their mean), can then be decomposed into a systematic component (due to underlying relationship between R_i and the return from the market index) and a nonsystematic component, μ_i, uncorrelated with the market index. We would expect investors to prefer those securities with low or negative β_i's. Other things being equal, a stock whose movements are not highly correlated with the market will tend to reduce the variability and thus, the risk of the stock portfolio. Of course, it should be emphasized that the covariances and variances that are being valued in the market are those perceived by investors and not some "true" set.

The final risk variable employed was a leverage variable measuring the "financial risk" of a company. As Franco Modigliani and Merton Miller have shown, leverage can be expected to decrease the price-earnings multiple by increasing the riskiness of the returns of common stock relative to their expected values. With a fully adequate measure for the risk associated with the stock, leverage should play no part. Otherwise, it may serve as a useful proxy for the expected variability of the returns stream. Indeed, if other risk measures apply to the instability of the operating earnings stream before fixed charges, and thus serve as estimates of the "business risk" of the firm, a leverage term may capture the additional financial risk of the firm.

Before ending this discussion of the general model underlying the study,[6] it is worth emphasizing that the model is cast entirely in terms of expectational variables. The critical dependence of share prices on expectational variables has proved to be a major obstacle for empirical investigators. Since only historical data have been available to most researchers, it has been difficult to isolate the true effect of the various variables influencing stock prices. A simple illustration should make this clear. The model described above indicates that we should expect that a *ceteris paribus* increase in the dividend-payout ratio should increase the price-earnings multiple of the shares.[7] Suppose, however, that the past

[6] In a forthcoming publication, the authors will present a thorough and integrated model of share valuation.

[7] We must be careful, however, not to interpret a positive dividend coefficient as indicating that an individual firm can increase the price-earnings ratio of its shares by raising the dividend-payout ratio. A higher dividend (lower retention rate) may lower the future

growth rate of earnings is a very imperfect substitute for the relevant expected growth rate security purchasers anticipate.[8] The dividend payout could actually serve as an alternative proxy for expected growth.

For example, investors may take a low dividend payout ratio as a signal that the firm has many profitable investment opportunities available and that a high rate of earnings growth can be expected. In such a case, the coefficient of the payout ratio will be biased downward.[9] Without the proper expectational variables, it will be impossible to untangle the true influence of the many factors influencing the structure of price-earnings multiples. The following section will discuss the actual data employed in the study and indicate how they were collected.

II. *A Description of the Data Employed*

The principal data used in the study consist of a small number of forecasts of the long-term growth rates of earnings for 178 corporations, as of the five year-end periods from 1961 through 1965. In addition, data were collected on security analysts' estimates of "normal" earnings for the preceding year, their forecasts of next year's earnings, and their expectations about the future variability of the earnings stream. Certain historical financial data were also used to provide a contrast with the expectations data. These included past growth rates of various financial variables, past dividend-payout ratios, and a number of calculated risk proxies.[10]

The expectations data were collected from 17 investment firms, most of which were members of The Institute for Quantitative Research in Finance.[11] Of the participating firms, four were brokerage houses doing a considerable amount of investment advisory and institutional business, five were banks heavily engaged in trust management, five were mutual-fund management companies, two were pension-fund managers, and the remaining participant was an insurance company. The sample of 178 corporations was selected on the basis of data availability. Companies were included in the sample only when several investment firms made estimates of future earnings growth. Since there tended to be considerable overlap in the coverage of the security analysts for the leading industrial and utility companies, our sample tends to contain the "blue-chip" group of companies in which investment interest is centered. A detailed description of the data used in the study follows:

(a) *Normalized Earnings*

It is well known that the market does not necessarily capitalize the reported accounting earnings for a firm during the preceding year. If, for example, reported earnings are affected unfavorably by such

growth rate per share by an amount sufficient to keep the price of the shares constant. Thus, the standard dividend model of share valuation is in no way inconsistent with the result of Miller and Modigliani that dividend policy cannot effect the value of the enterprise.

[8] It may be argued that one should not put so much reliance on either past or expected growth rates to explain security prices since there is considerable evidence that earnings growth is "higgledy piggledy." I. M. D. Little and Cragg and Malkiel have shown that both historic growth rates and even the forecasts of security analysts are little related to the growth that is actually achieved. This may be true and yet security analysts may continue to estimate the worth of shares and their anticipated future returns on the basis of the anticipated growth rate of the security's earnings. As is well known from work on the term structure of interest rates, expectations need not be correct to be an important determinant of the yield curve. Surely it is an empirical question whether or not the market actually does value shares consistently with the model presented here.

[9] For a full discussion of the pitfalls involved in isolating the effect of dividend policy on share prices, see Friend and Puckett.

[10] All historical data were taken from the *COMPUSTAT* tapes made available by Standard Statistics Corporation.

[11] The Institute is a consortium of 30 investment firms, organized to promote quantitative research in finance.

nonrecurring factors as strikes or flood damage, or by a cyclical contraction, it is likely that investors apply an appropriate price-earnings multiple to the amount they consider to represent the normal earning power of the company. Indeed, one of the first jobs of a security analyst is to adjust the firm's accounting earnings to arrive at an indication of true earning power (see B. Graham, D. L. Dodd, and S. Cottle ch. 34). Thus, the price-earnings ratios that are relevant for valuation may be the ratios of prices to normalized earnings rather than ratios of prices to reported earnings for the preceding accounting period. These normalized earnings are estimated to be the earnings that would obtain at a normal level of economic activity if the company were experiencing normal operations—that is, operations not affected by such nonrecurring items as strikes, natural disasters, and so forth. The normalized-earnings figures used in the present study were averages of estimates supplied by two of the participating firms.

(b) *Future Long-term and Short-term Growth Rates*

As was mentioned above, several theoretical models of stock valuation have all focused on the expected growth rates of earnings and dividends as a central explanatory variable. Most previous empirical studies, however, were forced to rely on past growth rates as a proxy for future growth rates. One of the major purposes of the present study was to ascertain whether the estimates of future growth rates from several securities firms can enable us to obtain a more satisfactory explanation for the structure of share prices.

In order to contrast the use of historical and expected growth rates, we first tried to find those historical growth rates that showed the closest correlation with market price-earnings multiples. Forty alternative growth rates were tried. These growth rates differed with respect to the period covered, the method of calculation, and the financial data upon which the growth rate was estimated. From the forty candidate growth rates, the following three were either clearly superior or, at least, no worse than any of the others. These were 1) the ten-year growth rate of earnings per share calculated as the geometric mean of first ratios, 2) the ten-year growth rate of cash earnings per share (i.e., earnings plus noncash charges) calculated as the geometric mean of first ratios, and 3) the ten-year growth rate of cash earnings plus taxes calculated from a regression of the logarithms of the earnings on time. The growth rate of cash earnings was slightly better than the other two in most of the five years studied, and was used in the regressions reported in this paper.

The expected growth rates were estimated by nine securities firms.[12] Each growth rate figure was reported as an average annual rate of growth of earnings per share expected to occur over the next five years. The figures used in the study were averages of the nine predictors.

In addition to these expectations of long-term growth rates, we also collected estimates of the following year's earnings from eleven securities firms.[13] We found, somewhat to our surprise, that the implicit forecasts of short-term (one-year) growth were not highly correlated with the long-term anticipations and we were able to use both sets of data in some of the empirical work presented later.

Obviously these expected growth rates are not the expectations of a wide cross-section of the buyers and sellers in the market. These expectations were formed,

[12] It should be noted that not all firms provided growth-rate estimates for each of the companies used in the sample during each of the five years, 1961–65.

[13] Three of these eleven firms also supplied long-term forecasts.

however, by professional security analysts for securities firms or for large institutional investors who are important participants in the market. Moreover, in many cases, these expectations were made to be provided to other investors whose own expectations may be influenced by their advisors. Finally, we should note that these expectations are not limited to published information. The security analysts involved frequently visit the companies they follow and discuss the company's prospects with its executives. Insofar as other security analysts follow the same sort of procedures as our participating firms, the growth-rate estimates of other institutional investors and securities firms may resemble those we have collected. Consequently, these predictions may well serve as acceptable proxies for market expectations and they surely seem worthy of detailed analysis.

(c) *Dividend Payout*

The measurement of the dividend-payout ratio also presents problems. If we simply take the ratio of dividends to earnings, short-run disturbances to reported earnings that do not produce equiproportional changes in dividends can make calculated payouts differ considerably from target or normal payout ratios. For this reason we chose two alternative methods of calculating the dividend payout. The first method was simply to divide the dividend by normalized rather than reported earnings. The second method, used in the regressions where only historic data were employed, was to average the actual payout ratios over the preceding seven years.

(d) *Risk Variables*

Several types of expectational risk variables were introduced to serve as proxies for the anticipated variance of individual security returns. We included such variables as the standard deviation of the forecasts of security firms, various types of subjective quality ratings, and an index of the expected instability of future earnings. These risk proxies all turned out to be highly correlated with each other and only the one most useful in explaining earnings multiples, the instability index, has been included in the regressions reported in this paper. This variable was collected from one of our participating firms and represented a measure of the past variability of earnings (around trend) adjusted by the security analyst to indicate anticipated future variability.

In order to contrast the use of expectations data with historical data, a number of risk proxies were calculated on the basis of the financial records of each company. These included statistics measuring the variance of past earnings and of other financial data, a leverage variable, and the conformance between returns of each individual security and that of a market index. The index of market conformance was obtained by estimating the slope, β_i, of a regression of the annual returns of each security on the annual returns from the Standard and Poor's Composite Index. Ten years of data were employed in obtaining the estimate. The most useful historic risk proxies for our present purposes were the semideviation of earnings around trend, the index of market conformance, and the leverage variable. In Table 1 we summarize the variables employed in the regressions.

Before turning to the regression results, a problem concerning the timing of the availability of the expectations and hisrorical data should be mentioned. Our study tries to explain differences among price-earnings multiples for a cross-section of securities as of December 31 in each of five years. While normal earnings per share (and expected growth rates) were estimated and, therefore, available at the end of each year, actual earnings per share for the 12 months to December 31 are not generally known until some time after the close of the year. Thus, the actual P/E ratios and the historic growth rates calculated to the end of the year, which we employed in the regressions estimated from historic data, were not available to investors on the dates for which equations were estimated, although rather close estimates of the earnings necessary for the calculations are usually well known by that time. In order to test whether our results might be strongly influenced by, in effect, assuming perfect foresight by the market regarding current-year's earnings, we performed an alternative set of runs using the most recent publicly available 12-months' earnings to calculate P/E ratios and historic growth rates. Since the regression results from the alternative set of runs were almost identical to those reported here, it seems safe to conclude that our assumptions regarding the timing of the availability of historic data had little influence on the results.

TABLE 1—VARIABLES USED IN VALUATION STUDY

P	End-of-year market price per share
D	Total dividends paid per share (adjusted to number of shares outstanding at year end)
E	Reported earnings per share (adjusted to exclude nonrecurring items)
$\overline{D/E}$	Average dividend-payout ratio over past 7 years
$\overline{NE}$	Average "normalized" earnings estimates of security analysts
$\bar{g}_p$	Average predicted future long-term growth rate of earnings per share, measured as an annual percentage rate of growth
g_H	Historic (10-year) growth rate of (cash) earnings per share measured as an annual percentage rate of growth
I_p	Predicted instability index of the future earnings stream
β	The slope of a regression of the annual returns from a company's shares on the annual returns from the market index
$I_{H,1}$	Calculated instability index of the historic earnings stream (semideviation of earnings around trend)
$I_{H,2}$	Calculated instability index of the historic operating earnings streams (semideviation of earnings plus financial fixed charges around trend)
$\bar{E}_{t+1}$	Average predicted earnings per share for the next year
$\frac{F}{E+F}$	Leverage variable (the ratio of fixed charges to earnings plus fixed charges)

III. *Regression Results*

In this section we first present a comparison of the regression results for equations including comparable historic and expectational variables. Then, the results for the most satisfactory expectational equations are shown and the stability of the coefficients over time is examined.

(a) *Comparison of Regressions Using Historical and Expectational Variables*

In Table 2 the results of regressions using only three variables calculated from readily available historical data are compared with regressions employing comparable expectations data.[14] In panel A of Table 2, the price-earnings multiple is regressed on the historic ten-year growth rate of cash earnings (calculated as the geometric mean of first ratios), the dividend-payout ratio (averaged over the preceding seven years), and an instability index of earnings (calculated as the semideviation from a regression of earnings over the past ten years). It will be noted that generally about half of the variance in price-earnings multiples is explained by the regressions. The growth-rate variable is highly significant in each of the years covered. The calculated payout and risk

[14] It will be noted that the sample size for each regression was usually less than the total sample of 178 companies. Companies had to be dropped from the sample whenever historic or expectational data were unavailable or could not be computed. In addition whenever a company's calculated historic growth rate was negative, the firm was dropped from the sample. This was done to make the regressions based on historic data as comparable as possible to those based on expectations data, where no negative growth rates were projected.

TABLE 2—COMPARISON OF REGRESSIONS USING HISTORICAL AND EXPECTATIONAL VARIABLES

A. REGRESSION RESULTS: HISTORIC VARIABLES

$$P/E = a_0 + a_1 g_H + a_2 \overline{D}/\overline{E} + a_3 I_{H,1}$$

Year	$\hat{a}_0$	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	R^2	F
1961	13.65	+1.87	−.26	−.65	.50	51.27
		(.17)	(6.14)	(1.37)		(3; 156)
		10.72	−.04	−.47		
1962	8.92	+1.06	+6.90	−.77	.45	44.78
		(.10)	(3.28)	(.68)		(3; 163)
		10.90	2.10	−1.14		
1963	9.39	+1.33	+5.22	−.96	.49	51.31
		(.12)	(3.73)	(.81)		(3; 161)
		11.29	1.40	−1.19		
1964	10.88	+.95	+4.85	−.69	.36	32.16
		(.11)	(3.52)	(.71)		(3; 170)
		8.65	1.38	−.96		
1965	5.74	+1.52	+6.64	+.35	.65	98.65
		(.10)	(3.55)	(.77)		(3; 162)
		15.23	1.87	.46		

B. REGRESSION RESULTS: COMPARABLE EXPECTATIONS VARIABLES

$$P/E = a_0 + a_1 \bar{g}_p + a_2 D/\overline{NE} + a_3 I_p$$

Year	$\hat{a}_0$	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	R^2	F
1961	4.73	+3.28	+2.05	−.82	.70	89.34
		(.23)	(4.33)	(.75)		(3; 115)
		14.47	.47	−1.09		
1962	11.06	+1.75	+.78	−1.61	.70	133.33
		(.13)	(2.47)	(.39)		(3; 174)
		13.99	.31	−4.11		
1963	2.94	+2.55	7.62	−.27	.75	174.51
		(.13)	(2.58)	(.39)		(3; 174)
		19.67	2.95	−.69		
1964	6.71	+2.05	+5.33	−.89	.75	168.46
		(.11)	(2.17)	(.36)		(3; 170)
		18.24	2.44	−2.48		
1965	.96	+2.74	+5.01	−.35	.85	317.52
		(.10)	(2.05)	(.30)		(3; 171)
		26.50	2.44	−1.14		

Note: Numbers in parentheses below coefficients are standard errors and numbers below parentheses are t-values. Numbers below the F-values are degrees of freedom.

measures usually have their expected signs but are not significant.[15]

In panel B of Table 2, the average growth rates and other expectational variables collected from the participating firms are used to explain price-earnings multiples. All coefficients have their expected signs. Moreover, the fits are very close for cross-sectional empirical work and are much better than those obtained with the historical data. About three quarters of the variability of price-earnings ratios is explained by the regressions. We should also mention that better fits were obtained by using the average growth rates of all predictors than by employing forecasts of a single analyst. This suggests that our survey was useful in getting closer to what might be considered the expectations of a "representative" investor.

(b) *Regression Results Employing a Covariance Risk Measure*

In Table 3 we present regression results employing a covariance risk measure. It will be noted that β, the index of market conformance, has the right sign in all cases except for the 1961 regression employing expectations data. Although it is significant in only two of the five years, the general consistency of the signs would suggest that market values do tend to reflect measures of past covariance with the market. It is also interesting that β had a particularly strong influence on price-earnings ratios at the end of 1962, following a large decline in stock prices. It would appear that investors particularly favor securities that tend to move relatively independently of the market during periods when the memory of sharply falling stock prices is clearly in mind.

Comparing Tables 2 and 3, the *t*-values associated with β tend to be slightly higher than those associated with either of the two previous risk variables.[16] When a variable measuring expected short-term growth is introduced, however, the predicted instability index tends to be somewhat superior, being "significant" in four out of the five years (see Table 5). The variables β and I_p cannot be used together in the same regression, because the two variables are highly correlated, and both become insignificant.[17]

(c) *Regression Results Employing a Combination of Expectations and Historic Data*

In Table 4, we present regression results involving a combination of expectations and historic data. The price-normalized earnings ratio is employed as the dependent variable. Independent expectational variables include anticipations of short- and long-term growth, and the dividend payout expressed as a percent of normalized earnings. Historic variables were an instability index and a leverage variable. In these regressions, the instability index was calculated from a time-series of earnings plus fixed charges. This measure should represent the instability of operating earnings and may serve as an acceptable proxy for business risk. We also included a leverage variable, which should indicate the additional financial risk borne

[15] As noted above, the positive sign on the dividend coefficient should not be interpreted as evidence that dividend policy can affect the value of the shares. This coefficient indicates only that a *ceteris paribus* change in dividend payout will increase the price of the shares. What the famous "dividend-irrelevancy" theorem of Modigliani and Miller says is that an increase in dividend payout (holding the firm's investment constant) will tend to reduce the growth rate of earnings per share since new shares will now have to be sold to make up for the extra funds paid out in dividends. A positive dividend coefficient is thus in no way inconsistent with the dividend-irrelevancy theorem.

[16] While it should be noted that these comparisons are based on regressions using somewhat different numbers of observations, the conclusions presented hold also for comparisons based on the smaller sample of companies for which all data were available.

[17] Correlation coefficients between β and I_p during the period studied are approximately 0.60.

Table 3—Regression Results Employing a Covariance Risk Measure

A. Historic Variables and Covariance Measure
$P/E = a_0 + a_1 g_H + a_2 \overline{D}/\overline{E} + a_3 \beta$

Year	$\hat{a}_0$	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	R^2	F
1961	15.52	1.82	−1.75	−1.53	.49	52.60
		(0.17)	(6.14)	(1.34)		(3; 161)
		10.54	−0.29	−1.15		
1962	12.42	1.02	4.28	−2.87	.54	65.86
		(0.09)	(2.94)	(0.60)		(3; 169)
		11.38	1.46	−4.76		
1963	9.20	1.28	6.84	−1.21	.48	51.69
		(0.11)	(3.67)	(0.88)		(3; 168)
		11.19	1.87	−1.38		
1964	14.37	0.96	3.29	−3.54	.44	44.76
		(0.10)	(3.18)	(0.72)		(3; 173)
		9.36	1.03	−4.92		
1965	7.47	1.52	5.58	−0.95	.64	99.49
		(0.10)	(3.34)	(0.79)		(3; 165)
		15.30	1.67	−1.20		

B. Comparable Expectations Variables and Covariance Measure
$P/E = a_0 + a_1 \bar{g}_p + a_2 D/\overline{NE} + a_3 \beta$

Year	$\hat{a}_0$	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	R^2	F
1961	3.63	3.29	3.24	0.97	.74	132.82
		(0.19)	(4.47)	(1.09)		(3; 140)
		17.20	0.73	0.89		
1962	9.79	1.87	2.25	−2.65	.72	148.29
		(0.11)	(2.23)	(0.47)		(3; 173)
		16.88	1.01	−5.69		
1963	3.47	2.57	7.17	−0.84	.75	176.82
		(0.12)	(2.47)	(0.61)		(3; 174)
		21.38	2.90	−1.37		
1964	6.16	2.10	5.87	−1.41	.76	184.63
		(0.10)	(2.04)	(0.53)		(3; 173)
		21.40	2.88	−2.67		
1965	0.25	2.86	5.01	−0.47	.86	352.19
		(0.10)	(2.00)	(0.49)		(3; 172)
		29.14	2.50	−0.96		

Note: Numbers in parentheses below coefficients are standard errors and numbers below parentheses are t-values. Numbers below the F-values are degrees of freedom.

by the shareholders. The specific measure employed was the ratio of fixed charges per share to earnings plus fixed charges per share.[18] In addition, a dummy variable was included that took the value unity for utility companies and zero for industrials. This variable was introduced to account

[18] For a discussion of the problems involved in using the debt-equity ratio itself, see A. Barges and R. Wippern.

TABLE 4—REGRESSION RESULTS EMPLOYING A COMBINATION OF EXPECTATIONS AND HISTORIC DATA

$$P/\overline{NE}=a_0+a_1\bar{g}_p+a_2\bar{E}_{t+1}/\overline{NE}+a_3D/\overline{NE}+a_4F/(E+F)+a_5Dum+a_6I_{H,2}$$

Year	$\hat{a}_0$	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	$\hat{a}_4$	$\hat{a}_5$	$\hat{a}_6$	R^2	F
1961	−41.19	+2.88	+44.88	+5.53	−12.34	+1.79	−4.93	.85	102.98
		(.20)	(5.24)	(4.53)	(4.06)	(1.69)	(9.21)		(6;106)
		14.07	8.57	1.22	−3.04	1.05	− .54		
1962	−1.41	+1.68	+9.89	+2.60	−7.53	+4.46	−7.69	.78	74.04
		(.13)	(2.72)	(2.50)	(2.07)	(.92)	(4.75)		(6;129)
		13.16	3.63	1.04	−3.65	4.87	−1.62		
1963	−12.94	+2.41	+15.29	+8.96	−6.20	+.71	−5.70	.81	90.72
		(.14)	(2.99)	(2.79)	(2.33)	(1.04)	(5.33)		(6;129)
		17.12	5.11	3.21	−2.66	.69	−1.07		
1964	−10.91	+1.89	+14.31	+7.70	−3.39	+3.62	+4.59	.80	83.42
		(.12)	(2.02)	(2.45)	(2.21)	(.94)	(5.28)		(6;128)
		15.65	7.09	3.14	−1.53	3.86	(.87)		
1965	−15.55	+2.64	+20.05	−2.04	−7.81	+2.64	−17.59	.84	118.41
		(.14)	(1.99)	(3.01)	(2.61)	(1.12)	(6.33)		(6;128)
		18.69	10.09	−.68	−2.99	2.37	−2.78		

Note: Numbers in parentheses below coefficients are standard errors and numbers below parentheses are t-values. Numbers below the F-values are degrees of freedom.

for differences in risk between the two classes of companies not captured by our other risk variables.

As can be seen from the table, the combination of historical and expectational variables works remarkably well in accounting for the structure of share prices. Most significant were the coefficients of the short- and long-term growth rates. It should be noted that while the coefficient of the "operating-risk" variable (the semi-deviation of earnings plus fixed charges around trend) usually was not statistically significant and had the "wrong" sign in 1964, the coefficient of the financial-risk variable (our measure of leverage) always had the "correct" sign and was significant in all but one year. This provides support for the Modigliani-Miller proposition that the required rate of return on equity should be an increasing function of leverage.

(d) *Regression Results Employing Expectations Data Alone*

In Table 5 we present additional regression results for the equations employing only expectations variables. The price-normalized earnings ratio is the dependent variable. Independent variables include expectations of short- and long-term growth, the dividend-payout ratio, and the expected instability index.[19]

We find that the long-term growth variable contributes most to an explanation of the structure of earnings multiples. The growth coefficient has a t-value over 13 in every year. The coefficient of short-term growth ($\bar{E}_{t+1}/\overline{NE}$) is also positive and highly significant. The coefficients of the payout ratio and the risk proxy are positive and negative, respectively, as ex-

[19] Fortunately, the correlations between the independent variables tended to be relatively low in all years. A sample correlation matrix (for the 1964 data) is presented below

	$\bar{g}_p$	$\bar{E}_{t+1}/\overline{NE}$	I_p	$D/\overline{NE}$
$\bar{g}_p$	1.00			
$\bar{E}_{t+1}/\overline{NE}$	.28	1.00		
I_p	− .32	.09	1.00	
$D/\overline{NE}$	− .34	− .07	− .37	1.00

TABLE 5—REGRESSION RESULTS: EMPLOYING EXPECTATIONS DATA

$$P/\overline{NE}=a_0+a_1\bar{g}_p+a_2\bar{E}_{t+1}/\overline{NE}+a_3D/\overline{NE}+a_4I_p \qquad R_{t+1}=a+b\left[\frac{P/\overline{NE}-\hat{P}/\overline{NE}}{\hat{P}/\overline{NE}}\right]$$

Year	$\hat{a}_0$	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	$\hat{a}_4$	R^2	F	$\hat{b}$	R^2	F
1961	−27.96	+2.91	+31.78	+4.57	−.58	.77	80.39	−.25	.09	9.47
		(.21)	(5.76)	(3.96)	(.70)		(4,96)	(.08)		(1;99)
		13.56	5.51	1.15	−.83			−3.08		
1962	+3.42	+1.61	+6.88	+3.21	−2.20	.79	129.14	.21	.03	3.73
		(.12)	(2.87)	(2.32)	(.41)		(4,138)	(.11)		(1;141)
		13.05	2.40	1.39	−5.44			1.93		
1963	−11.33	+2.29	+15.11	+8.11	−1.14	.80	139.82	−.20	.04	6.48
		(.14)	(2.82)	(2.70)	(.39)		(4,137)	(.08)		(1;140)
		16.30	5.35	3.01	−2.88			−2.55		
1964	−9.29	+1.87	+15.20	+7.03	−1.13	.78	120.00	−.00	.00	.00
		(.14)	(1.94)	(2.40)	(.41)		(4,134)	(.15)		(1;137)
		13.05	7.83	2.92	−2.75			−.00		
1965	−11.15	+2.42	+13.78	+4.22)	−.81	.83	162.21	—.01	.00	.01
		(.12)	(1.85)	(2.34)	(.38)		(4,136)	(.10)		(1;139)
		19.59	7.46	1.81	−2.14			−.11		

Note: Numbers in parentheses below coefficients are standard errors and numbers below parentheses are t-values. Numbers below the F-values are degrees of freedom.

pected, and are usually significant. While Tables 4 and 5 are not comparable because of different degrees of freedom, the regressions in Table 5 tend to produce slightly better fits adjusted for degrees of freedom.

It might be argued that the expectations data used as independent variables in the valuation equation may strongly reflect the $P/\overline{NE}$ ratio and, thus, we are in effect including the same variable on both sides of the valuation equation. The growth rates that we have collected are "supposedly" independent of market prices. The security analysts who have furnished the data claim that these estimates are ones that they use to calculate an "intrinsic" value of the shares, which is then compared with actual market prices in arriving at purchase or sale recommendations. In point of fact, however, the forecasted growth rates may still be strongly influenced by the market earnings multiples themselves.

Even if the anticipations data are strongly influenced by current market prices, however, this should not interfere with the basic purpose of this paper, which is to gain an understanding of the structure of share prices. The point is that the anticipations we have collected may simply be the security analysts' estimates of what the "average opinion" will continue to believe the reasonable expectations will be. The point is, of course, the familiar one about the Keynes beauty contest where the rational contestant would not pick those girls that he himself found prettiest, nor even those he deemed most likely to catch the fancy of the other contestants, but rather those that he anticipated the other contestants would believe the average opinion would consider prettiest.

Thus, if the $P/\overline{NE}$ ratio rises, and the security analyst believes that such a rise will continue to be justified by the average opinion, he may simply adjust his anticipated growth rate to a level that would justify the earnings multiple. In any case, what our valuation equation will measure is the relationship between growth rates and price-earnings multiples that security analysts believe the average opinion will continue to justify. Even in this event, our empirical results should still be useful in explaining and describing the structure of share prices at any given time.

(e) *Changes in the Valuation Relationship Over Time*

It is of some interest to examine whether the coefficients of the valuation equations are the same in each year or whether they change. This is of considerable importance to those who wish to use valuation equations in connection with assigned values of the independent variables to estimate the intrinsic worth of a security. Constancy of the relationship is also important if a firm is to seek to follow policies that will maximize the value of its shares. On the other hand, there is nothing in the theory of valuation to indicate that the equation need be constant over time.

An inspection of Table 5 indicates that the coefficients of our equation change considerably from year to year and in a manner that is consistent with the changing standards of value in vogue at the time. At the end of 1961 "growth stocks" were in high favor, and it is not surprising to find that the coefficient of the growth rate (2.91) is highest in this year. During 1962, however, there was a conspicuous change in the structure of share prices that was popularly called "the revaluation of growth stocks." This revaluation is reflected in the decline of the growth-rate coefficient for 1962 to 1.61, its lowest value for any of the five years. A similar set of observations can be made for the coefficient of the short-term growth rate ($\hat{E}_{t+1}/\overline{NE}$). On the other hand, the risk index has its most negative influence on earnings multiples in 1962, whereas the coefficient was smallest in 1961, and, while negative, it was not significantly different from zero.

In actually testing whether the coefficients of the valuation equation were the same over time, it had to be recognized that the residuals in different years might not be independent. Indeed, it is shown in the bottom panel of Table 6, which we will discuss below, that the residuals are fairly highly correlated. As a result, Arnold Zellner's seemingly unrelated regression version of Aitken's generalized least-squares model is appropriate, although it had to be modified to take account of the fact that we did not have observations for all corporations in all years.[20] Using this procedure, the hypothesis that the coefficients are the same in each year was rejected beyond the .0001 level.

IV. *Use of the Valuation Model for Security Selection*

One of the most intriguing questions concerning empirical valuation models is whether they can be used to aid investors in security selection. The empirical valuation equation shows us, at a moment in time, the average way in which variables such as growth, payout, and risk influence market price-earnings multiples. Given the values of these variables applicable to any specific company, we can compute an estimated normal price-earnings ratio based on the empirical valuation equation. It has been suggested that securities may be selected by comparing the actual market price-earnings ratio with the normal

[20] In using this procedure, the covariance matrix of the disturbances was estimated from the single-equation regression residuals. This procedure also produced more efficient estimates of the coefficients of the individual equations. Since these differed but little from those shown in Table 5, and had the same implications, we shall not present them here. The test reported is an F-test (asymptotically), which uses the vectors of independent and dependent variables, following transformation, in the usual way.

Table 6—Analysis of Lack of Forecasting Success

Description	Coefficient of Determination Residuals against 1964 Return	F-Value (and Degrees) of Freedom)
1963 Valuation equation with 1963 predictions	.04	6.48 (1; 140)
1964 Valuation equation with 1963 data. (Assume that next year's valuation relationship is known.)	.08	12.15 (1; 140)
1963 Valuation equation with realized growth rates. (Assumes perfect foresight regarding future long-term growth and next year's earnings.)	.12	18.140) (1; 14
1963 Valuation equation with 1964 predictions. (Assumes perfect foresight regarding market expectations next year.)	.24	41.75 (1; 140)

Correlations of Residuals over Years

Description	Coefficient of Determination
1962 vs. 1961	.46
1963 vs. 1962	.24
1964 vs. 1963	.13
1965 vs. 1964	.35

multiple predicted by the valuation equation. If the actual earnings multiple is greater (less) than the normal earnings multiple, we designate the security as "overpriced" ("underpriced") and recommend sale (purchase). Such a procedure was employed by Whitbeck and Kisor, who claimed that an underpriced group of securities selected by the above procedure consistently outperformed an overpriced group during the early 1960's.

Of course, even on a priori grounds, it is possible to think of many reasons why such a procedure would prove fruitless. For example, if high P/E (high growth rate) stocks tended to be overpriced during one particular period, the estimated growth-rate coefficient will be larger (by assumption) than that which is warranted. However, the recommended procedure will not indicate that high P/E stocks are overpriced because normal market-determined earnings multiples for these securities will themselves be higher than is warranted. Nevertheless, in view of the positive results reported by Whitbeck and Kisor, it would seem desirable to attempt to replicate their experiment with our data.

The results of some of our experiments are shown in the right-hand columns of Table 5. We measured the degree of over- or underpricing as the ratio of the residual from the prediction equation to the predicted earnings multiple, i.e., $[(P/\overline{NE}-\hat{P}/\overline{NE})/(\hat{P}/\overline{NE})]$. A percentage measure was chosen in view of the considerable variance in actual earnings multiples. If the model is useful in measuring underpricing, then underpriced securities, according to this criterion, ought to "outperform" overpriced issues over some subsequent period. We picked one year as the appropriate horizon and measured subsequent returns, in the normal manner, as

$$R_{t+1} = \frac{P_{t+1} - P_t + D_{t+1}}{P_t} \qquad (5)$$

If the empirical valuation model is successful in selecting securities for purchase, the percentage residual (degree of overvaluation) from the valuation equation ought to be negatively related to these subsequent returns. As the table indicates, in only three of the five years for which this experiment was performed was the relationship negative, and the degree of association was extremely low. In the other two years, there was either a positive or zero relationship. Supplementary tests conducted by industry and other groupings produced similar results. It should also be noted that the residuals from the equations employing historical data and from equations combining historical and expectational data were no more successful in predicting subsequent performance. Moreover, these results were unaltered when the subsequent returns were measured over alternative time periods such as one quarter ahead or two or more years ahead.

In Table 6 some statistics are presented which may be helpful in interpreting the reason for our predictive failures. We note that using the 1963 valuation equation as an example, the percentage degree of under- or overpricing is not highly correlated with subsequent returns. The coefficient of determination is only .04. It is possible, however, to isolate four reasons for our lack of forecasting success.

1) The first reason is that the valuation relationship changes over time. We might be unable to select truly underpriced securities because by the next year (the end of the horizon period) the norms of valuation have been significantly altered. Thus, what was cheap on the basis of 1963's relationship may no longer represent good value on the basis of the 1964 relationship. To test how important this factor might be, we performed the following experiment: We assumed that investors knew at the end of 1963 exactly what the market valuation relationship would be in 1964, i.e., we assumed perfect foresight regarding next year's valuation equation. Then, on the basis of the 1964 valuation equation, we utilized the 1963 data to calculate warranted $P/\overline{NE}$ multiples, which could then be compared with actual multiples to determine whether each security was appropriately priced. Correlating the percentage residuals with subsequent returns, we found that the coefficient of determination doubled, 8 percent of the variance in subsequent returns was explained.

2) A second reason for lack of success might be the quality of the expectations data employed. As was indicated in our 1968 article several of the growth-rate forecasts used in the present study were in fact shown to be rather poor predictors of realized earnings growth. To determine how much better off we would be with more accurate forecasts, we assumed perfect foresight regarding the future long-term growth rate of the company and regarding the next year's anticipated earnings. Thus, the 1963 empirical valuation equation was used to determine normal value, but in place of the variable $\tilde{E}_{64}/\overline{NE}_{63}$ we substituted the variable $E_{\text{actual }64}/\overline{NE}_{63}$, and in place of $\bar{g}_p$ we substituted the realized long-term growth rate through the end of 1966. Using these realized data to determine warranted price-earnings multiples, the percentage residuals therefrom were correlated with future returns. As expected, an even greater improvement in forecasting future returns was found. The R^2 rises to .12.

3) As a further experiment, perfect foresight was assumed not regarding the

actual rate of growth of earnings but rather regarding what the market expectations of growth would be next year. Calculating the degree of overpricing as before, we find a much greater improvement in prediction of future returns, 24 percent of the variability of future returns is explained, compared with 4 percent in the original experiment. We conclude that if one wants to explain returns over a one-year horizon it is far more important to know what the market will think the growth rate of earnings will be next year rather than to know the realized long-term growth rate. Of course this observation brings us back to Keynes' newspaper contest again. What matters is not one's personal criteria of beauty but what the average opinion will expect the average opinion to think is beautiful at the close of the contest.

4) A final source of error is that the valuation model does not capture all the significant determinants of value for each individual company. Despite our success in accounting for approximately 80 percent of the variance in market price-earnings multiples, there are likely to be special features applicable to many individual companies that cannot be captured quantitatively. For example, it turned out that the stock of Reynolds Tobacco always appeared to be underpriced. The reason for this is, of course, not difficult to conjecture. There is a risk of government sanctions against the tobacco industry, which weighs heavily in the minds of investors, but which is not related to the instability measure of Reynolds' earnings we have employed.

To indicate how important this problem of omitted variables might be, the residuals from our valuation equations from year to year were correlated. If certain factors specific to individual companies are consistently missing, the residuals from the valuation equations can be expected to be positively correlated over time. As the bottom half of Table 6 indicates, the residuals are significantly correlated over time. Thus, despite our success with expectations data in estimating a valuation equation which has far more explanatory ability than those based on historic information, it is clear that certain systematic valuation factors are still missing from the analysis.[21] Consequently, it cannot be said that all deviations of actual from predicted price-earnings ratios are simply manifestations of temporary over- or underpricing.

V. *Concluding Comments*

We have demonstrated that it is possible to explain, for several successive years, a large percentage of the variability in market price-earnings ratios with the variables included in this study and the specification suggested by the very simple model in Section I. The analysis was not successful, however, in isolating underpriced securities that might be expected to have above-average future returns. Needless to say, there are many additional factors that should be considered in a full valuation study. While it does not seem likely that this further work will provide direct answers to the problem of security selection, it may well shed further light on the logic of market valuations.

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[21] It would be possible, of course, to incorporate some sort of firm-effect variable to capture the systematic portion of the under- or overpricing. Such a procedure was attempted. As a firm effect we utilized the difference between the last year's actual $P/\overline{NE}$ multiple and and $\hat{P}/\overline{NE}$, the predicted earnings multiple. As might be expected, this procedure served to improve the goodness of fit substantially, but it did not affect the magnitude of the other regression coefficients. Unfortunately, however, it was not successful in improving the forecasts of future returns.

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