friedman & friedman PA
ATTORNEYS AT LAW

November 3, 2017

Carlotta S. Stauffer, Commission Clerk
Office of Commission Clerk
Florida Public Service Commission
2540 Shumard Oak Blvd.
Tallahassee, FL 32399-0850

RE: Docket No. 20170006-WS; Water and Wastewater Instrustry Annual Reestablishment of Authorized Range of Return on Common Equity for Water and Wastewater Utilities Pursuant to Section 367.081(4)(f), F. S.
Our File No. 30057.87

Dear Ms. Stauffer:

Enclosed are some corrected pages to the comments of Utilities, Inc. of Florida in the above-referenced Docket for the upcoming workshop which were filed October 31, 2017 (Document No. 09322-2017).

Should you or Staff have any questions, please do not hesitate to give me a call.

Very truly yours,

/s/ Martin S. Friedman

MARTIN S. FRIEDMAN
For the Firm

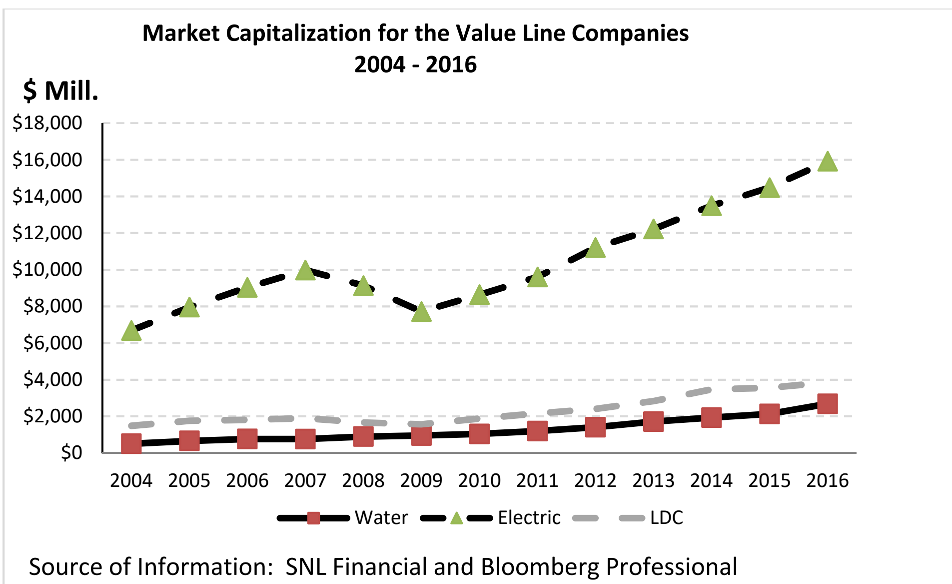
MSF/

cc: John Hoy (via e-mail)
Patrick Flynn (via e-mail)
Jared Deason (via e-mails)
Pauline Ahern (via e-mail)
Andrew Maurey (via email)
Mark Cicchetti (via email)

of electric and natural gas utilities. The market capitalization of electric utilities grew dramatically from just approximately $6.5B in 2004 to nearly $16.0B in 2016, while natural gas utilities grew much more dramatically from approximately $1.5B in 2004 to just nearly $4.0 in 2016. Since relative size is an indication of the relative investment risk between companies or groups of companies as recognized by the FPSC with inclusion of a "Small-Utility Premium" in its leverage formula, the significantly smaller size of water and wastewater utilities on average exacerbates their investment risk.

Later in these comments, size as a factor of risk will be discussed in more depth, as specifically related to the FPSC Natural Gas Index and a group of publicly traded water and wastewater utilities, as well as UIF and by inference the other small water and wastewater utilities to whom the leverage formula applies.

Chart 10



Therefore, I suggest that the FPSC consider including a Water and Wastewater Utilities Index in its annual Formula estimation, along with or replacing the Natural Gas Index. Since there are so few publicly traded water and wastewater utilities, I suggest that the eight publicly traded water utilities for which *Value Line* publishes a Rating and Report in its Standard Edition be used as the Water and Wastewater Utilities Index.

Table 4

	Market Capitalization (1) ($ Millions)	Times Greater than the Company
Utilities Inc. of Florida		
Based upon the Natural Gas Index	$96.251	
Based upon the Water & Wastewater Index	$134.608	
Natural Gas Index	$3,834.458	39.8X
Water & Wastewater Index	$3,339.931	22.8X

(1) From page 1 of Schedule UIF-3 (Corrected).

As shown above, UIF's estimated market capitalization of $96.251million based upon the Natural Gas Index is lower than the average market capitalization of that Index, $3.834 billion, or 39.8 times greater than UIF. Also, UIF's estimated market capitalization of $134.608 million based upon the Water and Wastewater Index is also lower than the average market capitalization of that index $3.339 billion, or 22.8 times greater than UIF.

Consequently, UIF has greater relative business risk because, all else being equal, size has a bearing on risk. Since Investors demand a higher return to compensate for assuming greater risk, UIF's greater relative business risk must be reflected in the cost of common equity derived from the market data of the less business risky Natural Gas and Water and Wastewater Indices.

An indication of the magnitude of an adjustment for the greater relative business risk due to smaller relative size is based upon the size premiums for the decile portfolios of New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and NASDAQ listed companies for

Derivation of Investment Risk Adjustment Based upon
Ibbotson Associates' Size Premia for the Decile Portfolios of the NYSE/AMEX/NASDAQ

Line No.		[1] Market Capitalization (1) (millions)	(times larger)	[2] Applicable Decile of the NYSE/AMEX/ NASDAQ (2)	[3] Applicable Size Premium (3)	[4] Spread from Applicable Size Premium (4)		
1.	Utilities Inc. of Florida							
	Based upon the Natural Gas Index	$ 96.251		10	5.59%			
	Based upon the Water and Wastewater Index	$ 134.608		10	5.59%			
2.	Natural Gas Index	$ 3,834.458	39.8 x	4 - 5	1.25%	4.34%	$ 349.651	2.89
3	Water and Wastewater Index	$ 3,065.625	22.8 x	5	1.51%	4.08%	$ 607.827	2.38%

	Decile	Number of Companies	Recent Total Market Capitalization (millions)	Recent Average Market Capitalization (millions)	Size Premium (Return in Excess of CAPM)		
Largest	1	191	$15,290,475.30	$80,054.84	-0.35%	1 - 2	$47,554.100
	2	200	$3,010,671.02	$15,053.36	0.61%	2 - 3	$11,510.776
	3	202	$1,609,575.62	$7,968.20	0.89%	3 - 4	$6,271.093
	4	221	$1,010,851.81	$4,573.99	0.98%	4 - 5	$3,778.449
	5	227	$677,120.07	$2,982.91	1.51%	5 - 6	$2,535.929
	6	259	$541,038.00	$2,088.95	1.66%	6 - 7	$1,723.148
	7	283	$384,129.20	$1,357.35	1.72%	7 - 8	$1,090.259
	8	361	$297,164.94	$823.17	2.08%	8 - 9	$629.871
	9	487	$212,609.64	$436.57	2.68%	9 - 10	$277.071
Smallest	10	790	$92,882.17	$117.57	5.59%		
	Mid-Cap (3-5)	660	$3,297,547.49	$4,996.28	1.02%		
	Low-Cap (6-8)	903	$1,222,332.14	$1,353.63	1.75%		
	Micro-Cap (9-10)	1,227.0000	$305,491.81	$248.97	3.67%		

*From 2017 Stocks, Bonds, Bills, and Inflation (SBBI) Yearbook

Notes:
(1) From page 2 of this Schedule.
(2) Gleaned from Column (D) on the bottom of this page. The appropriate decile (Column (A)) corresponds to the
(3) Corresponding risk premium to the decile is provided on Column (E) on the bottom of this page.
(4) Line No. 1 Column 3 – Line No. 2 Column 3. For example, the 4.% in Column 4, Line No. 2 is derived as follows 4.34% = 5.59% - 1.25%.

Market Capitalization of the
Natural Gas Index and the Water and Wastewater Index

Company	[1] Total Common Equity (millions)		[2] Market Capitalization (2) (millions)		[3] Market-to-Book Ratio (3)	
Utilities Inc. of Florida	$ 47.00	(1)				
Natural Gas Index			$ 96.251	(4)	204.8	% (5)
Water and Wastewater Index			$ 134.608	(4)	286.4	% (5)
Natural Gas Index						
Atmos Energy Corporation	$ 3,463.00		$ 8,374.60		241.8	%
Northwest Natural Gas Company	$ 850.50		$ 1,704.30		200.4	
WGL Holdings	$ 1,404.00		$ 4,217.20		300.4	
Southwest Gas Holdings	$ 1,661.00		$ 1,523.29		91.7	
Spire Inc.	$ 1,768.00		$ 3,352.90		189.6	
Average	$ 1,829.30		$ 3,834.46		204.8	%
Water and Wastewater Index						
Aqua America Inc	$ 1,850.07		$ 5,700.00		308.1	%
American Water Works Company	$ 5,218.00		$ 12,900.00		247.2	
American States Water	$ 494.30		$ 1,600.00		323.7	
California Water Service Group	$ 659.47		$ 1,700.00		257.8	
Connecticut Water Service Inc	$ 236.03		$ 600.00		254.2	
Middlesex Water Company	$ 218.44		$ 600.00		274.7	
SJW Group	$ 421.65		$ 975.00		231.2	
York Water Company	$ 114.06		$ 450.00		394.5	
Average	$ 1,151.501		$ 3,065.625		286.4	%

NA= Not Available

Notes:
(1) Company provided
Column 3 / Column 1.
(2) From Data Input Tab
(3) Column 2 / Column 1.
(4) If Utilities Inc. of Florida's common stock traded at a market-to-book ratio equal to the average market-to-book ratio of the Natural Gas Distribution Index, 164.8% , its market capitalization would be $77.433 million. If Utilities Inc. of Florida's common stock traded at a market-to-book ratio equal to that of the Water and Wastewater Index, 286.4%, its market capitalization would be $134.608 million.

(5) The market-to-book ratio of Utilities Inc. of Florida is assumed to be equal to the market-to-book ratio of the Natural Gas Distribution Index and the Water and Wastewater Index, respectively.

DOCKET NO. 20170006-WS

IN RE: WATER AND WASTEWATER INDUSTRY ANNUAL REESTABLISHMENT OF AUTHORIZED RANGE OF RETURN ON COMMMON EQUITY FOR WATER AND WASTEWATER UTILITIES PURSUANT TO SECTION 367.081(4)(F). F.S.

APPENDIX B

TO THE

COMMENTS

ON

FLORIDA LEVERAGE FORMULA

TO ESTABLISH THE ANNUAL AUTHORIZED

RANGE OF RETURNS FOR WATER & WASTERWATER UTILITIES

OF

PAULINE M. AHERN, CRRA

EXECUTIVE DIRECTOR

SCOTTMADDEN, INC.

ON BEHALF OF

UTILITIES, INC. OF FLORIDA

scottmadden

MANAGEMENT CONSULTANTS

2017 INFRASTRUCTURE REPORT CARD

OVERVIEW

Drinking water is delivered via one million miles of pipes across the country. Many of those pipes were laid in the early to mid-20th century with a lifespan of 75 to 100 years. The quality of drinking water in the United States remains high, but legacy and emerging contaminants continue to require close attention. While water consumption is down, there are still an estimated 240,000 water main breaks per year in the United States, wasting over two trillion gallons of treated drinking water. According to the American Water Works Association, an estimated $1 trillion is necessary to maintain and expand service to meet demands over the next 25 years.

CAPACITY AND CONDITION

The United States uses 42 billion gallons of water a day to support daily life from cooking and bathing in homes to use in factories and offices across the country. Around 80% of drinking water in the U.S. comes from surface waters such as rivers, lakes, reservoirs, and oceans, with the remaining 20% from groundwater aquifers. In total, there are approximately 155,000 active public drinking water systems across the country. Most Americans – just under 300 million people – receive their drinking water from one of the nation's 51,356 community water systems. Of these, just 8,674 systems, or 5.5%, serve more than 92% of the total population, or approximately 272.6 million people. Small systems that serve the remaining 17.4% of the population frequently lack both economies of scale and financial, managerial, and technical capacity, which can lead to problems of meeting Safe Drinking Water Act standards. Drinking water is delivered via one million miles of pipes across the country. Many of those pipes were laid in the early to mid- 20th century with a lifespan of 75-100 years. With utilities averaging a pipe replacement rate of 0.5% per year, it will take an estimated 200 years to replace the system – nearly double the useful life of the pipes.

Because America's drinking water infrastructure provides a critical service, significant new investment and increased efficiencies are needed as filtration plants, pipes, and pumps age past their useful life. Every day, nearly six billion gallons of treated drinking water are lost due to leaking pipes, with an estimated 240,000 water main breaks occurring each year. It is estimated that leaky, aging pipes are wasting 14 to 18% of each day's treated water; the amount of clean drinking water lost every day could support 15 million households.

To address deteriorating water infrastructure, asset management provides utility managers and decision-makers with critical information on capital infrastructure assets and timing of investments. Some key steps for asset management include making an inventory of critical assets; evaluating their condition and performance; developing plans to maintain, repair, and replace assets; and funding these activities.

FUNDING

While drinking water infrastructure is funded primarily through a rate-based system, the investment has been inadequate for decades and will continue to be underfunded without significant changes as the revenue generated will fall short as needs grow. According to the American Water Works Association, upgrading existing water systems and to meeting the drinking water infrastructure needs of a growing population will require at least $1 trillion.

The majority of funding for drinking water infrastructure comes from revenue generated by rate payers. In the nation's largest 50 cities, the rate users pay varies greatly; the lowest average monthly water bill is $14.74 in Memphis, while Seattle residents pay the most at $61.43. This large gap exemplifies the varied approaches to rate structure, as well as the contrast of need and investment across the country. While higher rates that reflect the true cost of service are important, public assistance programs should be considered for low income populations. Between 2009 and 2014, state and local governments decreased capital spending for both drinking water and wastewater by 22%; at the same time, federal capital spending did not change significantly.

The federal government offers financial support to local governments and utilities in the form of loans through the Drinking Water State Revolving Fund, which provides low-interest loans to state and local water infrastructure projects. The Environmental Protection Agency (EPA) provides an allotment of funding for each state, and each state provides a 20% match. Since the program's inception, $32.5 billion of low-interest loans have been allocated. However, with needs far surpassing the program's budget, it is unable to meet all investment needs or fund every deserving project.

In 2014, Congress authorized a new mechanism to fund primarily large water infrastructure projects over $20 million through the Water Infrastructure Finance and Innovation Act (WIFIA). In 2016 Congress appropriated $17 million in funds for the program. It is estimated that using WIFIA's full financial leveraging ability that a single dollar injected into the program can create $50 dollars for project lending. Under current appropriations, EPA estimates that current budget authority may provide more than $1 billion in credit assistance and may finance over $2 billion in water infrastructure investment.

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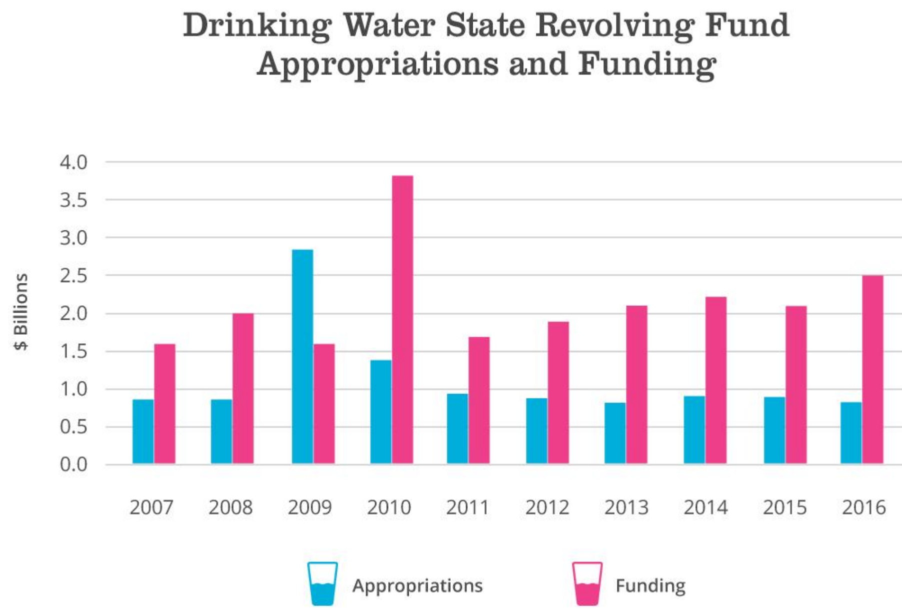

FUTURE NEED

Municipal drinking water consumption in the United States has declined by 5% this decade, marking the first time in nearly 40 years that water use at home has decreased. Total freshwater withdrawals this decade continue to decline in almost every sector including agriculture, industrial, domestic, and thermoelectric. This is primarily due to increased efficiencies and the reduction in withdrawals for retired coal-fired power plants.

Drinking water needed for public supply in the United States has been relatively flat since 1985 even as the population has increased by approximately 70 million people over the same period. Water conservation efforts, including through water efficient fixtures, have had a significant impact in reducing per capita water usage. Importantly, while per capita demand has fallen, population trends have significantly challenged how cities manage water. For example, the Government Accountability Office estimates that 99 of 674 midsized cities in the U.S. are shrinking. This poses significant challenges to utility managers; fewer rate payers and a declining tax base make it difficult to raise funds for capital infrastructure plans. To respond, utilities must raise rates, often in cities where jobs and pay have not kept pace with the economy, putting a burden on those who can least afford rate increases. Conversely, in areas of the country that are growing, such as the West and Southwest, water managers must respond to increased overall demand.

PUBLIC SAFETY

Drinking water quality in the United Sates remains the safest in the world. The EPA sets legal limits for over 90 contaminants in drinking water. The Safe Drinking Water Act (SDWA) allows states to set and enforce their own drinking water standards as long as the standards meet or exceed EPA's minimum national standards. Smaller systems that serve under 10,000 people report that a lack of resources and personnel can limit the frequency of testing, monitoring, maintenance, and technical capability in their systems. With sufficient funding and proper oversight, these risks can be mitigated and water quality can remain safe.

RESILIENCE AND INNOVATION

America's drinking water infrastructure doesn't stop at pipe, reservoir, pump station, and treatment plant upgrades; many threats to drinking water infrastructure can be attributed to the sources of drinking water, such as polluted water bodies, depleted aquifers, and inadequate storage. As watersheds continue to be impacted by shifting migration patterns, land use changes, consumption trends, and extreme weather, water infrastructure upgrades will be required to meet new demands. With proper planning, education, and conservation utilities are making strides to ensure demand is met for decades to come. Water conservation and improvements in water-use efficiency appear to have gained a general acceptance among water utilities as a sensible practice of water management.

According to the American Water Works Association, a majority of utilities –74%– have a formal conservation program, and 86% consider conserved water as one of their water supply alternatives. Additionally, many communities that have separate drinking water and wastewater departments are beginning to work together or even consolidate, creating "one water" utilities that manage water more holistically.

RECOMMENDATIONS TO RAISE THE GRADE

- Reinvigorate the State Revolving Loan Fund (SRF) program under the Safe Drinking Water Act through permanent reauthorization and tripling the amount of annual appropriations.
- Fully fund the Water Infrastructure Finance and Innovation Act (WIFIA) at its authorized level.
- Preserve tax exempt municipal bond financing. Low-cost access to capital helps keep lending for drinking water upgrades strong and accessible for communities large and small.
- Establish a federal Water Infrastructure Trust Fund to finance the national shortfall in funding of infrastructure systems under the Clean Water Act.
- Eliminate the state cap on private activity bonds for water infrastructure projects to bring an estimated $6 to $7 billion annually in new private financing.
- Encourage utilities to take regional approaches for water delivery to take advantage of economies of scale.
- Increase federal support and funding for green infrastructure, watershed permitting, and other programs that promote the concept of "one water" to protect source watersheds.
- Encourage utilities to conduct revenue forecasting models to determine the necessary rate revenues over a period of time and then institute rates that reflect the true cost of supplying clean, reliable drinking water.
- Encourage utilities to undertake asset management programs.
- Increase federal and local support for vocational training in the drinking water sector as engineers, operators, and maintenance staff begin to retire in large numbers.

- Support and advance conservation ballot measures that protect source water through dedicated funding to land and water protection.
- Utility managers must remain diligent to ensure science-based decisions control operations and facility function. While lead and other contaminants post significant health concerns when ignored, with proper funding safe and clean drinking water can be ensured.

DEFINITIONS

Non-community Water System is a public water system that is not a community water system and that regularly serves at least 25 of the same people over six months/year. These may include systems that provide water to schools, day care centers, government/military installations, manufacturers, hospitals or nursing homes, office buildings, and other facilities.

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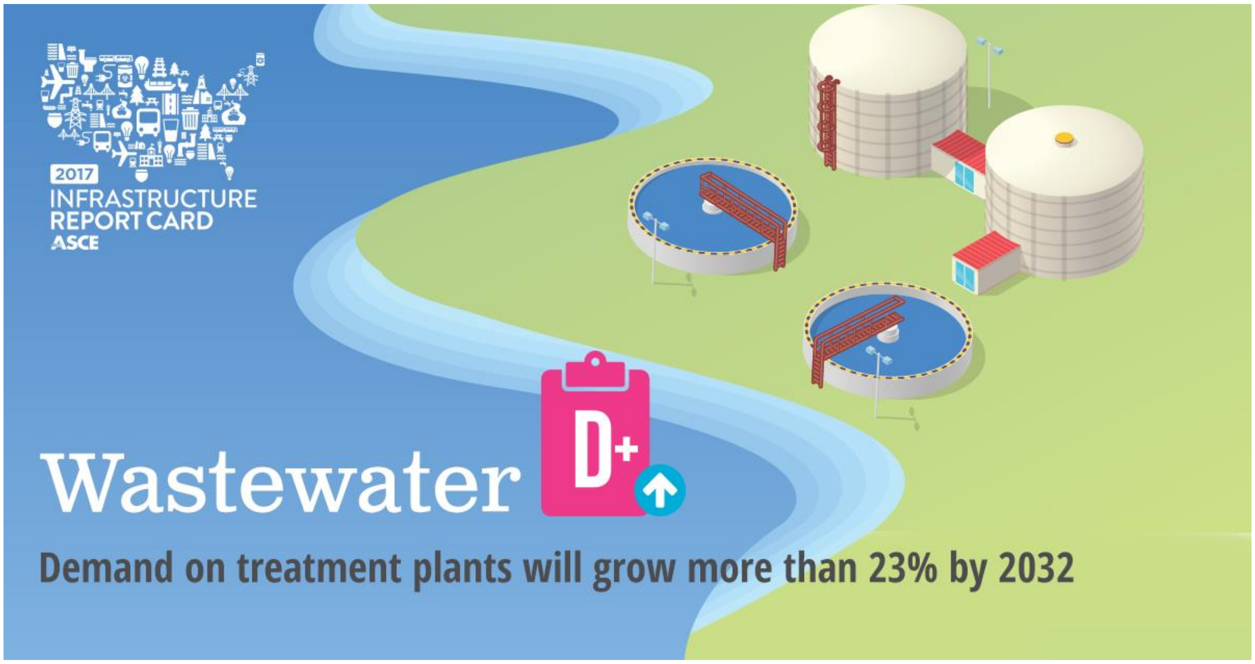


OVERVIEW

The nation's 14,748 wastewater treatment plants protect public health and the environment. Years of treatment plant upgrades and more stringent federal and state regulations have significantly reduced untreated releases and improved water quality nationwide. It is expected that more than 56 million new users will be connected to centralized treatment systems over the next two decades, and an estimated $271 billion is needed to meet current and future demands. Through new methods and technologies that turn waste into energy, the nation's 1,269 biogas plants help communities better manage waste through reuse.

CAPACITY & CONDITION

Wastewater removal and treatment is critical to protect public health. Wastewater treatment processes improve water quality by reducing toxins that cause harm to humans and pollute rivers, lakes, and oceans. Wastewater enters the treatment system from households, business, and industry through public sewer lines and, in many places across the country, stormwater drains.

Wastewater treatment is typically overseen by a community utility or public works department that ensures water quality standards are met before the treated water is discharged back into the environment. In most localities, all publicly-supplied water is treated to meet federal drinking water standards, regardless of whether it will be used for drinking. Nearly 240 million Americans – 76% of the population – rely on the nation's 14,748 treatment plants for wastewater sanitation. By 2032 it is expected that 56 million more people will connect to centralized treatment plants, rather than private septic systems – a 23% increase in demand. In the U.S., there are over 800,000 miles of public sewers and 500,000 miles of private lateral sewers connecting private property to public sewer lines. Each of these conveyance systems is susceptible to structural failure, blockages, and overflows. The U.S.

Environmental Protection Agency (EPA) estimates that at least 23,000 to 75,000 sanitary sewer overflow events occur in the United States each year.

As new users are connected to centralized treatment, older conveyance and treatment systems must manage increasing flow or new treatment facilities must be constructed. It is estimated 532 new systems will need to be constructed by 2032 to meet future treatment needs.

STORMWATER

Stormwater – runoff from rain or snow melt – also requires collection and treatment infrastructure. 39 states have one or more stormwater utility and seven states have 100 or more stormwater utilities. The number of communities with stormwater utilities or fees has grown from approximately 1,400 in 2013 to 1,600 in 2016.

In approximately 772 communities in the U.S., wastewater and stormwater drain into the same treatment system. These combined sewer systems can experience capacity issues following heavy rain events, resulting in overflows containing stormwater as well as untreated human and industrial waste, toxic substances, debris, and other pollutants. Called combined sewer overflows (CSOs), these occurrences can significantly impair water quality and impact public health and wildlife. After non-point source pollution (e.g., agricultural runoff and stormwater), combined sewer overflows are a leading source of water pollution in the U.S. The problem is exacerbated when communities have large amounts of impervious surfaces – concrete sidewalks, roads, parking lots, traditional roofs – that increase the amount of runoff entering the stormwater system.

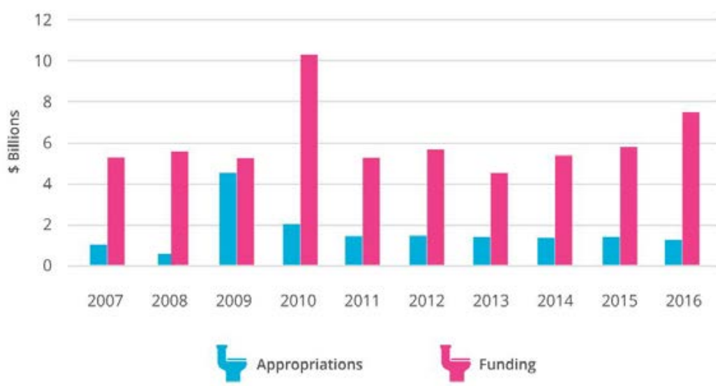
Data on stormwater infrastructure and CSOs are limited. In 2016, the EPA released a report to Congress on CSOs in the Great Lakes region. For the 184 CSO communities that discharge CSOs in the Great Lakes Basin, there were 1,482 CSO events in 2014, discharging an estimated 22 billion gallons of untreated wastewater into the Great Lakes Basin. Even these numbers were on the low side, as several communities did not report or have data available. In 2015, EPA finalized the National Pollutant Discharge Elimination System (NPDES) electronic reporting rule, requiring the filing of discharge monitoring reports; this will make more CSO data available to the public.

FUNDING & FUTURE NEED

The EPA estimates $271 billion is needed for wastewater infrastructure over the next 25 years. While the federal government provides some funding through the Clean Water State Revolving Fund (CWSRF), according to the U.S. Conference of Mayors 95% of spending on water infrastructure is made at the local level.

The federal government has provided on average $1.4 billion per year over the past five years to the 50 states and the District of Columbia through the Clean Water State Revolving Fund (CWSRF) programs. They, in turn, have provided on average a total of $5.8 billion per year in financial assistance to eligible recipients, primarily as discounted loans. In 2015 the annual assistance agreement for the CWSRF was $5.6 billion and in 2016 that number increased by $2 billion to $7.6 billion. Of the major infrastructure

Clean Water State Revolving Fund Appropriations and Funding



categories the federal government funds, water services receive less than 5%. It is estimated local governments spend $20 billion a year on capital sewer expenditures and $30 billion annually on O&M.

As cities continue to experience population growth, particularly in the south and west, new housing developments are constructed, and rural households switch from septic systems to public sewers, pressure on existing centralized systems and treatment plant infrastructure will require billions of dollars in new investment to meet federal regulatory requirements. 75% would go toward treatment plant improvements, conveyance system repairs, new conveyance systems, and recycled water distribution; 18% to CSO correction; and about 7% to stormwater management.

Cities and towns across the country report that complying with federal wastewater and stormwater regulations represents some of their costliest capital infrastructure projects. Local governments rely on a mix of funding, including sewer rates, dedicated fees such as stormwater or watershed restoration fees, local taxes, and the federal government. Approximately half of total annual expenditures in the wastewater sector go to operation and maintenance (O&M) and this share will likely rise further against capital investments. Since no federal funding may be used to pay for O&M, the full burden falls on rate payers.

Funding both capital projects and O&M is difficult because the public often does not see or appreciate the modern convenience of wastewater treatment, making it difficult to convey the need for sewer rate increases. Further, the rates charged on monthly bills are generally set by local governments and can be subject to political influence. As a result, wastewater rates often do not cover the full cost of service, particularly as needs rise due to aging systems, a growing number of users, and additional water quality measures. The majority of treatment facility expenses are supported by rate payers, however rising utility bills can present affordability issues. In a 2014 survey of the nation's 50 largest cities, average monthly sewer bills ranged from $12.72 in Memphis to $149.35 in Atlanta.

Through the Water Infrastructure Finance and Innovation Act (WIFIA) of 2014, Congress authorized a new mechanism to primarily fund large water infrastructure projects over $20 million. In December 2016, the WIFIA program received $20 million in appropriations and began releasing funding opportunities to prospective borrowers in January 2017. EPA estimates that this appropriation will result in approximately $1 billion in loans supporting approximately $2 billion in water and wastewater infrastructure investments.

RESILIENCE & INNOVATION

Treatment plants are typically located at the bottom of watersheds or coastal and riverine areas. Given these locations, many utilities have recently undertaken studies to assess vulnerability to more extreme flooding events and sea level rise. For instance, during Superstorm Sandy in 2012, several wastewater treatment plants in New York and New Jersey were inundated with storm surge, causing hundreds of millions of gallons of untreated sewage to spill into neighboring waterways. In the years since, many of these plants and others across the U.S. have developed resilience plans and increased infrastructure fortification against floods and storm surge.

Treatment plants are also rethinking biosolid disposal through nutrient recovery programs. Biosolids are the organic materials left over following the treatment process. Traditionally biosolids were considered waste and transferred to landfills. However, when properly treated and processed biosolids become nutrient rich organic material that can be applied as fertilizer or, through the use of anaerobic digesters and centrifuges, can be pelletized and incinerated at high pressure and temperature for use as energy. According to the American Biogas Council, there are currently 1,269 water resource recovery facilities using anaerobic digesters, with about 860 using biogas as a new energy source to reduce demand and costs from traditional, grid-supplied energy sources. More than 2,440 plants have been identified as ripe for future biogas development projects, which, when combined with other biogas sources such as agriculture, could produce enough energy to power 3.5 million American homes.

Through the advent of new treatment methods such as reverse osmosis, ozone, and ultraviolent light, treated water can be processed quicker than traditional chlorine contact methods. With less processing and holding time, plants can treat more wastewater and often discharge a cleaner, purer product back into the environment.

With heavy rain events in some regions of the country, and water shortages in others, wastewater and stormwater are increasingly reused. New methods and technologies of reusing water have allowed communities to better manage precious water supplies by treating wastewater products to levels required for commercial, irrigation, and industrial uses.

RECOMMENDATIONS TO RAISE THE GRADE

- Reinvigorate the State Revolving Loan Fund (SRF) under the Clean Water Act by reauthorizing the minimum federal funding of $20 billion over five years.
- Fully fund the Water Infrastructure Finance and Innovation Act (WIFIA) at its authorized level.
- Preserve tax exempt municipal bond financing. Low-cost access to capital helps keep lending for wastewater upgrades strong and accessible for communities large and small.
- Eliminate the state cap on private activity bonds for water infrastructure projects to bring an estimated $6 billion to $7 billion annually in new private financing.
- Establish a federal Water Infrastructure Trust Fund to finance the national shortfall in funding of infrastructure systems under the Clean Water Act.
- Preserve the status of tax-exempt bonds. These bonds have funded more than $1.9 trillion in infrastructure construction in the last decade alone.
- Raise awareness of the true cost of wastewater treatment.

- Achieve Clean Water Act compliance in a way that minimizes the impact on lower-income residents and on economic competitiveness through bill payment assistance; revisiting EPA affordability guidelines; renewed or enhanced federal and state aid; and redirecting other aid sources to sewer-mandate compliance.
- Support green infrastructure, which provides co-benefits such as water and air quality improvement, aesthetic value to communities, and cost competitiveness.

DEFINITIONS

Clean Water Act State Revolving Fund (CWSRF) — Program added to the Clean Water Act by Congress in 1987 to make funds available to drinking water systems to finance infrastructure improvements.

Clean Watersheds Needs Survey (CWNS) — A survey of wastewater infrastructure investment needs over 20 years undertaken by the Environmental Protection Agency's Office of Wastewater Management in conjunction with the states every four years. The CWNS is required by the Clean Water Act.

Sanitary Sewer Overflows (SSOs) — occasional unintentional discharges of raw sewage from municipal sanitary sewers due to blockages, line breaks, sewer defects that allow storm water and groundwater to overload the system, lapses in sewer system operation and maintenance, power failures, inadequate sewer design, and vandalism. EPA estimates that there are at least 23,000 to 75,000 SSOs per year.

Green Infrastructure — A man-made or natural system to prevent stormwater runoff that allows most precipitation to be absorbed or infiltrated into the ground where it replenishes aquifers, nourishes plants, and supplies water to nearby streams during low flows.

Large Community Water Systems — systems serving more than 50,000 people

Medium Community Water Systems — systems serving 3,301 to 50,000 people

Small Community Water Systems — systems serving 3,300 or fewer people

Water Infrastructure Finance Innovations Authority (WIFIA) — If enacted by Congress, a program that would access funds from the U.S. Treasury at Treasury rates and use those funds to support loans and other credit mechanisms for projects to repair or replace aging drinking water and wastewater infrastructure. The loans would be repaid to the Authority and then to the U.S. Treasury with interest.

Combined Sewer Systems — Sewers that are designed to collect rainwater runoff, domestic sewage, and industrial wastewater in the same pipe.

Combined Sewer Overflows (CSOs) — Contain not only stormwater but also untreated human and industrial waste, toxic materials, and debris when heavy rainfall produces a volume of water that exceeds the capacity of a combined sewer.

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2017 INFRASTRUCTURE REPORT CARD

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GAME CHANGERS

While all categories of American infrastructure require modernization and improvement, civil engineers, local communities, all levels of government, and the private sector have already started to develop innovative approaches to address our nation's significant infrastructure needs.

NEW REGULATORY FINANCE

Roger A. Morin, PhD

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Any forward-looking cost of capital calculation already embodies tax effects since investors price securities on the basis of after-tax returns. Besides, a very large proportion of trading is conducted by tax-exempt financial institutions (pension funds, mutual funds, 401K, etc.) for whom tax issues are largely immaterial.

The existence of a negative risk premium is highly unlikely, as it is at serious odds with the basic tenets of finance, economics, and law. Using proper definitions for expected rates of return of equity and debt, the preponderance of the evidence indicates that the negative risk premium does not exist. Several risk premium studies cited in this chapter have found positive risk premiums well in excess of 5% over the last decade. Risk premiums do narrow during unusually turbulent and volatile interest rate environments, but then return to normal levels. They are most unlikely to ever become negative.

4.7 Risk Premium Determinants

Fundamentally, the primary determinant of expected returns is risk. To wit, the various paradigms of financial theory, including the Capital Asset Pricing Model and the Arbitrage Pricing Model covered in subsequent chapters, posit fundamental relationships between return and risk. There are also secondary influences on the relative magnitude of the risk premium, however, including the level of interest rates, default risk, and taxes.

Interest Rates

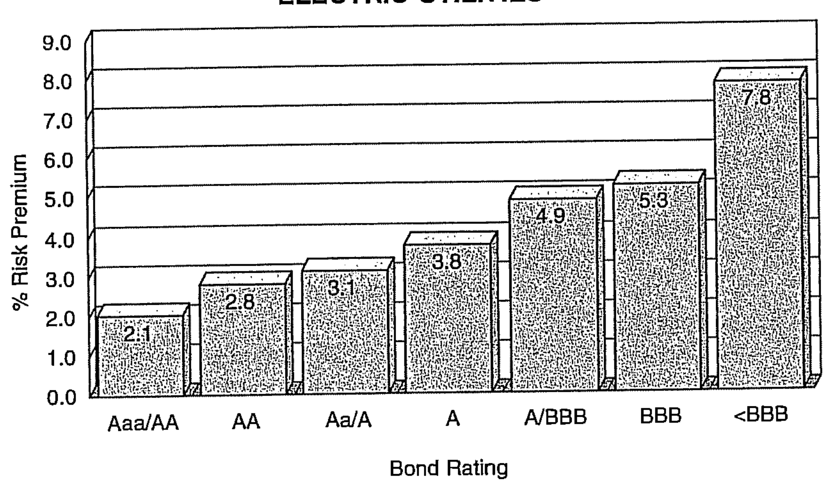
Published studies by Brigham, Shome, and Vinson (1985), Harris (1986), Harris and Marston (1992, 1993), Carleton, Chambers, and Lakonishok (1983), Morin, (2005), and McShane (2005), and others demonstrate that, beginning in 1980, risk premiums varied inversely with the level of interest rates—rising when rates fell and declining when interest rates rose. The reason for this relationship is that when interest rates rise, bondholders suffer a capital loss. This is referred to as interest rate risk. Stockholders, on the other hand, are more concerned with the firm's earning power. So, if bondholders' fear of interest rate risk exceeds shareholders' fear of loss of earning power, the risk differential will narrow and hence the risk premium will shrink. This is particularly true in high inflation environments. Interest rates rise as a result of accelerating inflation, and the interest rate risk of bonds intensifies more than the earnings risk of common stocks, which are partially hedged from the ravages of inflation. This phenomenon has been termed as a "lock-in" premium. Conversely in low interest rate environments, when bondholders' interest rate fears subside and shareholders' fears of loss of earning power dominate, the risk differential will widen and hence the risk premium will increase.

Harris (1986) showed that for every 100 basis point change in government bond yields, the equity risk premium for utilities changes 51 basis points in the opposite direction, for a net change in the cost of equity of 49 basis points. For example, a 100 basis point decline in government bond yields would lead to a 51 basis point increase in the equity risk premium and therefore an overall decrease in the cost of equity of 49 basis points, a result almost identical to the estimate reported in Morin (2005). As discussed earlier, similar results were uncovered by McShane (2005), who examined the statistical relationship between DCF-derived risk premiums and interest rates using a sample of natural gas distribution utilities.

The gist of the empirical research on this subject is that the cost of equity has changed only half as much as interest rates have changed in the past. The knowledge that risk premiums vary inversely to the level of interest rates can be used to adjust historical risk premiums to better reflect current market conditions. Thus, when interest rates are unusually high (low), the appropriate current risk premium is somewhat below (above) that long-run average. The empirical research cited above provides guidance as to the magnitude of the adjustment.

Risk premiums also tend to fluctuate with changes in investor risk aversion. Such changes can be tracked by observing the yield spreads between different bond rating categories over time. Brigham, Shome, and Vinson (1985) examined the relationship between risk premium and bond rating and found, unsurprisingly, that the risk premiums are higher for lower rated firms than for higher rated firms. Figure 4-5 shows the results graphically.

FIGURE 4-5
RISK PREMIUMS VS BOND RATINGS
ELECTRIC UTILITIES



Source: Brigham, Shome, and Vinson (1985).

Chapter 6
Alternative Asset Pricing Models

6.1 Empirical Validity of the CAPM

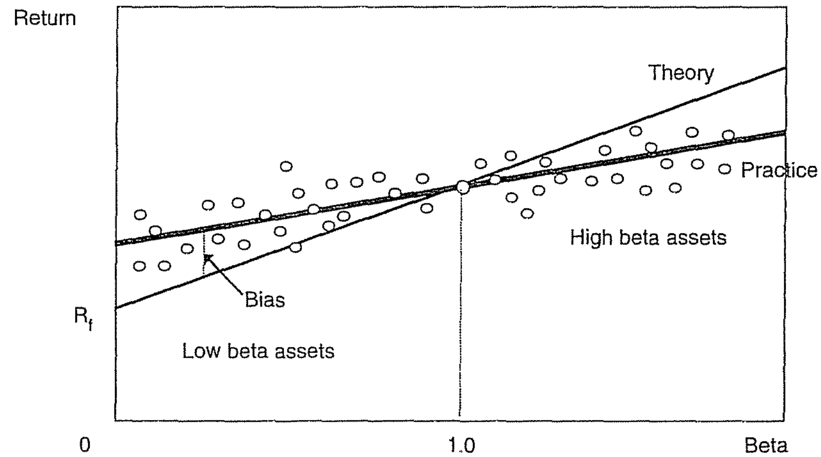
The last chapter showed that the practical difficulties of implementing the CAPM approach are surmountable. Conceptual and empirical problems remain, however.

At the conceptual level, the CAPM has been submitted to criticisms by academicians and practitioners. Contrary to the core assumption of the CAPM, investors may choose not to diversify, and bear company-specific risk if abnormal returns are expected. A substantial percentage of individual investors are indeed inadequately diversified. Short selling is somewhat restricted, in violation of CAPM assumptions. Factors other than market risk (beta) may also influence investor behavior, such as taxation, firm size, and restrictions on borrowing.

At the empirical level, there have been countless tests of the CAPM to determine to what extent security returns and betas are related in the manner predicted by the CAPM. The results of the tests support the idea that beta is related to security returns, that the risk-return tradeoff is positive, and that the relationship is linear. The contradictory finding is that the risk-return tradeoff is not as steeply sloped as predicted by the CAPM. With few exceptions, the empirical studies agree that the implied intercept term exceeds the risk-free rate and the slope term is less than predicted by the CAPM. That is, low-beta securities earn returns somewhat higher than the CAPM would predict, and high-beta securities earn less than predicted. This is shown pictorially in Figure 6-1. A CAPM-based estimate of cost of capital underestimates the return required from low-beta securities and overstates the return required from high-beta securities, based on the empirical evidence. Brealey, Myers, and Allen (2006), among many others,[1] provide recent empirical evidence very similar to the relationship depicted in Figure 6-1. This is one of the most

[1] For a summary of the empirical evidence on the CAPM, see Jensen (1972) and Ross (1978). The major empirical tests of the CAPM were published by Friend and Blume (1975), Black, Jensen, and Scholes (1972), Miller and Scholes (1972), Blume and Friend (1973), Blume and Husic (1973), Fama and Macbeth (1972), Basu (1977), Reinganum (1981B), Litzenberger and Ramaswamy (1979), Banz (1981), Gibbons (1982), Stambaugh (1982), Shanken (1985), Black (1993), and Brealey, Myers, and Allen (2006). Evidence in the Canadian context is available in Morin (1980, 1981).

FIGURE 6-1
PREDICTED VS. OBSERVED CAPM RETURN ESTIMATES



well-known results in finance. This result is particularly pertinent for public utilities whose betas are typically less than 1.00. Based on the evidence, as shown in Figure 6-1, a CAPM-based estimate of the cost of capital underestimates the return required from such securities.

The empirical evidence also demonstrates that the SML is highly unstable over short periods and differs significantly from the long-run relationship. This evidence underscores the potential for error in cost of capital estimates that apply the CAPM using historical data over short time periods. The evidence[2] also shows that the addition of specific company risk, as measured by standard deviation, adds explanatory power to the risk-return relationship.

In short, the currently available empirical evidence indicates that the simple version of the CAPM does not provide a perfectly accurate description of the process determining security returns. Explanations for this shortcoming include some or all of the following:

1. The CAPM excludes other important variables that are important in determining security returns, such as size, skewness, and taxes.
2. The market index used in the tests excludes important classes of securities, such as bonds, mortgages, and business investments. There is a further argument that the CAPM can never be really tested and that such a test is infeasible. This is because the market index proxy used

[2] See Friend, Westerfield, and Granito (1978) and Morin (1980).

in empirical tests of the CAPM is inadequate; since a true comprehensive market index is unavailable, such tests will be biased in the direction shown by the actual empirical results.[3] Moreover, the CAPM is a forward-looking expectational model and in order to test the model it is necessary to predict investor expectations correctly. Any empirical test of the CAPM is thus a test of the joint hypothesis of the model's validity and of the function used to generate expected returns from historical returns.

3. Constraints on investor borrowing exist contrary to the assumption of the CAPM.
4. Investors may value the hedging value of assets in protecting them against shifts in later investment opportunities. See Merton (1973) and Morin (1981).

Revised CAPM models have been proposed relaxing the above constraints, each model varying in complexity, each model attempting to inject more realism into the assumptions. Ross (1978), Tallman (1989), and more recently Guo (2004) present excellent surveys of the various asset pricing theories and related empirical evidence. These enhanced CAPMs produce broadly similar expressions for the relationship between risk and return and engender an SML that is flatter than the CAPM prediction, in line with the empirical evidence. Section 6.2 focuses on the more tractable extensions of the CAPM that possess some applicability to public utility regulation. Section 6.3 discusses the Empirical CAPM. Section 6.4 describes the Arbitrage Pricing Model, a viable alternative to the CAPM. Section 6.5 discusses the Fama-French Three-Factor Model of asset pricing. The Market-Derived Pricing Model is described in Section 6.6.

6.2 CAPM Extensions

Several attempts to enrich the CAPM's conceptual validity and to ameliorate its applicability have been advanced. One popular explanation of the CAPM's inability to explain security returns satisfactorily is that beta is insufficient and other systematic risk factors affect security returns. The implication is that the effects of these other independent variables should be quantified and used in estimating the cost of equity capital. The impact of the supplementary variables[4] can be expressed as an additive element to the standard CAPM equation as follows:

---

[3] See Roll (1977).

[4] The Arbitrage Pricing Model and the Fama-French three-factor asset pricing model, discussed in a later section, include factors other than the market that explain observed security returns.

The model is analogous to the standard CAPM, but with the return on a minimum risk portfolio that is unrelated to market returns, R_Z, replacing the risk-free rate, R_F. The model has been empirically tested by Black, Jensen, and Scholes (1972), who find a flatter than predicted SML, consistent with the model and other researchers' findings. An updated version of the Black-Jensen-Scholes study is available in Brealey, Myers, and Allen (2006) and reaches similar conclusions.

The zero-beta CAPM cannot be literally employed to estimate the cost of capital, since the zero-beta portfolio is a statistical construct difficult to replicate. Attempts to estimate the model are formally equivalent to estimating the constants, a and b, in Equation 6-2. A practical alternative is to employ the Empirical CAPM, to which we now turn.

6.3 Empirical CAPM

As discussed in the previous section, several finance scholars have developed refined and expanded versions of the standard CAPM by relaxing the constraints imposed on the CAPM, such as dividend yield, size, and skewness effects. These enhanced CAPMs typically produce a risk-return relationship that is flatter than the CAPM prediction in keeping with the actual observed risk-return relationship. The ECAPM makes use of these empirical findings. The ECAPM estimates the cost of capital with the equation:

$$K = R_F + \acute{\alpha} + \beta \times (MRP - \acute{\alpha}) \qquad (6\text{-}5)$$

where $\acute{\alpha}$ is the "alpha" of the risk-return line, a constant, and the other symbols are defined as before. All the potential vagaries of the CAPM are telescoped into the constant $\acute{\alpha}$, which must be estimated econometrically from market data. Table 6-2 summarizes[10] the empirical evidence on the magnitude of alpha.[11]

[10] The technique is formally applied by Litzenberger, Ramaswamy, and Sosin (1980) to public utilities in order to rectify the CAPM's basic shortcomings. Not only do they summarize the criticisms of the CAPM insofar as they affect public utilities, but they also describe the econometric intricacies involved and the methods of circumventing the statistical problems. Essentially, the average monthly returns over a lengthy time period on a large cross-section of securities grouped into portfolios are related to their corresponding betas by statistical regression techniques; that is, Equation 6-5 is estimated from market data. The utility's beta value is substituted into the equation to produce the cost of equity figure. Their own results demonstrate how the standard CAPM underestimates the cost of equity capital of public utilities because of utilities' high dividend yield and return skewness.

[11] Adapted from Vilbert (2004).

TABLE 6-2
EMPIRICAL EVIDENCE ON THE ALPHA FACTOR

Author	Range of alpha
Fischer (1993)	−3.6% to 3.6%
Fischer, Jensen and Scholes (1972)	−9.61% to 12.24%
Fama and McBeth (1972)	4.08% to 9.36%
Fama and French (1992)	10.08% to 13.56%
Litzenberger and Ramaswamy (1979)	5.32% to 8.17%
Litzenberger, Ramaswamy and Sosin (1980)	1.63% to 5.04%
Pettengill, Sundaram and Mathur (1995)	4.6%
Morin (1989)	2.0%

For an alpha in the range of 1%–2% and for reasonable values of the market risk premium and the risk-free rate, Equation 6-5 reduces to the following more pragmatic form:

$$K = R_F + 0.25\,(R_M - R_F) + 0.75\,\beta(R_M - R_F) \qquad (6\text{-}6)$$

Over reasonable values of the risk-free rate and the market risk premium, Equation 6-6 produces results that are indistinguishable from the ECAPM of Equation 6-5.[12]

An alpha range of 1%–2% is somewhat lower than that estimated empirically. The use of a lower value for alpha leads to a lower estimate of the cost of capital for low-beta stocks such as regulated utilities. This is because the use of a long-term risk-free rate rather than a short-term risk-free rate already incorporates some of the desired effect of using the ECAPM. That is, the

[12] Typical of the empirical evidence on the validity of the CAPM is a study by Morin (1989) who found that the relationship between the expected return on a security and beta over the period 1926–1984 was given by:

$$\text{Return} = 0.0829 + 0.0520\,\beta$$

Given that the risk-free rate over the estimation period was approximately 6% and that the market risk premium was 8% during the period of study, the intercept of the observed relationship between return and beta exceeds the risk-free rate by about 2%, or 1/4 of 8%, and that the slope of the relationship is close to 3/4 of 8%. Therefore, the empirical evidence suggests that the expected return on a security is related to its risk by the following approximation:

$$K = R_F + x(R_M - R_F) + (1 - x)\beta(R_M - R_F)$$

where x is a fraction to be determined empirically. The value of x that best explains the observed relationship Return = 0.0829 + 0.0520 β is between 0.25 and 0.30. If x = 0.25, the equation becomes:

$$K = R_F + 0.25(R_M - R_F) + 0.75\beta(R_M - R_F)$$

long-term risk-free rate version of the CAPM has a higher intercept and a flatter slope than the short-term risk-free version which has been tested. Thus, it is reasonable to apply a conservative alpha adjustment. Moreover, the lowering of the tax burden on capital gains and dividend income enacted in 2002 may have decreased the required return for taxable investors, steepening the slope of the ECAPM risk-return trade-off and bring it closer to the CAPM predicted returns.[13]

To illustrate the application of the ECAPM, assume a risk-free rate of 5%, a market risk premium of 7%, and a beta of 0.80. The Empirical CAPM equation (6-6) above yields a cost of equity estimate of 11.0% as follows:

$$K = 5\% + 0.25\,(12\% - 5\%) + 0.75 \times 0.80\,(12\% - 5\%)$$
$$= 5.0\% + 1.8\% + 4.2\%$$
$$= 11.0\%$$

As an alternative to specifying alpha, see Example 6-1.

Some have argued that the use of the ECAPM is inconsistent with the use of adjusted betas, such as those supplied by Value Line and Bloomberg. This is because the reason for using the ECAPM is to allow for the tendency of betas to regress toward the mean value of 1.00 over time, and, since Value Line betas are already adjusted for such trend, an ECAPM analysis results in double-counting. This argument is erroneous. Fundamentally, the ECAPM is not an adjustment, increase or decrease, in beta. This is obvious from the fact that the expected return on high beta securities is actually lower than that produced by the CAPM estimate. The ECAPM is a formal recognition that the observed risk-return tradeoff is flatter than predicted by the CAPM based on myriad empirical evidence. The ECAPM and the use of adjusted betas comprised two separate features of asset pricing. Even if a company's beta is estimated accurately, the CAPM still understates the return for low-beta stocks. Even if the ECAPM is used, the return for low-beta securities is understated if the betas are understated. Referring back to Figure 6-1, the ECAPM is a return (vertical axis) adjustment and not a beta (horizontal axis) adjustment. Both adjustments are necessary. Moreover, recall from Chapter 3 that the use of adjusted betas compensates for interest rate sensitivity of utility stocks not captured by unadjusted betas.

[13] The lowering of the tax burden on capital gains and dividend income has no impact as far as non-taxable institutional investors (pension funds, 401K, and mutual funds) are concerned, and such investors engage in very large amounts of trading on security markets. It is quite plausible that taxable retail investors are relatively inactive traders and that large non-taxable investors have a substantial influence on capital markets.

Portfolio Beta Coefficients

A portfolio consisting of low-beta securities will itself have a low beta, since the beta of any set of securities is a weighted average of the individual securities' betas:

$$b_p = \sum_{i=1}^{n} w_i b_i. \tag{6-5}$$

Here b_p is the beta of the portfolio, which reflects how volatile the portfolio is in relation to the market index; w_i is the fraction of the portfolio invested in the *i*th stock; and b_i is the beta coefficient of the *i*th stock.

If an investor holds a $100,000 portfolio consisting of $10,000 invested in each of 10 stocks, and if each stock has a beta of 0.8, then the portfolio will have $b_p = 0.8$. Thus, the portfolio is less risky than the market, and it should experience relatively narrow price swings and have small rate of return fluctuations.

Now suppose one of the existing stocks is sold and replaced by a stock with $b_i = 2.0$. This action will increase the riskiness of the portfolio from $b_{p1} = 0.8$ to $b_{p2} = 0.92$:

$$b_{p2} = \sum_{i=1}^{n} w_i b_i = 0.9(0.8) + 0.1(2.0) = 0.92.$$

Had a stock with $b_i = 0.2$ been added, the portfolio beta would have declined from 0.8 to 0.74. Adding this stock would, therefore, reduce the riskiness of the portfolio.

The Relationship between Risk and Rates of Return

In the preceding section, we saw that under the CAPM framework, beta is the appropriate measure of a stock's relevant risk. Now we must specify the relationship between risk and return—if beta rises by some specific amount, by how much must the stock's expected return increase to compensate for the increase in risk? To begin, let us define the following terms:

$\hat{k}_i$ = expected rate of return on the *i*th stock.

k_i = required rate of return on the *i*th stock. If $\hat{k}_i$ is less than k_i, then you would not purchase this stock, or you would sell it if you owned it.

R_F = riskless rate of return, generally measured by the rate of return on U.S. Treasury securities.

b_i = beta coefficient of the *i*th stock.

k_M = required rate of return on an average ($b = 1.0$) stock. k_M is also the required rate of return on a portfolio consisting of all stocks, or the market portfolio.

$RP_M = (k_M - R_F)$ = market risk premium. It is the additional return over the riskless rate required to compensate investors for assuming an "average" amount of risk.

$RP_i = b_i(k_M - R_F)$ = risk premium on the ith stock. The stock's risk premium is less than, equal to, or greater than the premium on an average stock, depending on whether its beta is less than, equal to, or greater than 1.0. If $b_i = 1.0$, then $RP_i = RP_M$.

The market risk premium, RP_M, depends on the degree of aversion that investors, in the aggregate, have to risk.[11] Let us assume that at the current time Treasury bonds yield $R_F = 8\%$, and an average share of stock has a required return of $k_M = 12\%$. Therefore, the market risk premium is 4 percent:

$$RP_M = k_M - R_F = 12\% - 8\% = 4\%.$$

It follows that, if one stock were twice as risky as some other, its risk premium would be twice as high, and, conversely, if its risk were only half as high, its risk premium would be half as high. Further, we can measure a stock's relative riskiness by its beta coefficient. Therefore, if we know the market risk premium, RP_M, and the stock's beta coefficient, b_i, we can find its risk premium as the product $b_i(RP_M)$. For example, if $b_i = 0.5$ and $RP_M = 4\%$, then RP_i is 2 percent:

$$\text{Risk premium for Stock i} = RP_i = b_i(RP_M) = 0.5(4\%) = 2.0\%. \quad (6\text{-}6)$$

To summarize, given estimates of R_F, k_M, and b_i, we can find the required rate of return on Stock i:

$$\begin{aligned} k_i &= R_F + b_i(k_M - R_F) = R_F + b_i(RP_M) \\ &= 8\% + 0.5(12\% - 8\%) = 8\% + 0.5(4\%) = 10\%. \end{aligned} \quad (6\text{-}7)$$

If some other stock, j, were more risky than Stock i and had $b_j = 2.0$, then its required rate of return would be 16 percent:

$$k_j = 8\% + 2.0(4\%) = 16\%.$$

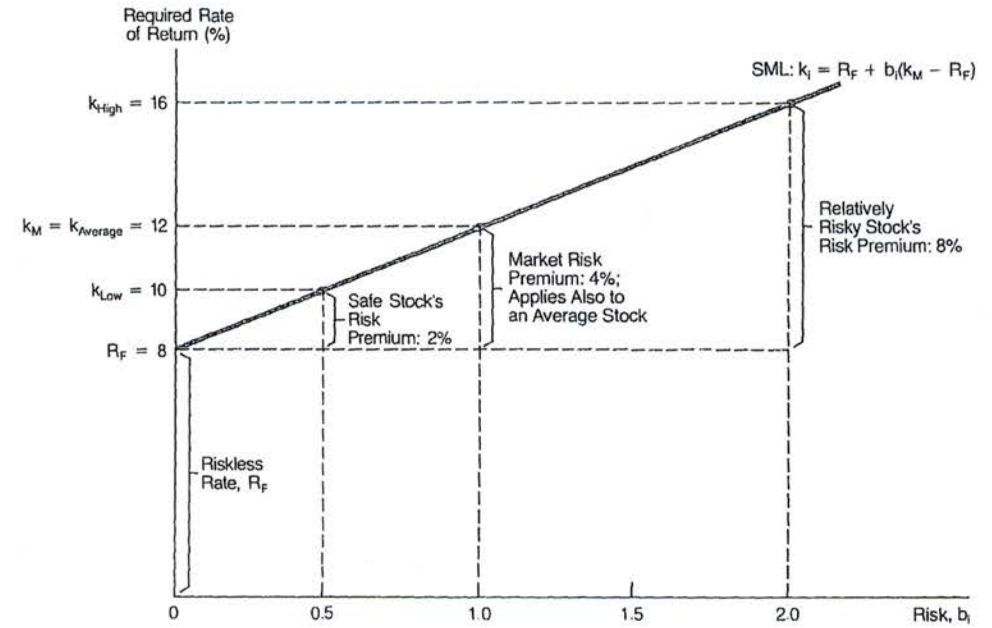
An average stock, with $b = 1.0$, would have a required return of 12 percent, the same as the market return:

$$k_{Average} = 8\% + 1.0(4\%) = 12\% = k_M.$$

Equation 6-7 is often expressed as a graph called the *Security Market Line (SML)*; Figure 6-9 shows the SML when $R_F = 8\%$ and $k_M = 12\%$. Note the following points:

[11] This concept is discussed in some detail in Appendix 6B. It should be noted that the risk premium of an average stock, $k_M - R_F$, cannot be measured with great precision because it is impossible to obtain precise values for k_M. However, empirical studies suggest that, where long-term U.S. Treasury bonds are used to measure R_F and where k_M is the expected return on the S&P 400 Industrial Stocks, the market risk premium varies somewhat from year to year, and it has generally ranged from 3 to 6 percent during the last 20 years.

Figure 6-9
The Security Market Line (SML)



1. Required rates of return are shown on the vertical axis, while risk as measured by beta is shown on the horizontal axis.

2. Riskless securities have $b_i = 0$; therefore, R_F appears as the vertical axis intercept.

3. The slope of the SML reflects the degree of risk aversion in the economy—the greater the average investor's aversion to risk, then (1) the steeper is the slope of the line, (2) the greater is the risk premium for any risky asset, and (3) the higher is the required rate of return on risky assets.[12] These points are discussed further in a later section.

[12]Students sometimes confuse beta with the slope of the SML. This is a mistake. As we saw earlier in connection with Figure 6-8, and as is developed further in Appendix 6A, beta does represent the slope of a line, but *not* the Security Market Line. This confusion arises partly because the SML equation is generally written, in this book and throughout the finance literature, as $k_i = R_F + b_i(k_M - R_F)$, and in this form b_i looks like the slope coefficient and $(k_M - R_F)$ the variable. It would perhaps be less confusing if the second term were written $(k_M - R_F)b_i$, but this is not generally done.

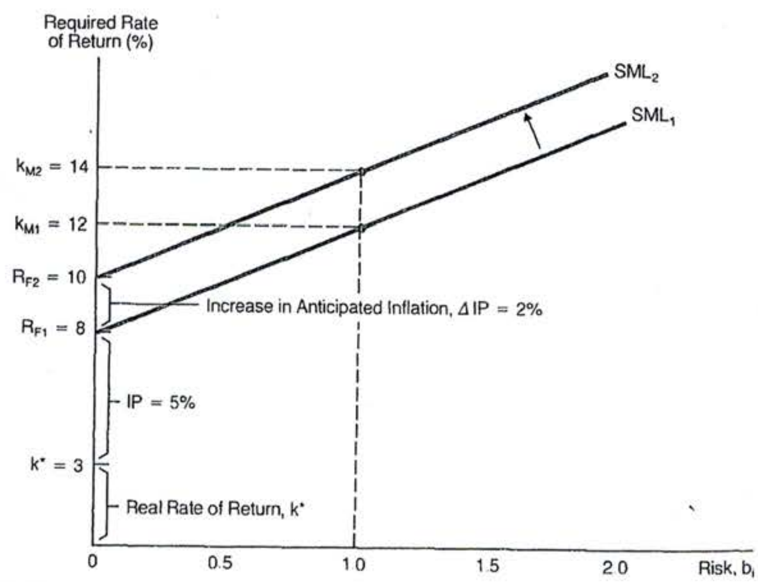
4. The values we worked out for stocks with $b_i = 0.5$, $b_i = 1.0$, and $b_i = 2.0$ agree with the values shown on the graph for k_{Low}, $k_{Average}$, and k_{High}.

The Security Market Line, and a company's position on the line, change over time as interest rates, investors' risk aversion, and individual companies' betas change. Such changes are discussed in the following sections.

The Impact of Inflation

As we saw in Chapter 3, interest amounts to "rent" on borrowed money, or the "price" of money. Thus, R_F is the price of money to a riskless borrower. The existing market risk-free rate is called the *nominal rate*, and it consists of two elements: (1) a *real*, or *inflation-free, rate of return*, k*, and (2) an *inflation premium*, IP, equal to the anticipated rate of inflation. Thus, $R_F = k^* + IP$. The real rate on risk-free government securities has, historically, ranged from 2 to 4 percent, with a mean of about 3 percent. Thus, if no inflation were expected, risk-free government securities would tend to yield about 3 percent. However, as the expected rate of inflation increases, a premium must be added to the real rate of return to compensate investors for the loss of purchasing

Figure 6-10
Shift in the SML Caused by an Increase in Inflation



expectations relative to history, historical growth rates become suspect as a measure of investor expectations.

Yet another issue associated with historical growth is that reliance on history to measure investor expectations renders the replication of that growth a self-fulfilling prophecy. Reliance on forecast growth rates avoids this inherent circularity.

The major point of all this is that it is perilous to apply historical growth when a utility is in a transition between growth paths. When payout ratios, equity return, and market-to-book ratios are changing, reliance on historical growth is hazardous. Such transitions can occur under variable inflation environments, and under fundamental structural shifts, such as deregulation.

Given the choice of variables, length of historical period, and the choice of statistical methodologies, the number of permutations and combinations of historical growth rates is such that other methods and proxies for expected growth must be explored. Historical growth rates constitute a useful starting point and provide useful information as long as the necessary conditions and assumptions outlined in this section are not dramatically violated. Although historical information provides a primary foundation for expectations, investors use additional information to supplement past growth rates. Extrapolating past history alone without consideration of historical trends and anticipated economic events would assume either that past rates will persist over time or that investors' expectations are based entirely on history.

9.4 Growth Estimates: Analysts' Forecasts

Since investor growth expectations are the quantities desired in the DCF model, the use of forecast growth published by investment services merits serious consideration. The growth rates assumed by investors can be determined by a study of the analyses of future earnings and projected long-run growth rates made by the investment community. The anticipated long-run growth rates actually used by institutional investors to determine the desirability of investing in different securities influence investors' growth anticipations.

Typically, growth forecasts are in the form of earnings per share over periods ranging from one to 5 years, and are supported by extensive financial analysis.[10]

[10] Analysts do not generally disseminate their methods of forecasting and do not generally recommend the purchase or sale of a security based on any single growth variable or growth estimating technique. A professional financial analyst is reluctant to reveal the premises and methods of his professional judgment and recommendations. Moreover, analysts' buy/sell recommendations result from complex judgments that cannot be reduced to a single variable or to simple mechanistic equations or models. Several methods and algorithms, involving both quantitative and qualitative factors, are likely to be used in arriving at a final growth forecast, including historical indicators.

The average growth rate estimate from all the analysts that follow the company measures the consensus expectation of the investment community for that company. In most cases, it is necessary to use earnings forecasts rather than dividend forecasts due to the extreme scarcity of dividend forecasts compared to the widespread availability of earnings forecasts. Given the paucity and variability of dividend forecasts, using the latter would produce unreliable DCF results. In any event, the use of the DCF model prospectively assumes constant growth in both earnings and dividends. Moreover, as discussed below, there is an abundance of empirical research that shows the validity and superiority of earnings forecasts relative to historical estimates when estimating the cost of capital.

The uniformity of growth projections is a test of whether they are typical of the market as a whole. If, for example, 10 out of 15 analysts forecast growth in the 7%–9% range, the probability is high that their analysis reflects a degree of consensus in the market as a whole. As a side note, the lack of uniformity in growth projections is a reasonable indicator of higher risk. Chapter 3 alluded to divergence of opinion amongst analysts as a valid risk indicator.

Because of the dominance of institutional investors and their influence on individual investors, analysts' forecasts of long-run growth rates provide a sound basis for estimating required returns. Financial analysts exert a strong influence on the expectations of many investors who do not possess the resources to make their own forecasts, that is, they are a cause of g. The accuracy of these forecasts in the sense of whether they turn out to be correct is not at issue here, as long as they reflect widely held expectations. As long as the forecasts are typical and/or influential in that they are consistent with current stock price levels, they are relevant. The use of analysts' forecasts in the DCF model is sometimes denounced on the grounds that it is difficult to forecast earnings and dividends for only one year, let alone for longer time periods. This objection is unfounded, however, because it is present investor expectations that are being priced; it is the consensus forecast that is embedded in price and therefore in required return, and not the future as it will turn out to be.

Empirical Literature on Earnings Forecasts

Published studies in the academic literature demonstrate that growth forecasts made by security analysts represent an appropriate source of DCF growth rates, are reasonable indicators of investor expectations and are more accurate than forecasts based on historical growth. These studies show that investors rely on analysts' forecasts to a greater extent than on historic data only.

Academic research confirms the superiority of analysts' earnings forecasts over univariate time-series forecasts that rely on history. This latter category includes many *ad hoc* forecasts from statistical models, ranging from the naive methods of simple averages, moving averages, etc. to the sophisticated time-series techniques such as the Box-Jenkins modeling techniques. The literature suggests that analysts' earnings forecasts incorporate all the public information available to the analysts and the public at the time the forecasts are released. This finding implies that analysts have already factored historical growth trends into their forecast growth rates, making reliance on historical growth rates somewhat redundant and, at worst, potentially double counting growth rates which are irrelevant to future expectations. Furthermore, these forecasts are statistically more accurate than forecasts based solely on historical earnings, dividends, book value equity, and the like.

Summary of Empirical Research

Important papers include Brown and Rozeff (1978), Cragg and Malkiel (1968, 1982), Harris (1986), Vander Weide and Carleton (1988), Lys and Sohn (1990), and Easterwood and Nutt (1999).

The study by Brown and Rozeff (1978) shows that analysts, as proxied by Value Line analysts, make better forecasts than could be obtained using only historical data, because analysts have available not only past data but also a knowledge of such crucial factors as rate case decisions, construction programs, new products, cost data, and so on. Brown and Rozeff test the accuracy of analysts' forecasts versus forecasts based on past data only, and conclude that their evidence of superior analyses means that analysts' forecasts should be used in studies of cost of capital. Their evidence supports the hypothesis that Value Line analysts consistently make better predictions than historical time-series models.

Using the IBES consensus earnings forecasts as proxies for investor expectation, Harris (1986) estimates the cost of equity using expected rather than historical earnings growth rates. In his review of the literature on financial analysts' forecasts, Harris concludes that a growing body of knowledge shows that analysts' earnings forecasts are indeed reflected in stock prices. Elton, Gruber, and Gultekin (1981) show that stock prices react more to changes in analysts' forecasts of earnings than they do to changes in earnings themselves, suggesting the usefulness of analysts' forecasts as surrogates for market expectations. In an extensive National Bureau of Economic Research study using analysts' earnings forecasts, Cragg and Malkiel (1982) present detailed empirical evidence that the average analyst's expectation is more similar to expectations being reflected in the marketplace than historical growth rates, and that it is the best possible source of DCF growth rates. The authors show that historical growth rates do not contain any information that is not already impounded in analysts' growth forecasts. They conclude that the expectations formed by Wall Street professionals get quickly and thoroughly impounded into the prices of securities and that the company valuations made by analysts are reflected in security prices.

Vander Weide and Carleton (1988) update the Cragg and Malkiel study and find overwhelming evidence that the consensus analysts' forecasts of future growth is superior to historically oriented growth measures in predicting the firm's stock price. Their results also are consistent with the hypothesis that investors use analysts' forecasts, rather than historically oriented growth calculations, in making stock buy-and-sell decisions. A study by Timme and Eiseman (1989) produced similar results.

Using virtually all publicly available analyst earnings forecasts for a large sample of companies (over 23,000 individual forecasts by 100 analyst firms), Lys and Sohn (1990) show that stock returns respond to individual analyst earnings forecasts, even when they are closely preceded by earnings forecasts made by other analysts or by corporate accounting disclosures. Using actual and IBES data from 1982–1995, Easterwood and Nutt (1999) regress the analysts' forecast errors against either historical earnings changes or analysts' forecasting errors in the prior years. Results show that analysts tend to underreact to negative earnings information, but overreact to positive earnings information.

The more recent studies provide evidence that analysts make biased forecasts and misinterpret the impact of new information.[11] For example, several studies in the early 1990s suggest that analysts either systematically underreact or overreact to new information. Easterwood and Nutt (1999) discriminate between these different reactions and reported that analysts underreact to negative information, but overreact to positive information. The recent studies do not necessarily contradict the earlier literature. The earlier research focused on whether analysts' earnings forecasts are better at forecasting future earnings than historical averages, whereas the recent literature investigates whether the analysts' earnings forecasts are unbiased estimates of future earnings. It is possible that even if the analysts' forecasts are biased, they are still closer to future earnings than the historical averages, although this hypothesis has not been tested in the recent studies. One way to assess the concern that analysts' forecasts may be biased upward is to incorporate into the analysis the growth forecasts of independent research firms, such as Value Line, in addition to the analyst consensus forecast. Unlike investment banking firms and stock brokerage firms, independent research firms such as Value Line have no incentive to distort earnings growth estimates in order to bolster interest in common stocks.

[11] Other relevant papers corroborating the superiority of analysts' forecasts as predictors of future returns versus historical growth rates include: Fried and Givoly (1982), Moyer, Chatfield and Kelley (1985), and Gordon, Gordon and Gould (1989).

Some argue that analysts tend to forecast earnings growth rates that exceed those actually achieved and that this optimism biases the DCF results upward. The magnitude of the optimism bias for large rate-regulated companies in stable segments of an industry is likely to be very small. Empirically, the severity of the optimism problem is unclear for regulated utilities, if a problem exists at all. It is interesting to note that Value Line forecasts for utility companies made by independent analysts with no incentive for over- or understating growth forecasts are not materially different from those published by analysts in security firms with incentives not based on forecast accuracy, and may in fact be more robust. If the optimism problem exists at all, it can be circumvented by relying on multiple-stage DCF models that substitute long-term economic growth for analysts' growth forecasts in the second and/ or third stages of the model.

Empirical studies have also been conducted showing that investors who rely primarily on data obtained from several large reputable investment research houses and security dealers obtain better results than those who do not.[12] Thus, both empirical research and common sense indicate that investors rely primarily on analysts' growth rate forecasts rather than on historical growth rates alone.

Ideally, one could decide which analysts make the most reliable forecasts and then confine the analysis to those forecasts. This would be impractical since reliable data on past forecasts are generally not available. Moreover, analysts with poor track records are replaced by more competent analysts, so that a poor forecasting record by a particular firm is not necessarily indicative of poor future forecasts. In any event, analysts working for large brokerage firms typically have a following, and investors who heed a particular analyst's recommendations do exert an influence on the market. So, an average of all the available forecasts from large reputable investment houses is likely to produce the best DCF growth rate.

Growth rate forecasts are available online from several sources. For example, Value Line Investment Analyzer, IBES (Institutional Brokers' Estimate System), Zacks Investment Research, Reuters, First Call, Yahoo Finance, and Multex Web sites provide analysts' earnings forecasts on a regular basis by reporting on the results of periodic (usually monthly) surveys of the earnings growth forecasts of a large number of investment advisors, brokerage houses, and other firms that engage in fundamental research on U.S. corporations. These firms include most large institutional investors, such as pension funds, banks, and insurance companies. Representative of industry practices, the Zacks Investment Research Web site is a central location whereby investors

[12] Examples of these studies include Stanley, Lewellen and Schlarbaum (1981) and Touche Ross Co. (1982).

are able to research the different analyst estimates for any given stock without necessarily searching for each individual analyst. Zacks gathers and compiles the different estimates made by stock analysts on the future earnings for the majority of U.S. publicly traded companies. Estimates of earnings per share for the upcoming 2 fiscal years, and a projected 5-year growth rate in such earnings per share are available at monthly intervals. The forecast 5-year growth rates are normalized in order to remove short-term distortions. Forecasts are updated when analysts formally change their stated predictions.

Exclusive reliance on a single analyst's growth forecast runs the risk of being unrepresentative of investors' consensus forecast. One would expect that averages of analysts' growth forecasts, such as those contained in IBES or Zacks, are more reliable estimates of investors' consensus expectations likely to be impounded in stock prices.[13] Averages of analysts' growth forecasts rather than a single analyst's growth forecasts are more reliable estimates of investors' consensus expectations.

One problem with the use of published analysts' forecasts is that some forecasts cover only the next one or two years. If these are abnormal years, they may not be indicative of longer-run average growth expectations. Another problem is that forecasts may not be available in sufficient quantities or may not be available at all for certain utilities, for example water utilities, in which case alternate methods of growth estimation must be employed.

Some financial economists are uncomfortable with the assumption that the DCF growth rates are perpetual growth rates, and argue that above average growth can be expected to prevail for a fixed number of years and then the growth rate will settle down to a steady-state, long-run level, consistent with that of the economy. The converse also can be true whereby below-average growth can be expected to prevail for a fixed number of years and then the growth rate will resume a higher steady-state, long-run level. Extended DCF models are available to accommodate such assumptions, and were discussed in Chapter 8.

Earnings versus Dividend Forecasts

Casual inspection of the Zacks Investment Research, First Call Thompson, and Multex Web sites reveals that earnings per share forecasts dominate the information provided. There are few, if any, dividend growth forecasts. Only Value Line provides comprehensive long-term dividend growth forecasts. The wide availability of earnings forecasts is not surprising. There is an abundance of evidence attesting to the importance of earnings in assessing investors'

[13] The earnings growth rates published by Zacks, First Call, Reuters, Value Line, and IBES contain significant overlap since all rely on virtually the same population of institutional analysts who provide such forecasts.

expectations. The sheer volume of earnings forecasts available from the investment community relative to the scarcity of dividend forecasts attests to their importance. The fact that these investment information providers focus on growth in earnings rather than growth in dividends indicates that the investment community regards earnings growth as a superior indicator of future long-term growth. Surveys of analytical techniques actually used by analysts reveal the dominance of earnings and conclude that earnings are considered far more important than dividends. Finally, Value Line's principal investment rating assigned to individual stocks, Timeliness Rank, is based primarily on earnings, accounting for 65% of the ranking.

Historical Growth Rates Versus Analysts' Forecasts

Obviously, historical growth rates as well as analysts' forecasts provide relevant information to the investor with regard to growth expectations. Each proxy for expected growth brings information to the judgment process from a different light. Neither proxy is without blemish; each has advantages and shortcomings. Historical growth rates are available and easily verifiable, but may no longer be applicable if structural shifts have occurred. Analysts' growth forecasts may be more relevant since they encompass both history and current changes, but are nevertheless imperfect proxies.

9.5 Growth Estimates: Sustainable Growth Method

The third method of estimating the growth component in the DCF model, alternately referred to as the ''sustainable growth'' or ''retention ratio'' method, can be used by investment analysts to predict future growth in earnings and dividends. In this method, the fraction of earnings expected to be retained by the company, b, is multiplied by the expected return on book equity, r, to produce the growth forecast. That is,

$$g = b \times r$$

The conceptual premise of the method, enunciated in Chapter 8, Section 8.4, is that future growth in dividends for existing equity can only occur if a portion of the overall return to investors is reinvested into the firm instead of being distributed as dividends.

For example, if a company earns 12% on equity, and pays all the earnings out in dividends, the retention factor, b, is zero and earnings per share will not grow for the simple reason that there are no increments to the asset base (rate base). Conversely, if the company retains all its earnings and pays no dividends, it would grow at an annual rate of 12%. Or again, if the company earns 12% on equity and pays out 60% of the earnings in dividends, the retention factor is 40%, and earnings growth will be 40% × 12% = 4.8% per year.

In implementing the method, both 'b' and 'r' should be the rate that the market expects to prevail in the future. If no explicit forecast of 'b' is available, it is reasonable to assume that the utility's future retention ratio will, on average, remain unchanged from its present level. Or, it can be estimated by taking a weighted average of past retention ratios as a proxy for the future on the grounds that utilities' target retention ratios are usually, although not always, stable.[14]

Both historical and forecast values of 'r' can be used to estimate g, although forecast values are superior. The use of historical realized book returns on equity rather than the expected return on equity is questionable since reliance on achieved results involves circular reasoning. Realized returns are the results of the regulatory process itself, and are also subject to tests of fairness and reasonableness. As a gauge of the expected return on book equity, either direct published analysts' forecasts of the long-run expected return on equity, or authorized rates of return in recent regulatory cases can be used as a guide. As a floor estimate, it seems reasonable for investors to expect allowed equity returns by state regulatory commissions to be in excess of the current cost of debt to the utility in question.

Another way of obtaining the expected 'r' is to examine its fundamental determinants. Since earnings per share, E, can be stated as dividends per share, D, divided by the payout ratio (1 − b), the earnings per share capitalized by investors can be inferred by dividing the current dividend by an expected payout ratio. Provided that a utility company follows a fairly stable dividend policy, the possibility of error is less when estimating the payout than when estimating the expected return on equity or the expected growth rate. Using this approach, and denoting book value per share by B, the expected return on equity is:

$$r = E/B = (D/(1 - b)) / B \qquad (9\text{-}9)$$

Estimates of the expected payout ratio can be inferred from historical 10-year average payout ratio data for utilities, assuming a stable dividend policy has been pursued. Since individual averages frequently tend to regress toward the grand mean, the historical payout ratio needs to be adjusted for this tendency, using statistical techniques for predicting future values based on this tendency of individual values to regress toward the grand mean over time.

An application of the sustainable growth method is shown in example 9-1.

[14] Statistically superior predictions of future averages are made by weighting individual past averages with the grand mean, with the variance within the individual averages and the variance across individual averages serving as weights.

EXAMPLE 9-1

Southeastern Electric's sustainable growth rate is required for upcoming rate case testimony. As a gauge of the expected return on equity, authorized rates of return in recent decisions for eastern U.S. electric utilities as reported by Value Line for 2005 and 2006 averaged 11%, with a standard deviation of 1%. In other words, the majority of utilities were authorized to earn 11%, with the allowed return on equity ranging from 10% to 12%. As a gauge of the expected retention ratio, the average 2006 payout ratio of 34 eastern electric utilities as compiled by Value Line was 60%, which indicates an average retention ratio of 40%, with a standard deviation of 5%. This was consistent with the long-run target retention ratio indicated by the management of Southeastern Electric. It is therefore reasonable to postulate that investors expect a retention ratio ranging from 35% to 45% for the company with a likely value of 40%. In Table 9-4 below, expected retention ratios of 35% to 45% and assumed returns on equity from 10% to 12% are multiplied to produce sustainable growth rates ranging from 3.8% to 5.4% with a likely value of 4.6%.

TABLE 9-4
SUSTAINABLE GROWTH METHOD ILLUSTRATION

Expected Retention Ratio (b)	Expected Return on Book Equity (r) 10%	11%	12%
35%	3.5%	3.9%	4.2%
40%	4.0%	4.4%	4.8%
45%	4.5%	5.0%	5.4%

It should be pointed out that published forecasts of the expected return on equity by analysts such as Value Line are sometimes based on end-of-period book equity rather than on average book equity. The following formula[15]

[15] The return on year-end common equity, r, is defined as $r = E/B_t$, where E is earnings per share, and B_t is the year-end book value per share. The return on average common equity, r_a, is defined as: $r_a = E/B_a$ where B_a = average book value per share. The latter is by definition: $B_a = (B_t + B_{t-1})/2$ where B_t is the year-end book equity per share and B_{t-1} is the beginning-of-year book equity per share. Dividing r by r_a and substituting:

$$\frac{r}{r_a} = \frac{E/B_t}{E/B_a} = \frac{B_a}{B_t} + \frac{B_t + B_{t-1}}{2B_t}$$

Solving for r_a, a formula for translating the return on year-end equity into the return on average equity is obtained, using reported beginning-of-the year and end-of-year common equity figures:

$$r_a = r\frac{2B_t}{B_t + B_{t-1}}$$

adjusts the reported end-of-year values so that they are based on average common equity, which is the common regulatory practice:

$$r_a = r_t \frac{2B_t}{B_t + B_{t-1}} \tag{9-10}$$

The sustainable growth method can also be extended to include external financing. From Chapter 8, the expanded growth estimate is given by:

$$g = br + sv$$

where b and r are defined as previously, s is the expected percent growth in number of shares to finance investment, and v is the profitability of the equity investment. The variable s measures the long-run expected stock financing that the utility will undertake. If the utility's investments are growing at a stable rate and if the earnings retention rate is also stable, then s will grow at a stable rate. The variable s can be estimated by taking a weighted average of past percentage increases in the number of shares. This measurement is difficult, however, owing to the sporadic and episodic nature of stock financing, and smoothing techniques must be employed. The variable v is the profitability of the equity investment and can be measured as the difference of market price and book value per share divided by the latter, as discussed in Chapter 8.

There are three problems in the practical application of the sustainable growth method. The first is that it may be even more difficult to estimate what b, r, s, and v investors have in mind than it is to estimate what g they envisage. It would appear far more economical and expeditious to use available growth forecasts and obtain g directly instead of relying on four individual forecasts of the determinants of such growth. It seems only logical that the measurement and forecasting errors inherent in using four different variables to predict growth far exceed the forecasting error inherent in a direct forecast of growth itself.

Second, there is a potential element of circularity in estimating g by a forecast of b and ROE for the utility being regulated, since ROE is determined in large part by regulation. To estimate what ROE resides in the minds of investors is equivalent to estimating the market's assessment of the outcome of regulatory hearings. Expected ROE is exactly what regulatory commissions set in determining an allowed rate of return. In other words, the method requires an estimate of return on equity before it can even be implemented. Common sense would dictate the inconsistency of a return on equity recommendation that is different than the expected ROE that the method assumes the utility will earn forever. For example, using an expected return on equity of 11% to determine the growth rate and using the growth rate to recommend a return on equity of 9% is inconsistent. It is not reasonable to assume that this regulated utility company is expected to earn 11% forever, but recommend a 9% return on equity. The only way this utility can earn 11% is that rates be set by the regulator so that the utility will in fact earn 11%. One is assuming, in effect, that the company will earn a return rate exceeding the recommended cost of equity forever, but then one is recommending that a different rate be granted by the regulator. In essence, using an ROE in the sustainable growth formula that differs from the final estimated cost of equity is asking the regulator to adopt two different returns.

The circularity problem is somewhat dampened by the self-correcting nature of the DCF model. If a high equity return is granted, the stock price will increase in response to the unanticipated favorable return allowance, lowering the dividend yield component of market return in compensation for the high g induced by the high allowed return. At the next regulatory hearing, more conservative forecasts of r would prevail. The impact on the dual components of the DCF formula, yield and growth, are at least partially offsetting.

Third, the empirical finance literature discussed earlier demonstrates that the sustainable growth method of determining growth is not as significantly correlated to measures of value, such as stock price and price/earnings ratios, as other historical growth measures or analysts' growth forecasts. Other proxies for growth, such as historical growth rates and analysts' growth forecasts, outperform retention growth estimates. See for example Timme and Eiseman (1989).

In summary, there are three proxies for the expected growth component of the DCF model: historical growth rates, analysts' forecasts, and the sustainable growth method. Criteria in choosing among the three proxies should include ease of use, ease of understanding, theoretical and mathematical correctness, and empirical validation. The latter two are crucial. The method should be logically valid and consistent, and should possess an adequate track record in predicting and explaining security value. The retention growth method is the weakest of the three proxies on both conceptual and empirical grounds. The research in this area has shown that the first two growth proxies do a better job of explaining variations in market valuation (M/B and P/E ratios) and are more highly correlated to measures of value than is the retention growth proxy.

THE PRICING OF COMMON STOCKS

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INTRODUCTION

The prices of common stocks differ among corporations due to differences among them in earnings per share, investment policies, financing policies, and business risk. Models which use related variables to explain differences in prices among stocks may be called intrinsic value models. There are at least two important uses for intrinsic value models. One is to discover the investment and financing policies which maximize the price of a company's stock. A related use is to discover the extent to which existing dividend, leverage and other policies depart from price maximizing policies. The other important use for intrinsic value models is to discover over or under-priced stocks. That may take place in either of two ways. One is to use the model to find the stocks that are mispriced on the basis of the current values of the model's independent variables. Alternatively, the model's parameters may be combined with values for the independent variables that are based on new private information that the analyst has obtained in order to discover the value of the information.

Intrinsic value models have had a very uneven history. Prior to the nineteen fifties we had simple heuristic models such as Graham and Dodd's in which price depended on some combination of earnings, dividends and book value. Then Gordon and Miller and Modigliani developed models of stock valuation which follow from plausible or theoretically interesting principles of asset valuation.[1] These models generated further theoretical work and considerable empirical work designed to implement and test them during the sixties and early seventies. However, the development of the Sharpe-Lintner-Mossin capital asset pricing model during the sixties soon captured the interest of researchers in the area of security valuation. Perhaps that is why there has been comparatively little progress over the last twenty years in theory and practice with respect to intrinsic value models of stock prices.

This paper is devoted to the use of intrinsic value models for the discovery of over or under-priced stocks. The motivation for the paper is the belief that developments over the last decade in data availability and some theoretical ideas raised here make further progress possible. The next section will review the Gordon and the MM models. The second section will critically evaluate the empirical adaptation of these models by their authors and certain other efforts at explaining the cross-section variation in price among stocks. Finally, the last section will discuss how certain data base developments and advances in theory can contribute to the advancement of practice in the area under consideration.

I. THEORETICAL MODELS

The current value of any asset is the present value of its expected future payments. In the case of a stock this expectation is the dividend for the coming period plus the end-of-period price. The expectation reduces to an infinite stream of dividends. Under the assumption that this dividend expectation can be represented with just two parameters, a current value and a growth rate, with the latter taken to be the same for every future period, I have shown that the price of a share is

$$P = D/(k-g). \qquad (1)$$

Here, P = current price per share, D = current dividend per share, k = expected or required return, and g = expected average rate of growth in the dividend.[2] Notice, the assumption that the growth rate is constant over time is consistent with the fact that the growth rate in the dividend may vary from one period to the next. We only assume that in pricing the stock future growth can be summarized with one number.

Eq. (1) can be given more economic content under the further restrictions that (1) the corporation is not expected to finance through the sale of new shares, and (2) dividend policy, capital structure, and return on investment can each be represented by one parameter. In other words, the value of each of these variables is not expected to change over time, and of course, their values can be estimated currently. Under these assumptions Eq. (1) becomes

$$P = (1-b)Y/(k-br). \qquad (2)$$

The additional variables are Y = normalized current earnings per share, b = fraction of earnings retained and invested, and r = return on equity investment.

In Eq. (2) the dividend becomes D = (1-b)Y, the growth rate becomes g = br, and dividend policy is investment policy.

The interesting economic content of Eq. (2) lies in what it says about dividend policy, which is represented with b, the fraction of earnings retained. It can be seen that as b rises P falls, on account of the fall in the dividend. On the other hand, P rises with b due to the rise in the growth rate. Whether P rises or falls on balance, and whether or not P is maximized at some retention rate depend on the relative levels of r and k and on how they change with k.

Eq. (2) is based on a number of more or less questionable assumptions. Of particular interest, academic if not practical interest, is the assumption that retained earnings is the sole source of equity funds.[3] In fact, the sale of stock is an alternative to retained earnings as a source of equity funds, but that does not render Eq. (2) useless. Let q be the sum of funds raised through retained earnings and the sale of stock expressed as a fraction of earnings. The value of q is independent of the relative amount of each source of equity funds. If stockholders looked on the sale of stock as a perfect substitute for retained earnings, we could substitute q for b in Eq. (2), and it would then tell us how the price of a share varies with the firm's equity financing rate.[4] However, we all know that taxes and transaction costs make retained earnings dominate the sale of stock as a source of funds. Hence, the assumption that retained earnings is the sole source of equity fund is not among our more questionable assumptions from a practical viewpoint. The more questionable assumptions will come up shortly.

Miller and Modigliani have shown that under the very strong assumptions of perfectly competitive capital markets, the price of a share is equal to the present value of the earnings on the existing equity plus the present value of the excess return on the expected future equity investment.[5] Under certain simplifying assumptions, we then have

$$P = \frac{Y}{k} + \frac{Yq(r-k)}{k(k-qr)} \, . \qquad (3)$$

The first term is the present value of the future earnings on the existing equity, and the second term is the present value of the excess return on all future investment. The future equity investment has an initial value of Yq, it has an excess return of r-k, it will grow at the rate qr, and it has a present value of Yq(r-k)/k(k-qr).

Notice that the same variables, Y, q, r, and k enter both the Gordon and MM models, and both models rely on the same simplifying assumptions with regard to their parameterization. Nonetheless, we end up with a fundamental difference between the two models. In the Gordon model the investor buys a dividend expectation, while in MM she buys an earnings expectation. Earnings do enter the Gordon model but only through their influence on the current value and the growth of the dividend expectation.

There are more fundamental differences between the Gordon and the MM models. In both models k, the expected or required return on a stock is equal to a risk-free interest rate plus a risk premium. The former is the same for all shares and the latter varies among shares, so that given the dividend or earnings expectations on a share, its price will vary depending on the share's risk attributes. However, MM imposed on their model the assumption that k is independent of the expected growth rate, qr, while Gordon allowed the risk of growth to make k an increasing function of qr. In addition, MM assumed that a firm does nothing to create investment opportunities, while Gordon's model has a firm's investment opportunities depend on its history.

II. EMPIRICAL MODELS

Prior to the above theoretical work Meader and Durand explained the variation in price among shares with models in which earnings, dividends and book value were the independent variables.[6] Meader's regressions were linear in the variables while Durand's were linear in their logs. Both obtained high coefficients of multiple correlation, but the regression coefficients were highly unstable from one year to the next, due no doubt to the very high correlations among the independent variables.

Turning back to the Gordon model, we see that it may be summarized with the statement that a stock's price is equal to the dividend divided by the dividend yield. Hence, the task in the econometric implementation of the model is to introduce variables that explain how the dividend yield varies among shares. Since the dividend yield is k - g, it varies with g and inversely with k. The latter as just stated is equal to a risk-free interest rate that is common to all stocks plus a risk premium that varies among stocks. The empirical adaptation of the model in Gordon was of the form

$$P = \alpha_0 D^{\alpha_1}(1+br)^{\alpha_2}(1+\sigma)^{\alpha_3}(1+h)^{\alpha_4}\ (S)^{\alpha_5}. \qquad (4)$$

The risk variables were σ, the variability in the rate of return on common, h, the leverage rate, and S the firm's size measured by its assets. Eq. (4) is linear in the logs, so that conventional econometric methods may be employed to estimate the α coefficients.

My empirical work employed similar models to Eq. (4) with P/B and P/D the dependent variables, B being the book value per share.[7] The objective with these dependent variables was to abstract from the correlation that may arise due to the variation in price with the dividend or book value among stocks. In all cases the models did an excellent job, explaining a large fraction of the variation in price among stocks for samples of food, machinery, utility and other classes of stocks in different years.[8] The econometric results are discussed in detail in the references cited.

It can be seen that Eq. (4) is a simple and direct representation of the theoretical model Eq. (2). With the constraint $\alpha_1=1$ (its actual value is close to one) price is a multiple of the dividend, the multiple increasing with the growth rate and varying inversely with the risk variables. Notice that the form of the relation in Eq. (4) makes each coefficient the elasticity of share price with the associated independent variable, so that the change in price with a variable depends upon the ratio of price to that variable. Finally, Eq. (4) provides a plausible explanation of how the dividend yield and the expected return, k=D/P+br vary among shares with growth and risk.

The most serious limitation of the Gordon model is the assumption that the dividend expectation can be represented with just two parameters, D and br. The model breaks down for corporations that are currently paying no dividend, and it can be seriously in error for a corporation that is currently paying a token dividend. In addition, financial statement data for b and r can result in a value for g that cannot be accepted as an average for the indefinite future.

The empirical adaptation of the MM model on how investors value stocks was carried out by MM[9] with the expression

$$\frac{V - \tau L}{A} = a_1 \frac{\bar{X}^{\tau} - \tau\bar{R}}{A} + a_2 \frac{\Delta A}{A} + a_3 \frac{10^7}{A} + \alpha_4 \frac{L}{A} + \alpha_5 \frac{D - \bar{D}}{A} . \qquad (5)$$

Here $V - \tau L$ = the market value of a firms's equity and debt less the value due to the tax advantage of debt,

A = book cost of total assets,

$\bar{X}^{\tau} - \tau\bar{R}$ = after tax earnings on common plus interest on debt,

$\Delta A/A$ = rate of growth in assets, L/A = ratio of debt to assets, and

$D-\bar{D}$ = excess of dividend on common over what it would have been if the firm's payout rate had been the industry average.

Notice that the variables in Eq. (5) except for the dividend variable refer not to the corporation's common stock but to the equity plus the debt.

MM's choice of variables in Eq. (5) was motivated by their special objective. It was not to explain the variation in price among common stock's but to test their theorems on capital structure and dividend policy. According to MM the value of a levered firm increases with debt by τL and it is independent of dividend policy. Hence, by subtracting τL from V in arriving at the dependent variable, it should be independent of the debt ratio as well as dividend policy. That is what they found, α_4 and α_5 not being significantly different from zero in their empirical results. This model, the empirical results and their interpretation by MM were subjected to considerable critical comment, and it will not be considered further here.[10]

A far simpler and more effective empirical adaptation of the MM theory and an important contribution in other respects was due to Malkiel and Cragg.[11] The regression equation they employed to explain the variation in price among stocks was

$$P/Y = \alpha_0 + \alpha_1 g + \alpha_2 (D/Y) + \alpha_3 \beta. \qquad (6)$$

Here, the dependent variable is the price-earnings ratio, g is the foecast rate of growth in earnings, D/Y is the dividend payout rate, and β is an index of systematic risk. Dividing both sides of MM's Eq. (3) by Y reveals more clearly the similarities between the MM and the Malkiel and Cragg models. It also reveals the difficulty of arriving at a faithful and plausible empirical adaptation of the MM model for pricing common stocks. Eq. (3) now is

$$\frac{P}{Y} = \frac{1}{k} + \frac{q}{k}\left[\frac{r-k}{k-qr}\right] \qquad (7)$$

It can be seen that α_0 in Eq. (6) is an estimate of 1/k under the unreasonable assumption that k does not vary among shares. The coefficient of g is also an estimate of 1/k under the same assumption, and g is an approximation of q(r-k)/(k-qr). In addition, D/Y is included among the independent variables on the assumption that dividend policy matters, and the presence of β among the independent variables (and other risk variables in some regressions) captures in some measure the variation in P/Y among shares due to risk. Finally, with Eq. (6) linear in the variables, the change in P/Y with each independent variable is independent of the values of the variables.

The Malkiel-Cragg model did a very good job of explaining the variation in price-earnings ratios among shares. The correlation with the payout rate as well as the growth rate was very strong, suggesting that dividend policy does influence price.

The major contribution of Malkiel and Cragg was to run their regressions with two alternative sets of data. In one case the earnings and growth variables were obtained from financial statements, with growth being the growth rate in earnings over the prior five years. In the other case, an average of the estimates or forecasts by a group of security analysts of the normalized earnings for the current year and the growth rate in earnings for the next five years were used. The regression results obtained with the data from the security analysts were much better than the results obtained with the historical financial statement data. Hence, estimates by security analysts can be an improvement on financial statement data for earnings and growth.

Malkiel and Cragg also investigated the use of their model for the discovery of over and under priced shares. They regressed the change in price over the following year on the difference between the actual and predicted price at the start of the year. Unfortunately, they only found very weak evidence in support of the hoped for relation. However, the ability of their model to discover over or under-priced stocks was improved by assuming additional information such as more recent parameter values and better estimates of growth.

III. A METHOD OF PRICING

Let us now turn to consideration of how it may be possible to make substantial advances in the use of econometric models to discover over and under priced stocks. By way of background let us review briefly how econometric models may serve that purpose. First, we establish a model that is considered a theoretically correct empirical representation of how investors price common stocks. Second, we obtain values of the variables for a sample of stocks and estimate the model's coefficients. These coefficients are then combined with the values of the independent variables for a stock that is in or out of the sample to provide the "correct" price for the stock based on the rules followed for estimating the independent variables. Third, for stocks with a difference between the actual and correct price that is large, the difference is a basis for a buy or sell decision. Finally, if the analyst has superior information which produces a different value for one or more independent variables of a particular stock, a new correct price is obtained with the coefficients on the basis of the superior values for these variables. The difference between the new correct price and the actual price is the basis for a buy or sell decision.

We have seen that earnings and growth estimates by security analysts were found by Malkiel and Cragg to be superior to data obtained from financial statements for the explanation of variation in price among common stocks. That is, better estimates are obtained for the coefficient of the various explanatory variables. Their results should be confirmed by further empirical work, but there is every reason to believe that the confirmation will be forthcoming. First, the estimates by security analysts available from sources such as IBES are far superior to the data available to Malkiel and Cragg. Secondly, the estimates by security analysts must be superior to the estimates derived solely from financial statements. For earnings we want normalized current earnings and for growth we want expected future growth. It is true that all our knowledge of the future is obtained from the past, and good estimates of Y and g can frequently be obtained from financial statement data. However, such data are available to security analysts, and they have additional information that can be incorporated in their estimates, so that an average over a number of security analysts which eliminates the bias of any one analyst should be superior to exclusive reliance on past financial statement data.

There are other more important ways in which the availability of IBES type data improves the usefulness of econometric models for the discovery of over and under-priced shares. Financial statement data are only available annually, since quarterly data has serious limitations due to seasonal and other distorting influences. With annual data the model's parameters can only be estimated annually, the annual data is not obtained at the same time for all firms, and it is out-of-date when it is obtained. By contrast with IBES or any other such service, the consensus of security analysts on such variables is available monthly. Hence, revised values for the independent variables and revised estimates of the models parameters may be obtained monthly instead of annually. We then have at any point in time a more accurate representation of how the market prices shares. Most important, with monthly data the discovery of over or under-priced shares can take place monthly instead of annually. Such data represent a critically important breakthrough in making models for pricing stocks useful to security analysts.[12]

Let us now turn to the problem of a model that provides a theoretically correct explanation of how stocks are priced. The model I recommend is

$$P/Y = \alpha_0(1+g)^{\alpha_1}(1+D/Y)^{\alpha_2}(1+L/B)^{\alpha_3}... \qquad (7)$$

with g = growth, D/Y = payout rate, L/B leverage rate and ..... signifying that one or more other risk variables may be added to the model. Eq. (7) is not as elegant as Eq. (4), but it has a good deal more intuitive appeal. It says that investors buy earnings, but what they will pay for a dollar of earnings increases with the extent to which the earnings are reflected in the dividend or in appreciation through growth. Hence, the price per dollar of earnings increases with both the growth rate and the dividend payout rate, and P/Y decreases as leverage or other sources of risk rise. Notice that we avoid having the model blow up because g, D/Y or L/B is equal to zero by using one plus each of these variables. In addition, having a model that is linear in the logs of the variables has the advantages mentioned earlier. Eq. (7) combines the best features of the Gordon and the Malkiel-Cragg models.

I am confident that Eq. (7) with values for earnings and growth based on a consensus of security analyst estimates will do an excellent job of explaining the variation in price among stocks. Whether or not the difference between the actual values of P/Y and the values predicted by the model will be useful for discovering over and under-priced stocks is open to question. The poor results obtained by Malkiel and Cragg are cause for doubt, but we now have the use of a better model and better data. Finally, there is no doubt that the model will be useful in conjunction with private estimates of earnings, growth and other independent variables. Such private estimates have been and will continue to be developed by security analysts. However, when the estimates are not combined with a sophisticated valuation model, there is no scientific basis for arriving at the impact on price of that information. Revised estimates of one or more independent variables combined with a good valuation model should be superior to the unaided use of such estimates in arriving at buy or sell recommendations.

FOOTNOTES

1 See M.J. Gordon and Eli Shapiro, "Capital Equipment Analysis: The Required Rate of Profit," Management Science (October 1956), pp. 102-110; M.J. Gordon, The Investment, Financing and Valuation of the Corporation, Homewood, IL, R.D. Irwin, 1962; F. Modigliani and M.H. Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment," American Economic Review (June 1958), pp. 261-297; M.H. Miller and F. Modigliani, "Dividend Policy, Growth, and the Valuation of Shares," Journal of Business (October 1961), pp. 411-433.

2 See M.J. Gordon, The Investment Financing ....., Ch. 4.

3 For an academic treatment of the subject, see M.J. Brennan, "A Note on Dividend Irrelevance and the Gordon Valuation Model," Journal of Finance (December 1971), pp. 1115-1122.

4 This is demonstrated in M.J. Gordon and L.I. Gould, "The Cost of Equity Capital: A Reconsideration," Journal of Finance (June 1978), pp. 849-861.

5 The assumptions common to the MM and Gordon-Gould models are no taxes, no transaction costs, and equal information. In addition, implicit in MM are the assumptions that a corporation's investment opportunities are independent of its history, and risk is independent of growth. For more on this see M.J. Gordon "Corporate Finance Under the MM Theorems," Financial Management (Summer 1989), pp. 19-28.

6 See J.W. Meader, "A Formula for Determining Basic Values Underlying Common Stock Prices," The Analyst Magazine of Finance, Commerce and Economics, Nov. 29, 1935 and June 27, 1940; David Durand, Bank Stock Prices and the Bank Capital Problem, New York: Occasional Paper 54, National Bureau of Economic Research, 1957.

7 See M.J. Gordon, The Investment, Financing ....., Chs. 11 and 12; and M.J. Gordon, The Cost of Capital to a Public Utility, East Lansing, MI, Michigan State University, 1974.

8 The various models experimented with other risk variables than those in Eq. (4). Their performance is not discussed, since the best combination and measurement of risk variables is beyond our purpose here.

9 See M.H. Miller and F. Modigliani, "Some Estimates of the Cost of Capital to the Electric Utility Industry, 1954-1957," American Economic Review (June 1966), pp. 333-391.

10 See the comments on their paper by Jean Crockett and Irwin Friend, M.J. Gordon, and A.A. Robichek, J.G. McDonald and

R.C. Higgins and their reply in the American Economic Review (December 1967), pp. 1258-1299.

11 See B.G. Malkiel and J.G. Cragg, "Expectations and the Structure of Share Prices," American Economic Review (September 1970), pp. 601-617.

12 For instance, with annual data they were compelled to assume that over or under-priced shares at a one point in time predict the change in price over the coming year, whereas the over or under-pricing may be eliminated over a shorter time period.

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GORDON MODEL

$$P = \frac{D}{K-G} = \frac{(1-B)Y}{K-BR}$$

P = SHARE PRICE

D = DIVIDEND PER SHARE

K = RETURN ON SHARE INVESTORS REQUIRE

G = EXPECTED GROWTH RATE IN DIVIDEND AND PRICE

Y = EARNINGS PER SHARE

B = FEACTION OF EARNINGS RETAINED

R = RATE OF RETURN ON INVESTMENT

MM MODEL

$$P = \frac{Y}{K} + \frac{YQ(R-K)}{K(K-QR)}$$

P = SHARE PRICE

Y = EARNINGS PER SHARE

K = RETURN ON SHARE INVESTORS REQUIRE

Q = EQUITY INVESTMENT/EARNINGS

R = RATE OF RETURN ON INVESTMENT

COMPARISON OF MM AND GORDON

EARNINGS VS DIVIDENDS

MM - INVESTOR BUYS EARNINGS

GORDON - INVESTOR BUYS DIVIDENDS

RISK AND REQUIRED RETURN

MM - THEY ARE INDEPENDENT OF GROWTH

GORDON - THEY INCREASE WITH GROWTH

RETURN ON INVESTMENT

MM - INDEPENDENT OF FIRM'S HISTORY

GORDON - DEPEND ON FIRM'S HISTORY

EMPIRICAL MODELS

GORDON

$$P = AO \bullet D^{A1} \bullet (1+G)^{A2} \bullet (1+LEV)^{A3} \bullet\bullet\bullet$$

$$LNP = LN\ AO + A1 \bullet LN\ D + A2 \bullet LN(1+G) + A3 \bullet LN\ (1+LEV) + \bullet\bullet\bullet$$

MALKIEL CRAGG

$$P/Y = A\ O + A1 \bullet G + A2 \bullet (D/Y) + A3 \bullet BETA + \bullet\bullet\bullet$$

P = PRICE D = DIVIDEND

Y = EARNINGS G = GROWTH

LEV = LEVERAGE BETA = RISK

AN INTRINSIC VALUE MODEL

$$P = AO \bullet Y^{A1} \bullet (1+G)^{A2} \bullet (1+D/Y)^{A3} \bullet (1+LEV)^{A4} \bullet BETA^{A5} \bullet\bullet\bullet$$

$$LPN = LN\ AO + A1 \bullet LN\ Y + A2 \bullet LN(1+G) + A3 \bullet LN\ (1+D/Y) + A4 \bullet LN(1+LEV) + A5 \bullet LN\ BETA + \bullet\bullet\bullet\bullet$$

BENEFITS FROM IBES TAPE

1. BETTER ESTIMATES OF MODEL'S PARAMETERS

   SECURITY ANALYST DATA FOR Y AND G

   MONTHLY REVISION OF PARAMETERS

2. DISCOVERY OF MISPRICED STOCKS ON

   BASIS OF CONSENSUS DATA MONTHLY

3. PRICE IMPLICATIONS OF PRIVATE

   INFORMATION CONTINUOUS ON BASIS

   OF CURRENT DATA

Principles of Public Utility Rates

Second Edition

by
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the literature with some commissions totally disregarding the new issue to those that apply an adjustment to the entire equity balance.

The Market to Book Ratio Issue

Introduction. One ongoing critical issue is whether the allowed rate of return should be designed to prevent the market prices of public utility stocks from rising to substantially above book value or falling to substantially below book value? A rigorous and literal application of a cost-of-capital-measure of a fair rate of return as outlined above would indicate that a commission should attempt to regulate rates so as to maintain the market value of a utility's stock on a par with its book value (or rate-base value) plus some allowance for underpricing. Yet such an attempt may be impractical or even impossible.

In the first place, commissions cannot forecast, except within wide limits, the effect their rate orders will have on the market prices of the stocks of the companies they regulate. In the second place, whatever the initial market prices may be, they are sure to change not only with the changing prospects for earnings, but with the changing outlook of an inherently volatile stock market. In short, market prices are beyond the control, though not beyond the influence, of rate regulation. Moreover, even if a commission did possess the power of control, any attempt to exercise it in the manner just suggested would result in harmful, uneconomic shifts in public utility rate levels. In addition, many utilities are regulated by more than one jurisdiction. Even if one commission were to attempt to regulate on the basis of market to book ratios, the commissions in the other jurisdictions would not be bound by its actions. Finally, even if regulators could put them in parity it may be undesirable following the theory of the second best if the comparable earnings exceed the cost of capital (see Kahn, 1970, pp. 52-53).

Two Facts. This situation is recognized even by supporters of a cost-of-capital standard of a fair rate of return, who undertake to meet the difficulty in two ways. First, the current cost of equity capital is rarely identified as a spot cost. Instead, it is taken to mean a normal or average capital-attracting rate of return characteristic of the recent market and typical of the market anticipated in the not distant future. Secondly, the estimated weighted average cost of capital resulting from the application of this normalized estimate of the current cost of equity may be characterized as a minimum allowance, subject to a reasonable upward adjustment perhaps justified on the basis of possible attrition.

It follows that the common stocks of public utilities which actually succeed in earning a fair rate of return as derived by a cost of capital approach may be expected to sell at a premium over their book values or rate-base values except in periods of a depressed stock market. The premiums may be greater than the modest allowance for underpricing associated with stock offerings sometimes granted by commissions. A question arises whether the prevalence of these premiums is persuasive evidence of a corporate earning power higher than required to give adequate assurance of the continued ability to attract needed capital on terms that do not impair the integrity of the existing capital. Conversely, when market to book ratios fall below one, the questions arise whether this is persuasive evidence that a utility is not earning its cost of capital.

Consistent with the opinion that regulation is simply powerless to set rates which insure any particular market to book ratio, the answer must be in the negative. Lacking this power, regulation should recognize the possibility of earnings liberal enough to permit market to book ratios of utilities to rise slightly above one. Some argue that these ratios should be roughly at the level of well-managed companies that actually succeed in realizing these earnings fairly continuously. For many years in the 1970s and 1980s utilities in general sold at market prices well below book. The call was for rates sufficient to produce market to book ratios of 1.1 to 1.2. Now the question of what constitutes a proper *degree* of liberality remains and has not received a convincing answer. We doubt whether a conclusive answer can ever be found under such an indefinite standard of a fair rate of return as that of a flexible rate designed to rise and fall with changes in the anticipated rates of income necessary to induce new investments of equity capital.

The Q-Ratio and Market to Book Ratio

One interpretation of the mandates of the Supreme Court, and one consistent with a present-value standard of reasonable rates rather than with an original-cost standard, is that regulated enterprises should be permitted to earn on the current values of their corporate assets, as based on replacement-cost appraisals, rates of return similar to the rates actually being earned by unregulated enterprises on the values of their assets, similarly appraised. This is a mere attempt to spell out a criterion which the Supreme Court itself has never undertaken to rid of its ambiguities.

Expectations and the Structure of Share Prices

John G. Cragg and
Burton G. Malkiel

The University of Chicago Press

Chicago and London

4 Empirical Connection of the Growth Forecasts with Share-Valuation Models

We suggested in chapter 3 that a relationship should exist between the earnings growth expectations we have collected and the market values of the corresponding shares. The present chapter reports on our empirical investigation of this relationship. This investigation may be regarded in one of two ways. Assuming that growth-rate expectations are a major input used by investors to form expected security returns, our empirical work tests the validity of the valuation models. Conversely, if we maintain the validity of the valuation models, we may be regarded as testing the hypothesis that earnings growth expectations do play a major role, along with the other specified variables, in investors' evaluations of expected security returns.

We begin by investigating the expected rate of return measure suggested by equation (3.3-14) and obtained by using the averages of the long-term expected growth rates. We are particularly concerned with whether the relationship between expected return and the systematic risk variables represented by various regression coefficients holds when expected return is measured with our analysts' forecasts. First, in section 4.1 we specify more precisely exactly what measures of risk will be employed. Next, in section 4.2, we examine the prima facie evidence in favor of hypotheses suggested by the diversification model. Section 4.3 then adopts a more structural approach, which takes into account some econometric problems that were discussed in section 3.4. We switch in section 4.4 to the alternative specification (3.3-15), which we suggested might also give a good representation of the model. This price-earnings ratio formulation allows us to enquire whether other growth forecasts might give a closer explanation of valuation relationships than the expectations data we collected. Failure to find such improvement allows us to conclude that our growth measures are closest to the actual expectations that enter market valuation.

Having a model for prices also allows us to investigate whether knowledge of the model and access to the expectations data would have allowed superior stock selection. The fact that they would not comes as no surprise, but the reasons are of considerable interest. These are the subject of section 4.5. The various findings of these investigations are summarized in section 4.6.

4.1 The Risk Measures Used

It is not clear from the diversification model exactly what measures of risk would be most appropriate. We did provide, in section 3.4, a theoretical justification for the general approach that we shall take. Nevertheless, some empirical investigation is needed before we can ascertain what specific measures are most appropriate: that is, we need to select the exact form of the regression equation whose estimated coefficients will stand for the factor coefficients. We begin by exploring relationships between security returns and some economic variables that are of interest whatever valuation model is appropriate. Once we have established the variables to be used, we proceed to explore the valuation relationships suggested by the theory.

The first set of variables employed are measures of so-called market risk derived from the regressions of the realized rates of return on various market-wide variables.[1] We experimented with several market indicators including the Standard & Poor's 500 Stock Index, the Dow Jones Industrial Average (of 30 stocks), and the (value) weighted and unweighted indexes made available by the University of Chicago's Center for Research in Security Prices (CRSP). The realized rates of return were obtained from the CRSP. Our results turned out not to be sensitive to use of the alternative market indexes, so we report here only the results for the CRSP weighted index. This index tended to give results as strong as any in terms of r^2 for the regressions of company returns on the index and provided coefficients which were marginally stronger for the subsequent simple regressions reported in section 4.2.

Correlation with other types of variables may also yield needed risk measures whether the extended CAPM (involving nonmarketable income streams) or the diversification model is assumed. We selected three such additional variables. They are the rate of change of National Income (NI), the short-term interest rate measured by the ninety-day Treasury Bill rate, and the rate of inflation measured by the increase of the Consumer Price Index.[2] These may be considered typical measures of

1. These are the "beta" coefficients often calculated allegedly to give content to the CAPM.

2. We used alternatively the rate of change of GNP as opposed to NI; the long rate as opposed to the short; and the GNP deflator as opposed to the CPI. The alternative series were so highly correlated that it made little difference which we employed.

some risks to which investors are subject, stemming from variation in other sources of income, from changes in interest rates, and from changes in inflation.

The period over which the regression coefficients should be calculated is not clear a priori. It is not even clear that only past values should be used. The theory involves the covariances of returns with various quantities in the future. These parameters could safely be estimated from past data if they did not change or if investors did not perceive change. Such stability is unlikely. Changes in the nature and type of activities that corporations pursue and alterations in the structure of the economy make it likely that the appropriate regression coefficients change through time. Insofar as investors can perceive and even anticipate these changes, they are unlikely simply to extrapolate past betas into the future. Indeed, many of the popular "beta services" in the financial community explicitly adjust the betas calculated from past data, on the basis of changes that are known to have occurred in the structure of the business. Thus, in calculating the relevant betas at any time, it might be sensible to use values estimated with data following the time at which the valuation took place. Fortunately, our expectations data are not based on calculations using the realizations over the forecast period so we do not have to worry about spurious correlations being found between the expected return and these future values.

We adopted a compromise approach after some experimentation. The regression coefficients are calculated using quarterly observations over ten-year periods. The periods used covered the three years prior to the valuation date and the seven years following it. The results reported in the next section are not very sensitive to variations in the details of this procedure. Almost the same results were obtained, for example, when we took five years before and after the valuation date. Nevertheless, we did find that use of data entirely from past periods gave less satisfactory results than those obtained by including some future data. Extending the estimation period into the future improved the values of r^2 and was particularly important for obtaining some precision in evaluating the effect of inflation.

We also tried monthly rather than quarterly observations and shorter time periods over which to make the calculations of covariances with the market index. Again we found that the results improved when future data were included in the calculations, i.e., when some foresight regarding the future was assumed. However, the use of the shorter period made no substantial difference to the results. Since it is desirable to calculate all the regression coefficients over the same period so that the variance-covariance matrices of these estimates can be easily obtained for use in testing certain hypotheses, and since National Income is available only quarterly, we pursued the quarterly calculations.

4.2 Association of Expected Return and Risk

4.2.1 Strength of Individual Measures

The first question we investigate is the relationship between expected return and each of the various risk measures. The critical questions are whether the regression coefficients specified in the previous section are related to expected return and whether other types of risk measures (not suggested by the CAPM) are more important.

The expected return variable we use is suggested by equation (3.3-14). Let $\bar{g}_{jt}$ be the average of the long-term predicted (percentage) rates of growth available for company j at time t, D_{jt+1} be the dividends expected to be paid per share in the course of the next year (as estimated by the predictor which furnished data in all years), and P_{jt} be the end-of-year closing price (ex dividend where appropriate) for the shares of company j. Then the expected percentage rate of return, $\bar{\rho}_{jt}$, is calculated as

$$\bar{\rho}_{jt} = \bar{g}_{jt} + 100(D_{jt+1}/P_{jt}). \tag{4.2-1}$$

Simple regressions of this expected return measure on the various risk proxies are summarized in table 4.1. The sort of cross-sectional data we are using makes us vulnerable to heteroscedasticity, which can produce some seriously misleading results from our data if the problem is ignored. To avoid the difficulties produced by heteroscedasticity, we calculated the standard errors of the coefficients in the way advocated by White (1980) that allows for any heteroscedasticity that may be present. We report in table 4.1 the asymptotic t-values for the regression coefficients calculated in this way. Because of the adjustment for heteroscedasticity, the coefficient of determination r^2 is not a monotonic transformation of these t-values. The values of r^2 did nevertheless tend to parallel the t-values.

The first risk measure is the regression coefficient of the (excess) rate of return of each security on the (excess) rate of return to the CRSP value-weighted market index. It is denoted by $\hat{\beta}_{Mj}$ and was obtained by estimating the equation

$$\pi_{jt} - \rho_t = \beta_{Mj}(\pi_{Mt} - \rho_t) + u_{jt} \tag{4.2-2}$$

for each company j over forty quarters, that is, forty values of t. Here π_{jt} is the ex post return to company j, ρ_t is the short-term (ninety-day) Treasury Bill rate taken to represent the risk-free rate of interest, and π_{Mt} is the rate of return of the CRSP index. This β_{Mj} coefficient is, of course, the measure suggested by the CAPM if one ignores the problem that the market index must provide complete coverage of marketable securities. We then proceed to estimate the equation

$$\bar{\rho}_{jt} = a_{0} + a_1 \hat{\beta}_{Mj} + v_{jt}. \tag{4.2-3}$$

Table 4.1 Risk Measures and Naive Expected Return (asymptotic t-values adjusted for heteroscedasticity)

A. Using Regression Coefficients

Year	$\dot{\beta}_M$	$\dot{\beta}_Y$	$\dot{\beta}_r$	$\dot{\beta}_p$
1961	4.04	2.37	−.59	−1.13
1962	2.01	1.82	.92	−.54
1963	1.74	.96	−.33	−.59
1964	2.21	.77	−1.45	−1.08
1965	1.92	1.48	−1.52	−1.40
1966	3.99	2.48	−4.04	−4.33
1967	3.11	2.93	−4.44	−3.83
1968	3.91	1.98	−4.27	−4.02

B. Using Variance Measures

Year	s_g^2	s_g	s_e	s_f^2
1961	1.90	.99	2.89	1.68
1962	3.63	3.63	1.56	−.32
1963	2.39	2.09	.52	1.51
1964	6.47	2.42	.83	−3.14
1965	4.76	3.30	1.21	−.91
1966	2.21	2.76	1.60	
1967	2.82	3.91	1.35	
1968	8.21	6.98	2.68	

β_M = coefficient of the CRSP value weighted index.
β_Y = coefficient of the rate of change of National Income.
β_r = coefficient of the Treasury Bill rate.
β_p = coefficient of the rate of change of prices.
s_g^2 = variance of the long-term growth predictions.
s_g = standard deviations of the long-term growth predictions.
s_e = standard error of regression of return on four variables.
s_f^2 = variance of the short-term growth predictions.

This equation is estimated separately for each year t on the basis of all companies j for which we had data in that year. The resulting t-values for a_1 appear in table 4.1.

The t-values obtained from estimating equation (4.2-3) are positive and usually significant. The strength of the association is not great, however: the value of r^2 corresponding to the highest t-value is only 0.16. The weakness of these associations could arise from the particular market index and periods used. However, as noted above, the results did not vary substantially if alternative indexes were used in place of the CRSP weighted index, and seemed more apt to be weaker than stronger. They also were not substantially changed by using the coefficient obtained by regressing individual returns on the market return rather than using excess returns in each case. Moreover, the results were not very sensitive to changing the period over which the coefficients were estimated, provided that at least some observations following the date at which the growth forecasts were made were included.

Although the regression coefficients with the CRSP index give significant results, strong t-values (and coefficients of determination) are sometimes obtained from using the regression coefficients of the securities' returns on the rate of change of National Income, indicated by $\hat{\beta}_Y$ in table 4.1, in place of $\hat{\beta}_M$ in estimating equation (4.2-3). These t-values are not, however, as strong as those for the coefficient of the CRSP index.

Our next risk measures come from estimating the regression of each security's rate of return on the rate of inflation ($\hat{\beta}_p$) and on the Treasury Bill rate ($\hat{\beta}_r$). Systematic relationships between security returns and inflation and interest rates are consistent with the wider specification of returns being associated with a variety of factors, as we argued in chapter 3. Table 4.1 indicates that these alternative risk measures do not do as well as the standard $\hat{\beta}_M$ measure during the early years. They do, however, tend to have a much stronger influence later in the 1960s when inflation rates and interest rates begin to soar. The signs of β_r and β_p can be expected to be negative if they do not also stand as proxies for other risk measures. A higher value of β_p indicates that a stock provides a better inflation hedge, which is a desirable attribute. Similarly, a positive value of β_r indicates that the stock does well when interest rates rise and hence is negatively correlated with realized returns from fixed income securities.

These results clearly indicate that the various regression coefficients are indeed related to expected return. The next question is whether other types of risk measure have still closer associations. Part B of table 4.1 summarizes the results obtained by using various variance measures for risk instead of regression coefficients.

The first of these alternative risk measures is the variance of the predictions of long-term growth, s_g^2. This quantity may possibly be interpreted as a measure of own variance and thus of specific risk. Nevertheless, the decomposition shown in equation (3.4-14) suggests that it may instead be a particularly good expectational proxy for systematic risk. For the years 1962 through 1965, when our sample was widest, s_g^2 gives stronger results than any of the regression risk measures. It also shows positive associations with expected rates of return in other years, which are clearly significant except in 1961.

Equation (3.4-14), which provides the basis for the possible interpretation of s_g^2 as representing systematic risk, also indicates that s_g^2 would be a quadratic rather than a linear combination of the factor coefficients γ_{jk}. This might suggest that the standard deviations of growth forecasts might be stronger measures of systematic risk than the variances. However, as the column of table 4.1 headed s_g shows, there was no reliable tendency for this to be the case.

If s_g^2 should represent specific risk rather than systematic risk, one might expect a better measure to be provided by the residual variances or standard errors of estimate of the regressions of the rates of return on the various systematic variables. Our findings do not, however, support this supposition. The standard errors from the regression of return on the four variables used to calculate the β coefficients produced weaker results than did s_g^2. They are shown in the column of table 4.1 headed s_r. The residual variances, that is, s_r^2, gave no stronger results.

The success of the variance of the long-term predictors makes one wonder whether the variance of the short-term growth predictions could also be used to provide a useful measure. This did not prove to be the case. The results, given in the final column of table 4.1, show mixed signs and are generally not significant. This risk measure quite clearly is weaker than the variance of the long-term predictions.

4.2.2 Use of Several Risk Measures

These results already have some interesting implications despite the simplistic approach used. There is, however, no reason to limit ourselves to only one risk measure. We now turn to the wider specification where in the first step the realized rate of return is regressed on all the suggested variables.[3] Before looking in the next section at the more structural aspects of this specification, we examine the prima facie case that all these variables are relevant to valuation, even though these inferences may turn out to be influenced by errors-in-variables difficulties.

The coefficients were obtained from the multiple regression of the rate of return of each security on the CRSP value-weighted index (M), on the rate of change of National Income (DY), on the Treasury Bill rate (r), and on the rate of inflation (DP). The equation fitted for each company is

(4.2-4) $$\pi_{jt} = \delta_{0j} + \delta_{Mj}M_t + \delta_{Yj}DY_t + \delta_{rj}r_t + \delta_{pj}DP_t + u_{jt},$$

and the estimated regression coefficient $\hat{\delta}_{ij}$ serves as risk measures. The cross-section specification for $\bar{\rho}_{jt}$ is expanded from (4.2-3) to

(4.2-5) $$\bar{\rho}_{jt} = a_0 + a_1\hat{\delta}_{Mj} + a_2\hat{\delta}_{Yj} + a_3\hat{\delta}_{rj} + a_4\hat{\delta}_{pj}.$$

Estimates of this equation are given in table 4.2.

A number of findings indicated by table 4.2 are worth emphasizing. Of most importance, each type of coefficient is significant in some years. In the first part of the period only the market coefficient is significant. However, toward the end of the period other coefficients tend to be important, especially those measuring systematic relationships with inflation and interest rates. When these results are taken at face value, two

3. These are the regressions from which the standard errors of estimate referred to in table 4.1 were obtained.

Table 4.2 Regression Estimates for Extended Model for the Expected Rate of Return (asymptotic *t*-ratios adjusted for heteroscedasticity)

Year	Constant	$\dot{\delta}_w$	$\dot{\delta}_Y$	$\dot{\delta}_o$	$\dot{\delta}_p$	R^2
1961	7.01	1.58	.23	−.02	−.03	.15
	(12.75)	(2.78)	(1.47)	(−.73)	(−.59)	
1962	7.82	1.19	.20	.02	−.02	.10
	(10.52)	(1.59)	(1.61)	(.54)	(−.29)	
1963	6.94	1.63	.05	−.03	−.01	.07
	(14.85)	(3.04)	(.44)	(−.46)	(−.12)	
1964	6.00	2.58	−.06	−.08	−.06	.12
	(10.22)	(3.80)	(−.74)	(−1.27)	(−.68)	
1965	8.31	.79	.11	−.07	−.10	.07
	(18.31)	(1.57)	(1.14)	(−1.25)	(−1.04)	
1966	9.85	.90	.17	−.09	−.19	.19
	(21.18)	(2.11)	(1.92)	(−3.25)	(−3.60)	
1967	9.82	1.26	.25	−.15	−.30	.26
	(17.46)	(2.11)	(2.55)	(−4.19)	(−3.67)	
1968	8.83	3.98	.42	−.24	−.52	.28
	(11.70)	(4.69)	(3.28)	(−4.19)	(−3.77)	

explanations for them come to mind. First, in the more stable early part of the period, estimates of the δ coefficients may be sufficiently imprecise that in the subsequent estimation of equation (4.2-5) the relatively greater errors of measurement lead to lack of significance. Second, investors may have become more concerned about the other sources of risk, such as inflation and interest-rate instability, as the decade proceeded.[4] Overall, the results suggest strongly that all influences play a role, though it is an open question whether this is because they act as proxies for other variables.

The signs of the coefficients tend to be the same across the different equations. Although with errors in variables we must be cautious in attaching much importance to the signs of particular coefficients, the patterns obtained do usually conform to the signs suggested by intuition. Positive association with either the market return or income raises the expected rate of return. Correspondingly, positive partial correlation with the rate of inflation, indicating that the stock tends to act as a hedge against inflation, lowers the expected rate of return. Finally, the coefficient for the Treasury Bill rate usually has the expected negative sign. There is, however, a good deal of correlation across securities (roughly about 0.6) between the coefficients for the Treasury Bill rate and for the rate of inflation so that one may be partly serving as an additional proxy for the other. This correlation is sufficiently low, however, that one cannot legitimately presume that variations in the rate of change of prices and in the short-term rate of interest necessarily represent the same

4. Inflation, as measured by the annual rate of change in the Consumer Price Index, remained below the 2 percent level through 1965. Later in the decade, inflation increased to the 6 percent level.

variable. Except for this fairly mild correlation, multicollinearity problems are small, making it less plausible that all the different measures serve as proxies for some single variable.

Inclusion of all these different regression coefficients does not account for the strength we found earlier for the variance of the predictions. When that variable was included in (4.2-5) along with the four $\hat{\delta}$ variables measuring various systematic risks, it usually was highly significant with a positive coefficient. The a coefficients for the four $\hat{\delta}$ variables tended to retain the same signs, though with lessened significance. The apparent importance of s_g^2 may in part result from errors-in-variables problems or misspecification. Nevertheless, it may also indicate that s_g^2 is a particularly useful expectational proxy for several of the systematic risk measures. What is important is that the values of R^2 are sufficiently high and so very highly significant that there is no question about there being some underlying systematic association among the variables included in the specification.

4.3 Structural Relations between Expected Return and Risk Coefficients

The results reported in the previous section may arise because the market actually takes a multifaceted approach to risk. In contrast, they may simply be the outcome of using poor data. To investigate this question, we proceed in two stages. First, we examine the extent to which our risk coefficients exhibit the linear structure that we indicated in section 3.4 would be found if there were fewer factors than the number of independent variables used in the regressions in which the $\hat{\delta}_j$ coefficients were calculated. Establishment of the number of factor coefficients is also needed in order to proceed to take account of the errors of estimation of the $\hat{\delta}$ coefficients. The second stage involves estimating the valuation model allowing for the presence of these errors.

4.3.1 The Number of Factor Coefficients

We showed in equation (3.4-12) that the variance-covariance matrix of the regression coefficients has a particular structure under the common-factor model for rates of return. Let $\bar{\delta}$ be the average of the $\hat{\delta}_j$ vectors, and let $\bar{\alpha}$ be the average of the α_j vectors whose elements α_{jk} are the coefficients of the common K factors in the (true) rate-of-return equation (3.2-16). Letting $\bar{h} = \Sigma_{j=1}^{J} h_j/J$, where h_j is the residual variance, we can rewrite equation (3.4-12) as

$$(4.3\text{-}1) \qquad V = E\left[\sum_{j=1}^{J} (\hat{\delta}_j - \bar{\delta})(\hat{\delta}_j - \bar{\delta})'/J\right]$$
$$= \Xi'\left[\sum_{j=1}^{J} (\alpha_j - \bar{\alpha})(\alpha_j - \bar{\alpha})'/J\right]\Xi + \bar{h}(X'X)^{-1}.$$

Table 4.3 Significance Levels for the Hypothesis That More Than Specified Numbers of Factors Are Present in the Regression Coefficients

	Number of Factors			
Years	0	1	2	3
1959–68	.000	.816	.594	.174
1960–69	.000	.134	.266	.126
1961–70	.000	.890	.784	.303
1962–71	.000	.935	.339	.951
1963–72	.000	.767	.789	.305
1964–73	.000	.001	.059	.694
1965–74	.000	.068	.196	.992
1966–75	.000	.005	.065	.398
1967–76	.000	.006	.053	.317

Since $(X'X)$, the cross-product matrix of the variables used to estimate the coefficients, is known,[5] we can investigate the hypothesis that this common-factor structure does apply[6] to the variance-covariance matrix of the estimated coefficients calculated for the different companies. Assuming that the coefficients are normally distributed across companies, we performed likelihood-ratio tests of a variety of hypotheses. In doing so we used the value of $\bar{h}$, the average of the estimates coming from the estimates of the individual regressions, rather than jointly estimating this parameter in the factor analysis. No substantial differences in results occur when instead $\bar{h}$ is estimated from the $\hat{\delta}$ data.

The regression coefficients used for different years are far from being independent, since thirty-six of the quarterly observations are the same in regressions for adjacent years. Nevertheless, the patterns that occur over time are of interest. When we tested the hypothesis that there are less than four factors represented by the four regression coefficients, the data strongly supported the hypothesis that there are fewer factors. These tests are summarized in table 4.3 in terms of the smallest significance levels at which one could reject the (null) hypothesis of only zero, one, two, and three factors over the alternative hypothesis of at least four different factors being present.[7]

The hypothesis of only one factor is very strongly indicated in the early part of the period. However, when observations from the 1970s begin to

5. Of course, when the $\hat{\delta}$ vector being investigated does not contain the constant term, the appropriate row and column are first removed from $(X'X)^{-1}$.

6. Specifically, the procedure involves the principal components of $\Sigma_{j=1}^{J} (\hat{\delta}_j - \bar{\delta})(\hat{\delta}_j - \bar{\delta})'/J$ in the metric of $(X'X)^{-1}$. See Anderson and Rubin (1956) for a discussion of maximum likelihood estimates of the model. The fact that $\bar{h}(X'X)^{-1}$ is known makes more factors identifiable than would usually be the case.

7. Qualitatively similar results are obtained when we test three versus four factors, two versus three, etc.

play an important part. the data indicate that at least two factors are present and would reject at the 0.10 level the hypothesis of two factors in favor of three factors for some of the estimations.

The reason for the success of a one common-factor model in the early estimates was not that the correlations of different quantities, which themselves all varied significantly, could be fully attributed to a single factor. Rather, it was the case that some of the estimated coefficients varied so little across companies, relative to their errors of estimation, that both the variances across companies of their true values, δ_{jk}, and their correlations with other coefficients could be treated as zero.

This problem is illustrated by the data from the 1960s shown in table 4.4. There we present the matrices

$$\sum_{j=1}^{J} (\hat{\delta}_j - \bar{\delta})(\hat{\delta}_j - \bar{\delta})'/J$$

and

$$\left[\sum_{j=1}^{J} (\hat{\delta}_j - \bar{\delta})(\hat{\delta}_j - \bar{\delta})'/J - \bar{h}(X'X)^{-1}\right].$$

All the variances of the $\hat{\delta}_r$ and $\hat{\delta}_p$ coefficients can be attributed to estimation errors, and the hypothesis that the variance across companies in the true coefficients was zero could not be rejected. Indeed, all the variance can be so attributed for $\hat{\delta}_p$, the coefficients of inflation. Later, as interest rates and inflation rates themselves showed more variation, this ceased to be the case and all coefficients showed variation across companies significant beyond the 0.05 level. As noted earlier, while short-term interest rates and inflation may primarily reflect the same factor (as might be the case if the real rate of interest is constant), the magnitude of measurement errors in each variable must then be very substantial since collinearity problems in the data were mild and do not clearly account for the

Table 4.4 Covariance Matrices of the Regression Coefficients Fitted for 1960–69

	$\dot{\delta}_w$	$\dot{\delta}_Y$	$\dot{\delta}_r$	$\dot{\delta}_p$
A. Unadjusted				
$\dot{\delta}_w$	.09			
$\dot{\delta}_Y$	.17	4.46		
$\dot{\delta}_r$	.47	−3.78	58.9	
$\dot{\delta}_p$	−.03	1.27	−22.9	19.3
B. After Subtraction of Estimation Error				
$\dot{\delta}_w$	.05			
$\dot{\delta}_Y$	.13	1.08		
$\dot{\delta}_r$	.61	1.63	5.13	
$\dot{\delta}_p$	−.25	−1.21	4.73	−4.25

difficulties. Furthermore, the results about the number of factors were repeated when we dropped the interest-rate variable from the original regressions. The 1964–73 period and later ones indicated the presence of at least two and possibly three factors. Prior to that period, the variance-covariance matrices suggest only a single factor.

Earlier investigations of the appropriateness of the common-factor model to security returns suggested that several factors would be found. King (1966) as well as Roll and Ross (1980) each found support for such a hypothesis. Hence one may suspect that our results for the early years reflect the peculiarities of the data on some of the independent variables in that period.

These tests have involved the variance-covariance matrices of the regression coefficients. This was appropriate in view of our desire to use the adjusted matrices subsequently in estimation where it is necessary to avoid using singular matrices. However, the original hypothesis applies also to the averages (across companies) of the coefficients, that is, to

$$E\left[\sum_{j=1}^{J} \hat{\delta}_j \hat{\delta}_j'/J - \bar{h}(X'X)^{-1}\right].$$

When we investigated the number of factors, recognizing that the means of the regression coefficients should have the same factor structure, we found evidence for two factors rather than only one in the early years. That is, the hypothesis of only one factor can be rejected well beyond the 0.05 level, but not that of there being only two factors. The results for the later years did not change appreciably. We can still conclude that there are certainly two, and possibly three, common factors.

4.3.2 Results Allowing for Estimation Error

The previous findings about the number of factor coefficients present in the rate of return regressions pose a dilemma for the next part of our investigation. We suspect that the reason for finding only one factor in the early years is that the other factors happened to have very little variation in the 1960s. However, if the risk was still present that they would vary, then their coefficients should still enter the valuation equation. Using a one-factor model would then involve misspecification. Testing the hypothesis that more than one factor is actually present does require that the data clearly involve more than one factor. A procedure developed in Cragg (1982) that allows for estimation errors in $\hat{\delta}$ involves the use of

$$\left[\sum_{j=1}^{J} (\hat{\delta}_j - \bar{\delta})(\hat{\delta}_j - \bar{\delta})'/J - h(X'X)^{-1}\right]^{-1}.$$

The procedure makes sense only if the matrix is clearly positive definite. When this is the case, we can allow for the estimation error to see what inferences stand up even when its effects are recognized. In doing so, we shall use the simplification, discussed in Cragg (1982), in which the u_{jt} of equation (4.2-4) are assumed to be normally distributed.

We resolve the dilemma posed by our findings about the structure of the $\hat{\delta}_j$ coefficients by fitting two types of model, allowing in each case for the estimation errors of the regression coefficients. First, we estimate the equations for the expected rate of return using only the regression coefficient for the market and the variance of the long-term predictors; i.e., we fit the equation

$$\bar{\rho}_{jt} = a_0 + a_1\beta_{Mj} + a_2 s^2_{gj}. \qquad (4.3\text{-}2)$$

Here, the β_{Mj} are based on the three years before and the seven years after the valuation. Second, we use the coefficients for the 1966–75 period, estimated without the interest-rate variable; that is, we estimate

$$\bar{\rho}_{jt} = b_0 + b_1\gamma_{Mj} + b_2\gamma_{Yj} + b_3\gamma_{pj} + b_4 s^2_{gj}, \qquad (4.3\text{-}3)$$

where the γ_j are calculated from the regression

$$\pi_{jt} = \gamma_{0j} + \gamma_{Mj}\pi_M + \gamma_{Yj}DY + \gamma_{pj}DP + v_{jt} \qquad (4.3\text{-}4)$$

for the period 1966–75. As we noted, there γ coefficients do support (though not strongly) the conclusion that a three-factor model is appropriate.

The first approach does little to resolve the puzzle. In the early part of the period, β_M was not significant while s^2_g was always stronger and usually significant. For 1966 and subsequent years, when the number of predictors available on which to base s^2_g becomes small, β_M is highly significant, and positive, as is s^2_g in the last two years. These results suggest that s^2_g is not simply another proxy for the systematic risk measured with considerable estimation error by β_M. Instead, it suggests that a model with two or more factors is appropriate—or that there is another relevant risk concept proxied by s^2_g.

The results of the second approach shed quite a bit more light on the matter. When adjustment was made for errors in variables and allowance was made for heteroscedasticity, it usually turned out that none of the coefficients was significantly different from zero. At best, but one would be, and then only just at the 0.05 level. This was true whether s^2_g was included or not. Overall, however, when s^2_g was included in the equation, the hypothesis that all γ_j parameters had zero coefficients in equation (4.3-3) could be rejected beyond the 0.01 level, except in 1963 and 1965. When s^2_g was not included, the hypothesis could sometimes be rejected at the 0.10 level and sometimes not.

Part of the difficulty stems from multicollinearity. As lack of certainty about the number of underlying factors indicated, the "corrected" $\hat{\gamma}_j$ coefficients are correlated with each other. Moreover, there is some correlation with s^2_g, though it is small. The technique used involves much more complicated standard errors than ordinary regression, and for a given covariance matrix of explanatory variables these standard errors are considerably larger. More coherent results were obtained when the γ_Y coefficient for National Income was eliminated from (4.3-3). A pattern then emerged in which the coefficient of inflation and the variance of the predictors were significant, but the coefficient for the market index was not. Eliminating this coefficient as well as the one for national income then produced the results shown in table 4.5.

The results shown in table 4.5 are similar in nature for the different years. The risk variable s_g^2 has a positive and usually significant effect. The notable change in its magnitude in 1966 corresponds to the change in the number of predictors from which the forecast data were collected. The sensitivity of the security's rate of return to the rate of inflation as measured by γ_p had a negative effect as we would expect.

These results suggest that at least two factors are relevant in valuation. One may be equated broadly to inflation and its associated effects. The other, possibly representing market risk, seems to be better represented by the variance of the predictions of long-term growth than by any of the regression coefficients. Its exact nature therefore remains a bit of a puzzle. The first factor has a negative sign and is usually significant at the 0.10 level. This was true even in the early years when the experienced variations in the inflation rate were very small. The second factor is very strongly positive and highly significant.

Table 4.5 Equation for Expected Rates of Return Allowing for Estimation Error in γ_i (asymptotic t-values adjusted for heteroscedasticity)

Year	Constant	γ_p	s_g^2	"r^2"**	r_e*
1961	9.26	−1.13	.63	.56	—
	(12.62)	(−1.72)	(6.30)		
1962	8.40	−.46	.67	.38	.88
	(30.87)	(−1.70)	(4.96)		
1963	8.18	−.61	.72	.34	.89
	(32.30)	(−1.58)	(2.98)		
1964	8.55	−.74	.63	.54	.84
	(21.17)	(−1.92)	(18.77)		
1965	9.01	−.74	.62	.61	.90
	(24.20)	(−1.99)	(29.39)		
1966	10.72	−.20	.05	.08	.68
	(28.48)	(−.73)	(1.48)		
1967	11.35	−.53	.03	.25	.67
	(24.88)	(−1.65)	(2.05)		
1968	11.93	−.75	.05	.70	.48
	(17.74)	(−1.82)	(7.44)		

**"r^2" is 1 − (estimated residual variance)/(variance of ρ_{it}).

*r_e is correlation of residuals with previous year's residuals.

These results have been corrected for the errors of measurement in the regression coefficients, but errors in $s_{\bar{g}}^2$ have been ignored. The interpretation we have been giving to that variable means that we cannot calculate the variance of errors in its measurement by assuming that it is simply the sampling variance of predictions which all have the same mean for each firm. We did, however, attempt to deal with this measurement error by the use of instrumental variables while continuing to allow for the estimation errors in the regression coefficients. To do so, we used as instruments the regression coefficients γ_M and γ_Y and the residual variances s_{e}^2, whose usefulness we explored earlier, in table 4.1.

The main difficulty with the instrumental-variable approach in this case was that the proposed instruments are not closely associated with $s_{\bar{g}}^2$. The value of R^2 obtained from regressing $s_{\bar{g}}^2$ on all the instruments and γ_p varied from 0.05 to 0.31. The main effect of this weakness on the estimates of the equations for expected return was to reduce the standard errors of the coefficients of $s_{\bar{g}}^2$ sharply. These findings strengthen the impression that $s_{\bar{g}}^2$ contains relevant information about risk not readily available in other forms. However, the significance levels of γ_p were not affected by the use of instrumental variables, and the results were qualitatively much the same as those shown in table 4.5 in terms of the signs and magnitudes of the coefficients.

4.3.3 Constancy over Time

One of the interesting questions about valuation equations is whether the coefficients remain the same each year or whether they change. There is nothing in the valuation theory to suggest that they should be constant. The opportunity sets faced by investors, extending beyond simply the financial securities available to them, probably change and so may their preferences and concerns about various types of risk. The results of tables 4.2 and 4.5 give an impression of considerable variation. We now test for variability explicitly.

The residuals from the equations shown in table 4.5 for different years are correlated even after allowance is made for the effects of estimation errors of $\hat{\gamma}_p$. Problems of missing observations mean that we can simultaneously calculate the equations for a common set of companies in all years only at the expense of losing a large number of companies. Pairwise comparisons indicated that the residuals for adjacent years are quite highly correlated. The correlations of these residuals are recorded in table 4.5 in the column headed r_e. It gives the correlations of the residuals in one year with those of the year immediately preceding. The quantities tabulated are the correlations of residuals using a common set of companies to estimate the regression coefficients in the two years. The exact values of the coefficients used differ slightly from those shown in table 4.5 because of the reduced number of observations used in their calculation.

The correlations of residuals, which are highly significant, complicate the problem of inquiring into the stability of the regression coefficients over time. Zellner's (1962) "seemingly unrelated regression technique" can be adapted in a straightforward way to the estimation of our equations even when allowing for estimation error of the original regression coefficients as well as for heteroscedasticity. To avoid the extensive loss of observations involved when all equations are fitted simultaneously, only pairs of equations were fitted.

Pairwise estimation of the equations usually produced significant differences in the coefficients of the valuation equation for different years. The main exceptions, where rejection did not occur even at the 0.10 level, are the 1964–65 comparison and the 1962–63 one. The coefficient for 1963 did differ from that for 1964 significantly at the 0.01 level even though the values shown in table 4.5 indicate the same qualitative findings in the sense that the coefficients are of similar magnitude.

The different estimation procedure used in these tests, which involve estimating the coefficients of each of two years jointly, did not change the conclusions about risk that were derived from our regressions in section 4.3.2 for the individual years. Indeed, these estimates indicated stronger support than the ones in table 4.5 for the hypothesis that two types of risk measures are indicated by the data.

4.3.4 Average Realized Return and Risk

The constant term $\hat{\delta}_{0y}$ obtained when equation (4.2-4) was fitted to obtain the other δ coefficient contains implicitly another estimate of the expected rate of return. It is the average rate of return realized over the period, which many empirical studies of valuation presume corresponds to the return expected *ex ante* by investors. We can use this estimate to investigate the *ex post* validity of the APT, or diversification model, which suggests that we should find the same number of factors in the $\hat{\delta}_y$ vector when $\hat{\delta}_{0y}$ is included as when it is not. This consideration induces us to repeat the investigations carried out in section 4.3.1 with the other coefficients, but now including the constant $\hat{\delta}_{0y}$ as well.[8]

The estimates for the earlier periods included in our investigation tend to confirm the model fully in the sense that exactly the same number of factors is significantly present in the covariance matrix including the constant as we found when only the regression coefficients were used. This support for the model is less than might appear to be the case, however. As was the case for some of the coefficients, significant variation across companies was not present in the average rates of return in the

8. All independent variables are measured as deviations from their averages, so the constant term is also the average quarterly rate of return in the period over which the regression coefficients are calculated.

early years. In the final two years, the wider covariance matrix indicated that at least five factors were needed to account for the covariances of the constants with the other coefficients.

With the companies altering their natures over time and with the market valuation of risk quite possibly changing substantially over the decade of the seventies, such a finding should not be surprising even if the common-factor model is a correct description of security returns. However, it does not seem feasible to use these "objective," *ex post* measures of returns to obtain comparisons with the very successful results obtained from the *ex ante* measures we have employed. These estimated average *ex post* returns are not closely correlated with the *ex ante* measures derived from using the long-term growth predictions. The strong and interesting results we have obtained with these *ex ante* measures of expected returns and the fact that the *ex post* ones are not closely related to them emphasize the importance of using genuinely *ex ante* expectations of returns for studying security valuation.

4.4 An Alternative Valuation Specification

The derivation of the valuation model in chapter 3 suggested that the expected return formulation we have been investigating is only one approximation to the underlying model and that an alternative model may also be usefully estimated. The alternative approximation produces a more traditional formulation in which the price-earnings ratio is the dependent variable and earnings (dividend) growth, the payout ratio, and our various risk measures are treated as explanatory variables. The expected return formulation is particularly convenient for focusing on the risk structure suggested by the diversification model. The alternative allows us to ask whether growth-rate expectations are more relevant for valuation than other measures. It also allows us to investigate the role of the short-term growth predictions as well as to examine again which risk measures appear to be strongest.

An empirical analysis of the price-earnings model is also desirable because of an ambiguity of interpretation of the expected return models we have been studying. The results of the return model indicate partly that predicted earnings growth is connected with the regression coefficients giving the associations of rates of return to various economic indicators. Recall, however, that we found evidence in chapter 2 that a common-factor model may fit the growth predictions of security analysts. Our findings for the expected rates of return may reflect this feature of the data, even though the expected rate of return includes the dividend yield as well as the expected growth rate. Thus it is not entirely clear that we have actually been investigating a valuation relationship.

Implementation of the alternative model involved dividing both end-

Table 4.6 Risk Measures in Stock Price Regressions (asymptotic *t*-values for alternative risk variables in equation (4.4-1))

Year	β_M	β_Y	β_r	β_p	s_g^2
1961	−.32	1.10	1.25	.01	−.41
1962	−4.54	−.71	.59	2.58	−5.79
1963	−.33	.74	−.43	.28	−2.37
1964	−2.38	−2.76	−.88	1.65	−9.75
1965	1.43	1.32	−.70	−.43	−1.24
1966	−1.49	−.87	.14	.33	−.19
1967	1.34	.74	−2.29	−2.34	−8.67
1968	2.29	−.22	−1.41	−2.74	−1.12

of-year prices (P) and the dividends projected to be paid (D) by average normalized earnings[9] ($\overline{NE}$) to give the equation

$$P/\overline{NE} = a_0 + a_1\bar{g}_p + a_2 D/\overline{NE} + a_3 RISK. \qquad (4.4\text{-}1)$$

where *RISK* stands for the various risk variables used.

4.4.1 Risk Measures

We begin our investigation of equation (4.4-1) by treating each of the risk measures we have been using as alternatives, just as we did when considering equation (4.2-1). In these regressions, both the average expected five-year growth rate and the dividend payout ratio almost always had positive and significant coefficients throughout the sample period.

The pattern for the risk measures is more complicated than earlier. Table 4.6 corresponds to table 4.1. In these regressions, a negative sign should be expected for the risk measures based on covariance with the market index and with national income, since higher risk should, ceteris paribus, lower price-earnings multiples. Although both $\hat{\beta}$ measures have the correct negative values more often than not, the *t*-values indicate that they are only occasionally significant. Positive signs should be expected for the risk measures based on reported inflation and interest rates. As was found in the regressions in table 4.1, these risk measures are only significant toward the end of the period studied, but their signs are often incorrect in these valuation regressions.

These findings indicate the difficulties of using the simple regression coefficients as risk measures in a specification also containing several other variables. In contrast to these ambiguous results, the variance of

9. The "normalized" earnings were furnished by two of the forecasters and were described in chapter 1. When more than one forecaster's estimates of "normalized" earnings were available for a company, the estimates were averaged. The results are little different (but a bit poorer) if reported earnings over the most recent twelve-month period are substituted for "normalized" earnings.

Table 4.7 P/NE Regression Estimates of Equation (4.4-1) (asymptotic t-ratios adjusted for heteroscedasticity)

Year	Constant	$\bar{g}_e$	$D/\overline{NE}$	$s_{\bar{g}}^2$	R^2
1961	1.88	3.91	1.22	−.57	.81
	(.64)	(7.51)	(.24)	(−.41)	
1962	3.30	2.23	8.41	−1.17	.75
	(1.75)	(16.69)	(2.91)	(−5.79)	
1963	2.85	2.70	6.71	−.59	.77
	(.94)	(11.20)	(1.75)	(−2.37)	
1964	2.53	2.15	13.16	−1.09	.77
	(1.73)	(23.94)	(6.13)	(−9.71)	
1965	1.76	2.32	4.73	−.66	.67
	(.62)	(6.98)	(1.14)	(−1.24)	
1966	.22	1.74	7.42	−.01	.57
	(.09)	(9.62)	(2.79)	(−.19)	
1967	1.88	2.35	−1.05	−.09	.69
	(.67)	(13.28)	(−.35)	(−8.67)	
1968	2.18	1.78	5.13	−.04	.52
	(.56)	(8.10)	(.99)	(−1.12)	

the predictions always has a negative sign. Its significance does vary considerably across years, primarily reflecting variation in the magnitude of its coefficient. The important point, which agrees with our previous results with the expected return measures, is that $s_{\bar{g}}^2$ provided a better single risk proxy than the regression coefficients based on more objective calculations. It also provided a more significant and consistent measure than the residual variances of the regressions, s_{ϵ}^2.

Table 4.7 shows the full estimates of equation (4.4-1) using $s_{\bar{g}}^2$ as the risk variable. The growth-rate variable is highly significant in each of the years covered. The payout ratio has the expected sign except in one year but is usually insignificant.[10] As we have already noted, the risk variable always has the correct negative sign and is often significant.

4.4.2 Alternative Growth Measures

The extent to which using truly expectational data is important for valuation models is indicated in table 4.8. Here we show the values of R^2

10. The positive sign of the dividend coefficient should not be interpreted as evidence that dividend policy can affect the value of the shares. This coefficient indicates only that a ceteris paribus change in dividend payout will increase the price of the shares. Among the things held constant in this equation is the growth rate of earnings and dividends per share. A positive dividend coefficient thus indicates only that given the future growth rate in earnings and dividends, the price of a share should be higher, the higher is the current percentage of earnings that can be paid out. The famous "dividend irrelevancy" theorem of Miller and Modigliani (1961) says that an increase in dividend payout will tend to reduce the growth rate of earnings per share since new shares will now have to be sold to make up for the extra funds paid out in dividends. A positive dividend coefficient is thus in no way inconsistent with the dividend irrelevancy theorem.

Table 4.8 Values of R^2 for Alternative Specifications of the Valuation Equation

	Specification		
Year	1	2	3
1961	.42	.45	.81
1962	.50	.53	.75
1963	.49	.50	.77
1964	.37	.43	.77
1965	.29	.31	.67
1966	.31	.44	.57
1967	.32	.36	.69
1968	.33	.41	.52

NOTE. See text for specifications.

for various combinations of historical and expectational data. The first specification (column 2) involved regressing the price-earnings multiple on three historic figures: the past ten-year growth rate of cash earnings, the average (over the preceding seven years) historic dividend-payout rate, and β_M, estimated using only previous data. The third column substitutes the expectational variable $s_{\bar{g}}^2$ for the β_M coefficient. The fourth column repeats the specification of equation (4.4-1) with $s_{\bar{g}}^2$ as the risk variables, $\bar{g}_p$ and $D/\overline{NE}$ in place of historic growth and payout, and $P/\overline{NE}$ as the dependent variable in place of P/E. These r^2 values are the same as in table 4.7.

The dramatic change in the value of r^2 for the valuation equation occurs when $\bar{g}_p$ is used for the growth rate. Other variations have comparatively minor effects. There are, of course, a large number of ways of calculating past growth. Our findings hold up for the wide variety of historical growth rate we tried as well as the one reported in table 4.8. Using the average predicted growth rates substantially improves the fit of the regression. It is therefore safe to conclude that insofar as the market does value growth, the growth rates involved are far better represented by actual predictions made by security analysts than by any mechanically calculated rate.

One may wonder whether we would have done better to use only one forecaster rather than the average we have employed. Problems of missing observations again hinder this investigation. One of the advantages of using the average is that it allows us to include most of the companies in the regressions. However, it is also the case that closer fits tended to be obtained by using the average growth rates of all predictors than by employing the forecasts of any single firm. This suggests that our survey was useful in getting closer to what might be considered the expectations of a "representative" investor.

4.4.3 Role of Short-Term Predictions

In addition to the long-term growth estimates, which have played such an important role in our empirical valuation work thus far, we also collected short-term predictions for earnings in the next year. These were described and analyzed in chapters 1 and 2. Given the long-term growth rate, a stock should sell for a higher price if more of that growth is expected to be realized earlier in the period. Therefore we augmented our valuation equation (4.4-1) to include the term $\bar{E}_{t+1}/\overline{NE}$, the ratio of next year's average predicted earnings ($\bar{E}_{t+1}$) to average normalized earnings (for the present period). Equation (4.4-1) then becomes

$$P/\overline{NE} = a_0 + a_1 g_p + a_2 \bar{E}_{t+1}/\overline{NE} + a_3 D/\overline{NE} + a_4 s_g^2. \qquad (4.4\text{-}2)$$

The results obtained with this specification are presented in table 4.9. The addition of a term for short-term growth does add some explanatory power to the regression, although the significant t-statistic for the coefficient of $\bar{E}_{t+1}/\overline{NE}$ comes partly at the expense of the long-term growth coefficient. The dividend and risk terms generally retain their usual signs, though they are often not significant.

4.4.4 Variations of Specification

The success of the short-term growth variable raises the question whether more generally a nonlinear specification might be appropriate. As we noted in section 3.4, the linear form of the equation is only an approximation to some more complicated true form. To investigate this

Table 4.9 $P/\overline{NE}$ Regression Estimates of Equation (4.4-2) (asymptotic t-values adjusted for heteroscedasticity)

Year	Constant	$\bar{g}_p$	$\bar{E}_{t+1}/\overline{NE}$	$D/\overline{NE}$	s_g^2	R^2
1961	−35.02	3.07	41.31	−1.58	−.71	.88
	(−4.16)	(11.94)	(4.78)	(−.35)	(−.75)	
1962	−3.36	1.99	8.57	6.96	−1.00	.75
	(−.82)	(14.05)	(2.15)	(1.97)	(−4.20)	
1963	−11.43	2.58	13.66	7.22	−.53	.81
	(−2.61)	(12.25)	(4.33)	(1.57)	(−2.16)	
1964	−7.21	2.13	8.56	13.19	−.84	.81
	(−2.46)	(18.67)	(3.90)	(5.41)	(−2.52)	
1965	−14.53	2.82	10.53	8.20	−1.09	.78
	(−1.89)	(7.12)	(1.73)	(1.82)	(.99)	
1966	−7.67	1.83	6.51	8.94	−.02	.58
	(−1.94)	(10.41)	(2.00)	(3.59)	(−.28)	
1967	−8.55	2.31	9.33	1.15	−.08	.72
	(−1.41)	(12.78)	(1.67)	(.33)	(−7.18)	
1968	−15.77	1.57	18.20	4.66	−.03	.55
	(2.54)	(6.74)	(3.12)	(.96)	(−.86)	

possibility, we used a quadratic specification for the growth and dividend-payout variables. That is, we added the squares of $\bar{g}_{\rho}$ and of $D/\overline{NE}$ and their cross-product to the specification (4.4-2).

Use of these nonlinear terms did little to improve the explanatory power of the equation, though in some instances they did have significant coefficients. Stability was found neither in which variables were significant nor in their signs. Since undoubtedly our variables have substantial measurement errors, these findings may well represent little more than the problems such errors produce.

It is not surprising in view of these findings that we sometimes found that breaking the sample into various groups produced significant differences between the groups. Thus, when the equation was run separately for low-dividend/high-growth and high-dividend/low-growth companies, (where the dividing lines are the medians of the variables), we did find some significant differences in coefficients. Similarly, fitting the equation for different industry groups produced some significant differences across industries in the coefficients (e.g., dividends were more highly valued in public utility companies). Since in each case the classifications tended to reduce the variances of the independent variables, the significant differences may arise simply from the changed importance of the variances of the measurement errors relative to the variances of the true underlying variables.

4.4.5 Measurement and Estimation Error

Allowing for errors of estimation in calculating the regression coefficients did not relieve the problems we encountered when we introduced the risk measures (based on regression coefficients) directly in estimating equation (4.4-1). Using either $\hat{\beta}_M$ or the $\hat{\gamma}$ coefficients defined in equation (4.3-4), whether alone or in conjunction with $s_{\bar{g}}^2$, produced neither stable nor significant coefficients for these variables when they were added to (4.4-2). It is far from clear that the reason for this finding was that such risk terms do not also play a role in valuation; in other words, we cannot conclude that a model with only one factor is appropriate. Instead, we may ascribe the findings, at least partially, to multicollinearity, particularly with the payout ratio. When these regression coefficients were added to the specification, the coefficient of $D/\overline{NE}$ usually became completely insignificant and it was highly correlated with the coefficients for β_M or for the $\hat{\gamma}_j$ coefficients. As we noted earlier, the growth variable $\bar{g}_{\rho}$ is also somewhat correlated with these risk proxies. In this connection, it is interesting to note that Rosenberg and Guy (1976) have suggested that both dividend payout and growth potential are important systematic risk variables.

Measurement errors are far from being confined to the risk variables. Clearly our growth variables are subject to error and the payout variable also is only an approximation to what the market could perceive to be the payout rate. These errors may account for some of the problems we have encountered.

As was also the case when we sought instruments for s_g^2, finding good instruments for the growth rate and the payout variables was not easy. We have already seen that $\bar{g}_e$ contains useful information not available from mechanically calculated growth rates. As a result, satisfactory instruments for it are unlikely to be found. We tried using past four- and ten-year calculated growth rates as instruments for $\bar{g}_e$ and the lagged value of $D/\overline{NE}$ for the current value of this variable. When we used the specification (4.4-1), we also included $\bar{E}_{t-1}/\overline{NE}$ as an instrumental variable. We could also take advantage of some of the correlations of risk with growth and payout by treating $\hat{\gamma}_m$ and $\hat{\gamma}_Y$ as additional instruments when only $\hat{\gamma}_e$ and s_g^2 were used as risk measures.

Using instrumental variables to deal with these measurement errors did not substantially alter our findings. What we obtained were equations qualitatively similar to those shown in tables 4.7 and 4.9, but with much larger standard errors for the coefficients. This finding may be taken to indicate, at least, that errors in variables have not produced seriously misleading results in those tables. When the problems of multicollinearity of the growth and dividend variables with the risk ones were combined with the complicated variances of the coefficients that were the result of making allowance for the estimation error of the risk parameters, it is small wonder that more precise results could not be obtained about the precise specification of risk.

4.4.6 Stability over Time

We found earlier that the coefficients of the expected return model varied over time. The question of the constancy of the valuation equation is particularly interesting in the present form, where prices are the dependent variable. Stability of the coefficients is also important to those who wish to make practical use of valuation equations in connection with assigned values of the independent variables to estimate the "intrinsic worth" of a security. Furthermore, constancy of the relationship is important if a firm is to seek to follow policies that will maximize the values of its shares, since it will find it hard to please investors if their desires are changing.

An inspection of tables 4.7 and 4.9 indicates that the coefficients of our equations do change considerably from year to year, and in a manner that is consistent with the changing standards of value in vogue at the different times. We may illustrate this finding by the regression results of table 4.9. At the end of 1961, "growth stocks" were in high favor, and it is not surprising to find that the coefficient of the growth rate (3.07) is highest in this year. During 1962, however, there was a conspicuous change in the structure of share prices that was popularly called "the revaluation of growth stocks." This revaluation is reflected in the decline of the growth-rate coefficient for 1962 to 1.99. At the same time, dividend payout became more highly valued in 1962 than it had been in 1961, the dividend coefficient rising from −1.58 to 6.96. Nineteen sixty-two was also the year when the coefficient of the risk measure was most strongly negative.

In order to test formally whether the coefficients of the valuation equation were the same over time, we again had to recognize that the residuals in different years were not independent. The correlations, which are shown in table 4.10, are somewhat smaller than those found in section 4.3 when we were investigating the expected rates of return, but they are significantly different from zero. They again raise the need to use an appropriate technique for assessing the stability of the coefficients and the problem that calculating all the equations simultaneously for a common set of companies entails the loss of a large proportion of the observations.

Using the seemingly unrelated regression technique for a pair of years, we could reject the hypothesis of equality of the coefficients in each pair of years at least at the 0.01 level. When all years were considered simultaneously, rejection occurred beyond the 0.0001 level despite the large loss of observations. Thus it seems clear that valuation relationships do change over time. While this finding may, of course, be due to problems with the data being used, it certainly lends no credence to the proposition that the parameters do not change.

4.5 Use of the Valuation Model for Security Selection

One of the most intriguing questions concerning empirical valuation models is whether they can be used to aid investors in security selection. The estimated valuation equation shows us, at a moment in time, the average way in which variables, such as growth, payout, and risk, influence market price-earnings multiples. Given the value of these vari-

Table 4.10 Correlations of Residuals in Adjacent Years and with Subsequent Returns

Year	Residuals from (4.4-1)	Residuals from (4.4-2)	Residuals of (4.4-2) with Future Returns
1961/62	.52	.62	−.20
1962/63	.56	.57	.09
1963/64	.41	.46	−.25
1964/65	.30	.39	−.06
1965/66	.37	.32	.06
1966/67	.50	.48	−.03
1967/68	.60	.64	−.10
1968/69	—	—	.20

ables applicable to any specific security, we can compute an estimated price-earnings ratio based on the empirical valuation equation. The next step is to compare the actual price-earnings multiple with that predicted by the valuation equation. If the actual multiple is greater than the predicted one, we might suppose that the security is temporarily over-priced and recommend sale. If the actual price-earnings multiple is less than the predicted multiple, we might designate the security as temporarily underpriced and recommend its purchase.

Even on a priori grounds, it is possible to think of many reasons why such a procedure would prove fruitless. For example, if high growth-rate stocks tended to be overpriced during one particular period, the estimated growth-rate coefficient would be larger (by assumption) than that which is warranted. However, the recommended procedure will not indicate that these stocks are overpriced because "normal" market-determined earnings multiples for these securities will be higher than is warranted. Nevertheless, in view of the popularity of these techniques with some practitioners, it seems worthwhile to try some experiments using our data.

The results of some of our experiments are shown in table 4.10. We measured the degree of "over-" or "underpricing" as the predicted ratio of the residual from the valuation equation (4.4-2) to the predicted earnings multiple, that is, as $(P/\overline{NE} - \hat{P}/\overline{NE})/(\hat{P}/\overline{NE})$. A percentage measure was chosen in view of the considerable variance in actual earnings multiples. If the model is useful in measuring underpricing, then underpriced securities, determined according to this criterion, ought to outperform overpriced issues over some subsequent period. We picked one year as the appropriate horizon and measured subsequent returns in the usual manner as

$$(4.5\text{-}1) \qquad P_{t+1} = (P_{t+1} - P_t + D_{t+1})/P_t.$$

If the empirical valuation model is successful in selecting securities for purchase, the percentage residual (degree of overvaluation) from the valuation equation ought to be negatively related to these subsequent returns. As the fourth column of table 4.10 indicates, in only five of the eight years for which this experiment was performed was the relationship negative, and the degree of association was low. There was a positive relationship for the other three years.[11] Two of these correlations are significant at the 0.05 level: the negative one in 1963/64 and the positive one for 1968/69. The 1961/62 correlation just misses significance at this level. We would not consider these significant correlations as representing forecasting success. As we argue below, we suspect strongly that we

11. We were no more successful at finding wrongly priced securities using expectations data for the individual predictors rather than the average expectations of the particular group.

have left out some common factors and that this omission could lead to correlations over particular periods of time. Unless one can forecast these changes in a way not already available to the general market participant, one can hardly exploit these changes. It is therefore particularly indicative that one of the significant correlations had the "wrong" sign.

Supplementary tests conducted by the type of equation or industry and other groupings produced similar results. For example, subsequent returns were still unrelated to the residuals when we first split the sample into high and low growth and dividend groupings. Similar results were obtained when the experiment was attempted for separate industries. We also found that the residuals from the equations employing historical data in place of our expectational data were no more successful in predicting subsequent performance. Moreover, these results were unaltered when the subsequent returns were measured over alternative time periods such as one-quarter ahead or two or more years ahead. The technique simply did not produce excess returns in any consistent or reliable fashion over any time period in the future. These findings are what we should expect in a reasonably efficient market.

Some statistics are presented in table 4.11 that may be helpful in interpreting the reason for our predictive failures. We note, using the 1963 valuation equation as an example, that the percentage degree of under- or overpricing is not highly correlated with subsequent returns, the coefficient of determination being only 0.06. It is possible to isolate four reasons for our lack of forecasting success.

1. The first reason is that the valuation relationship changes over time. We might be unable to select truly underpriced securities because by the next year the norms of valuation have been significantly altered. Thus what was cheap on the basis of the 1963 relationship may no longer represent good value on the basis of the 1964 equation. To test how important this change might be, we performed the following experiment: We assumed that investors knew at the end of 1963 exactly what the

Table 4.11 *Analysis of Lack of Forecasting Success*

Year	Description	r^{2*}
1963	Valuation equation with 1963 predictions	.06
1964	Valuation equation with 1963 data (assumes next year's valuation relationship is known)	.10
1963	Valuation equation with realized growth rates (assumes perfect foresight regarding future long-term growth and next year's earnings)	.14
1963	Valuation equation with 1964 predictions (assumes perfect foresight regarding market expectations next year)	.27

*Percent residuals versus 1964 return.

market valuation relationship would be for the end of 1964; that is, we assumed perfect foresight regarding next year's valuation equation. Then, on the basis of the 1964 valuation equation, we used the 1963 data to calculate warranted $P/\overline{NE}$ multiples, which could then be compared with actual multiples to determine whether each security was appropriately priced. Correlating the percentage residuals with subsequent returns, we found that the coefficient of determination nearly doubled, 10 percent of the variance in subsequent returns now being explained.

2. A second reason for lack of success might be the quality of the expectations data employed. As indicated in chapter 2, the growth-rate forecasts used in the present study were not accurate predictors of realized growth. To determine how much better off we would have been with more accurate forecasts, we assumed perfect foresight regarding the future long-term growth rate of the company. Thus the 1963 empirical valuation equation was used to determine "normal" value, but in place of $\bar{g}_p$ we substituted the realized long-term growth rate through 1968. Using these realized data to determine warranted price-earnings multiples, we correlated the percentage residuals therefrom with future returns. As expected, an even greater improvement in forecasting future returns was found. The r^2 rises to 0.14.

3. As a further experiment, perfect foresight was assumed not about the actual rate of growth of earnings but rather regarding what the market expectations of growth would be next year, that is, about $\bar{g}_p$ next year. Calculating the degree of overpricing as before, we find a much greater improvement in prediction of future returns. Twenty-seven percent of the variability of future returns is now explained, compared with only 6 percent in the original experiment. We conclude that if one wants to explain returns over a one-year horizon, it is far more important to know what the market will think the growth rate of earnings will be next year rather than to know the realized long-term growth rate. This observation brings us back to Keynes's celebrated newspaper contest. What matters is not one's personal criteria of beauty but what the average opinion will expect average opinion to think is beautiful at the close of the contest.

4. A final source of error is that the valuation model does not capture all the significant determinants of value for each individual company. Despite our success in accounting for approximately three-quarters of the variance in market price-earnings multiples, there are likely to be special features applicable to many individual companies that cannot be captured quantitatively. For example, it turned out that the stock of many tobacco companies always appeared to be underpriced. The reason for this is not difficult to conjecture. There is a risk of government sanctions against the tobacco industry that weighs heavily in the minds of investors, but that is not related to the risk measures we have employed. Such an explanation is not at variance with the underlying approach to risk

valuation that we have been using. The common susceptibility of the tobacco companies to an identifiable but ignored hazard is simply an important factor which we have omitted from our data.

This problem of omitted variables may account for the correlations of residuals which we found in the equations. If certain factors specific to individual companies were consistently missing, the residuals from the valuation equations could be expected to be positively correlated over time. This is exactly what we found in table 4.10. Thus, despite our success in using expectations data to estimate a valuation equation which has far more explanatory ability than those based on historic information, it is still quite clear that certain systematic valuation factors are missing from the analysis. Consequently, it cannot be said that all deviations of actual from predicted price-earnings ratios are simply manifestations of temporary over- or underpricing.

4.6 Conclusion

Our investigations of valuation models, while not without some ambiguous results, suggest several notable conclusions. These conclusions concern the role in market valuation of the sort of earnings forecasts we have collected, the nature of risk valuation, and the efficiency of the market.

4.6.1 Valuation of Expected Growth

One of our major findings is that the average of the expected long-term growth rates, together with the risk measure provided by the variance of the growth-rate predictions, gives a closer account of the valuation of common stocks than do alternatives. These growth rates were clearly superior in accounting for prices to any of the simple alternatives we considered. More closely fitting equations are the results that one would expect from smaller errors of measurement or from using data that contain more relevant information in place of less germane measures. Hence one can safely presume that our data are more similar to the expectations being valued in the market than are measures based on *ex post* realized growth or regression coefficients. This conclusion, based on the ability to "explain" prices, is buttressed by noticing that the overall risk-free expected rates of return suggested by the estimates of the expected return regressions are of plausible orders of magnitude.

The finding that prices reflect expected growth occurred in spite of the difficulties we encountered from the large variations in which companies were covered by each of the various predictors. Earlier we saw that there is a great deal of diversity of expectations among forecasters, an aspect of reality with which valuation models do not usually cope. We also found that, while hardly being strong predictions, the expectations data appear to yield forecasts at least as accurate as, and often better than, naive forecasts based on *ex post* realizations. Furthermore, we found that we could not calculate a linear combination of different types of forecasts whose superior forecasting performance continued over time.

Efficient market hypotheses suggest that valuation should reflect the information available to investors. Insofar as analysts' forecasts are more precise than other types we should therefore expect their differences from other measures to be reflected in the market. It is therefore noteworthy that our regression results do support the hypothesis that analysts' forecasts are needed even when calculated growth rates are available. As we noted when we described the data, security analysts do not use simple mechanical methods to obtain their evaluations of companies. The growth-rate figures we obtained were distilled from careful examination of all aspects of the companies' records, evaluation of contingencies to which they might be subject, and whatever information about their prospects the analysts could glean from the companies themselves or from other sources. It is therefore notable that the results of their efforts are found to be so much more relevant to the valuation than the various simpler and more "objective" alternatives that we tried.

We saw in section 3.2.3 that diversity of expectations together with market imperfections might invalidate the valuation model. However, we also argued that there were theoretical grounds for supposing that the model would still hold for the average of investors' expectations. It is therefore of particular interest that our empirical results do support the hypothesis that prices reflect average expectations.

It is no surprise that we found roles for both short- and long-term expected rates of growth. Models of valuation using only long-term growth rates are clearly only simplifications of the more complicated processes that earnings and dividends follow over time, and we would expect market valuation to reflect the more complicated processes.

4.6.2 Risk Measures and Valuation

The results did not provide wholly unambiguous support for the specific valuation models developed here. A number of aspects of our results about risk are particularly intriguing. It is clear from our results that expected returns do seem to be related to various systematic risk factors. Equally clearly, our results do not give straightforward support to the simple form of the CAPM. It would appear that systematic risk is not entirely captured by single measures of covariance with the market index. This has important implications for those who attempt to use the modern investment technology in practical problems of portfolio selection. One such suggestion, which had attracted a considerable following in the investment community by the 1980s, was the proposal for a yield-tilted index fund.

The reasoning behind the yield-tilted index fund seems appealingly plausible. Since dividends are generally taxed more highly than capital gains and since the market equilibrium is presumably achieved on the basis of after-tax returns, the equilibrium pretax returns for stocks that pay high dividends ought to be higher than for securities that produce lower dividends and correspondingly higher capital gains. Hence the tax-exempt investor is advised to buy a diversified portfolio of high-dividend-paying stocks. In order to avoid the assumption of any greater risk than is involved in buying the market index, the tax-exempt investor is also advised to purchase a yield-tilted index fund, that is, a very broadly diversified portfolio of high-dividend-paying stocks that mirrors the market index in the sense that it has a beta coefficient β_M precisely equal to unity.

Even on a priori grounds one might question the logic of the yield-tilted index fund. Many of the largest investors in the market are tax-exempt (such as pension and endowment funds) and others (such as corporations) actually pay a lower tax on capital gains than on dividend income.[12] Thus it is far from clear that the marginal investor in the stock market prefers to receive income through capital gains rather than through dividend payments. Our theoretical arguments in chapter 3 also indicated that great care must be taken with arguments involving "marginal" investors and pointed out that the diversification theory gives no presumption that dividends and capital gains will be valued differently. But apart from these a priori arguments, our empirical results can be interpreted as providing another argument against the yield-tilted index fund.

If the traditional beta calculation (β_M) does not provide a full description of systematic risk, the yield-tilted index fund may well fail to mirror the market index. Specifically, during periods when inflation and interest rates rise, it may well be the case that high-dividend stocks are particularly vulnerable: that is, they have high δ_p and δ_r coefficients. Public-utility common stocks are a good example. While they are known as "low-beta" stocks, they are likely to have high systematic risk with respect to interest rates and inflation. This is so not only because they are good substitutes for fixed-income securities, but also because public utilities are vulnerable to a profits squeeze during periods of rising inflation because of regulatory lags and increased borrowing costs. Hence the yield-tilted index fund with $\beta_M = 1$ may not mirror the market index when inflation accelerates.

The actual experience of yield-tilted index funds during the 1979–80 period shows that these funds did not live up to expectations and their performance was significantly worse than the market. Of course, we should not reject a model simply because of its failure over any specific short-term period. Nevertheless, we believe that an understanding of the wider aspects of systematic risk, such as those analyzed here, would have

12. For corporate investors, 85 percent of dividend income is excluded from taxable income while capital gains are taxed at normal gains rates.

helped prevent what turned out to be (at least over the short term) some serious investment errors.

Our findings on systematic risk still leave some major and intriguing perplexities. We found in both versions of the valuation model that the most important aspect of risk for valuation was that represented by the extent to which forecasters were not in agreement about the future growth of the company. Exactly what is the basis for this finding is not clear.

It might be quite reasonable to interpret s_g^2 as representing specific risk. In that case, the findings go against most recent models of valuation including both the CAPM and the APT. On the other hand, it may indirectly measure sensitivity to underlying common factors and thus serve as a very effective proxy for a variety of systematic risks. Finally, it may arise from technical difficulties having to do with undetected biases in our data. It seems unlikely that this would fully account for the strength we found for this variable, but it cannot be ruled out. Further investigation probably requires a data set less beset by problems of missing observations and an adequately specified model of earnings. Overall, our results do suggest that risk undoubtedly has dimensions not fully captured by the covariances with market indexes or other variables that have dominated recent work on valuation. They also suggest that the variance of analysts' forecasts may represent the most effective risk proxy available.

4.6.3 Efficient Markets

We find it encouraging that we were unable to use the expectations data to select securities with subsequent above- or below-average performance characteristics. We would not expect that analysts' forecasts would be sounder than those apparently used by the market or that they would be irrelevant to market valuations. Apparently, the expectations formed by Wall Street professionals get quickly and thoroughly impounded into the prices of securities. Implicitly, we have found that the evaluations of companies that analysts make are the sorts of ones on which market valuation is based. Thus, while our work raises questions about some currently popular valuation theories, it strongly supports the view that the market is reasonably efficient in incorporating into present prices whatever information there is about the future.

Third Edition

STOCKS FOR THE LONG RUN

The Definitive Guide to Financial Market Returns and Long-Term Investment Strategies

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whereas investors regarded stocks as the best investment to protect against the eroding value of money. As early as September 1958, *BusinessWeek* noted that "the relationship between stock and bond yields was clearly posting a warning signal, but investors still believe inflation is inevitable and stocks are the only hedge against it."[3]

Yet many on Wall Street were still puzzled by the "great yield reversal." Nicholas Molodovsky, vice president of White, Weld & Co. and editor of the *Financial Analysts Journal*, observed:

> Some financial analysts called . . . [the reversal of bond and stock yields] a financial revolution brought about by many complex causes. Others, on the contrary, made no attempt to explain the unexplainable. They showed readiness to accept it as a manifestation of providence in the financial universe.[4]

Imagine value-oriented investors who pulled all their money out of the stock market in August of 1958 and put it into bonds, vowing never to buy stocks again unless dividend yields rose above those on high-quality bonds. Such investors would still be waiting to get back into stocks. After 1958, stock dividend yields never again exceeded those of bonds. Yet, from August 1958 onward, overall stock returns overwhelmed the returns on fixed-income securities over any long-term period.

Benchmarks for valuation are valid only as long as economic institutions do not change. The chronic postwar inflation, resulting from the switch to a paper money standard, changed forever the way investors judged the yields on stocks and bonds. Investors who clung to the old ways of valuing equity never participated in the greatest bull market for stocks in history.

VALUATION OF CASH FLOWS FROM STOCKS

The fundamental sources of stock valuation are the dividends and earnings of firms. In contrast to a work of art—which can be bought both for an investment and for its viewing pleasure—stocks have value only because of the potential cash flows that investors receive. These cash flows can come from any distribution (such as dividends or capital gains realized on sale) that stockholders expect to receive from their share of ownership of the firm, and it is by forecasting and valuing

[3] "In the Markets," *BusinessWeek*, September 13, 1958, p. 91.
[4] "The Many Aspects of Yields," *Financial Analysts Journal* 18(2)(March–April 1962):49–62.

these expected future cash flows that one can judge the investment value of shares.[5]

The value of any asset is determined by the discounted value of all expected future cash flows. Future cash flows from assets are *discounted* because cash received in the future is not worth as much as cash received in the present. The reasons for discounting are (1) the innate *time preferences* of most individuals to enjoy their consumption today rather than wait for tomorrow, (2) *productivity*, which allows funds invested today to yield a higher return tomorrow, and (3) *inflation*, which reduces the future purchasing power of cash received in the future. These factors also apply to both stocks and bonds and are the foundation of the theory of interest rates. A fourth reason, which applies primarily to the cash flows from equities, is the *uncertainty* associated with the magnitude of future cash flows.

SOURCES OF SHAREHOLDER VALUE

For the equity holder, the source of future cash flows is the earnings of firms. Earnings are the cash flows that remain after the costs of production are subtracted from the sales revenues of the firm. The costs of production include labor and material costs, interest on debt, corporate taxes, and allowance for depreciation.

Earnings create value for shareholders by the:

- Payment of cash dividends
- Repurchase of shares
- Retirement of debt
- Investment in securities, capital projects, or other firms

If a firm repurchases its shares (known as *buybacks*), it reduces the number of shares outstanding and thus increases future *per-share* earnings. If a firm retires its debt, it reduces its interest expense and therefore increases the cash flow available to shareholders. Finally, earnings that are not used for dividends, share repurchases, or debt retirement are referred to as *retained earnings*. Retained earnings may increase future cash flows to shareholders if they are invested productively in securities, capital projects, or other firms.

[5] There might be some psychic value to holding a controlling interest above and beyond the returns accrued. In such a case, the owner values the stock more than minority shareholders.

Some people argue that shareholders most value stocks' cash dividends. But this is not necessarily true. In fact, from a tax standpoint, share repurchases are superior to dividends. Cash dividends are taxed at the highest marginal tax rate to the investor; share repurchases, however, generate capital gains that can be realized at the shareholder's discretion and at a lower capital gains tax rate. Recently, there have been an increasing number of firms who engage in share repurchases. As will be discussed in the next chapter, the shift from dividends to share repurchases is one factor that has raised the valuation of some equities.

Others might argue that debt repayment lowers shareholder value because the interest saved on the debt retired generally is less than the rate of return earned on equity capital. They also might claim that by retiring debt, they lose the ability to deduct the interest paid as an expense (the interest tax shield).[6] However, debt entails a fixed commitment that must be met in good or bad times and, as such, increases the volatility of earnings that go to the shareholder. Reducing debt therefore lowers the volatility of future earnings and may not diminish shareholder value.[7]

Many investors claim that the fourth factor, the reinvestment of earnings, is the most important source of value, but this is not always the case. If retained earnings are reinvested profitably, value surely will be created. However, retained earnings may tempt managers to pursue other goals, such as overbidding to acquire other firms or spending on perquisites that do not increase the value to shareholders. Therefore, the market often views the buildup of cash reserves and marketable securities with suspicion and frequently discounts their value.

If the fear of misusing retained earnings is particularly strong, it is possible that the market will value the firm at less than the value of its reserves. Great investors, such as Benjamin Graham, made some of their most profitable trades by purchasing shares in such companies and then convincing management (sometimes tactfully, sometimes with a threat of takeover) to disgorge their liquid assets.[8]

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[6]Whether debt is a valuable tax shield depends on whether interest rates are bid up enough to offset that shield. See Merton H. Miller, "Debt and Taxes," Papers and Proceedings of the Thirty-Fifth Annual Meeting of the American Finance Association, Atlantic City, NJ, September 16–18, 1977, *The Journal of Finance* 32(2)(May, 1977):261–275.

[7]Meeting interest payments also may be a good discipline for management and reduce the tendency to waste excess profits. See Michael Jensen, "The Takeover Controversy: Analysis and Evidence." In John Coffee, Louis Lowenstein, and Susan Rose-Ackerman (eds.), *Takeovers and Contests for Corporate Control* (New York: Oxford University Press, 1987).

[8]Benjamin Graham, *The Memoirs of the Dean of Wall Street* (New York: McGraw-Hill, 1946), Chap. 11.

One might question why management would not employ assets in a way to maximize shareholder value, since managers often hold a large equity stake in the firm. The reason is that there may exist a conflict between the goal of the shareholders, which is solely to increase the return on the company's shares, and the goals of management, which may include prestige, control of markets, and other objectives. Economists recognize the conflict between the goals of managers and shareholders as *agency costs*, and these costs are inherent in every corporate structure where ownership is separated from management. Payment of cash dividends or committed share repurchases often lowers management's temptation to pursue goals that do not maximize shareholder value.

In recent years dividend yields have fallen to 1½ percent, less than one-third of their historic average. The major reasons for this are the tax disadvantage of dividends and the increase in employee stock options, where capital gains and not dividends figure into option value. Nevertheless, dividends historically have served the function of showing investors that the firms' earnings were indeed real. Recent concerns about aggressive accounting policies and the integrity of earnings following the Enron debacle may bring back this once-favored way of delivering investor value.[9]

DOES THE VALUE OF STOCKS DEPEND ON DIVIDENDS OR EARNINGS?

Management determines its dividend policy—the fraction of earnings it will pay out to shareholders—by evaluating many factors, including the tax differences between dividend income and capital gains, the need to generate internal funds to retire debt or invest, and the desire to keep dividends relatively constant in the face of fluctuating earnings. Since the price of a stock depends primarily on the present discounted value of all expected future dividends, it appears that dividend policy is crucial to determining the value of the stock.

However, this is not generally true. It does not matter how much is paid as dividends and how much is reinvested *as long as* the firm earns the same return on its retained earnings that shareholders demand on its stock.[10] The reason for this is that dividends not paid today are reinvested by the firm and paid as even larger dividends in the future.

[9]Jeremy J. Siegel, "The Dividend Deficit," *Wall Street Journal*, February 13, 2002, p. A20.
[10]This ignores differential taxation between capital gains and dividend income that favors reinvestment. This is explored in Chapter 4.

Of course, management's choice of dividend payout ratio, which is the ratio of cash dividends to total earnings, does influence the timing of the dividend payments. The lower the dividend payout ratio, the smaller the dividends will be in the near future. Over time, however, dividends will rise and eventually will exceed the dividend path associated with a higher payout ratio. Moreover, assuming that the firm earns the same return on investment as the investors require from its equity, the present value of these dividend streams will be identical no matter what payout ratio is chosen.

Note that the price of the stock is always equal to the present value of all future *dividends* and not the present value of future earnings. Earnings not paid to investors can have value only if they are paid as dividends or other cash disbursements at a later date. Valuing stock as the present discounted value of future earnings is manifestly wrong and greatly overstates the value of a firm.[11]

John Burr Williams, one of the greatest investment analysts of the early part of last century and author of the classic *The Theory of Investment Value*, argued this point persuasively in 1938. He wrote:

> Most people will object at once to the foregoing formula for valuing stocks by saying that it should use the present worth of future earnings, not future dividends. But should not earnings and dividends both give the same answer under the implicit assumptions of our critics? If earnings not paid out in dividends are all successfully reinvested at compound interest for the benefit of the stockholder, as the critics imply, then these earnings should produce dividends later; if not, then they are money lost. Earnings are only a means to an end, and the means should not be mistaken for the end.[12]

LONG-TERM EARNINGS GROWTH AND ECONOMIC GROWTH

Since stock prices are the present value of future dividends, it would seem natural to assume that economic growth would be an important factor influencing future dividends and hence stock prices. However, this is not necessarily so. The determinants of stock prices are earnings and dividends on a *per-share* basis. Although economic growth may influence *aggregate* earnings and dividends favorably, economic growth does not necessarily increase the growth of per-share earnings or dividends. It is earnings per share (EPS) that is important to Wall Street be-

[11]Firms that pay no dividends, such as Warren Buffett's Berkshire Hathaway, have value because their assets, which earn cash returns, can be liquidated and disbursed to shareholders in the future.
[12]John Burr Williams, *The Theory of Investment Value* (Cambridge, MA: Harvard University Press, 1938), p. 30.

cause per-share data, not aggregate earnings or dividends, are the basis of investor returns.

The reason that economic growth does not necessarily increase EPS is because economic growth requires increased capital expenditures and this capital does not come freely. Implementing and upgrading technology requires substantial firm investment. These expenditures must be funded either by borrowing in the debt market (through banks or trade credit or by selling bonds) or by floating new shares. The added interest costs and the dilution of profits that this funding involves place a burden on the firm's bottom line.

Can earnings increase without increasing capital expenditures? In the short run, this may occur, but the long-run historical evidence suggests that it will not. One of the signal characteristics of long-term historical data is that the level of the capital stock—the total value of all physical capital such as factories and equipment, as well as intellectual capital, that has accumulated over time—has grown in proportion to the level of aggregate output. In other words, a 10 percent increase in output requires a 10 percent increase in the capital stock.

Many investors believe that investment in productivity-enhancing technology can spur earnings growth to permanently higher levels. However, "cost-saving investments," frequently touted as a source of increasing profit margins, only temporarily affect bottom-line earnings. As long as these investments are available to other firms, competition will force management to reduce product prices by the amount of the cost savings, and extra profits will quickly be competed away. In fact, capital expenditures often are undertaken not necessarily to *enhance* profits but rather to *preserve* profits when other firms have adopted competitive cost-saving measures.

Table 6-1 shows the summary statistics for dividends per share, earnings per share (EPS), and stock returns from 1871 through Septem-

TABLE 6–1

Long-Term Growth of GDP, Earnings, and Dividends, 1871–2001

	Real GDP Growth	Real Per-Share Earnings Growth	Real Per-Share Dividend Growth	Dividend Yield*	Payout Ratio*
1871-2001	3.91%	1.25%	1.09%	4.54%	58.75%
1871-1945	4.51%	0.66%	0.74%	5.07%	66.78%
1946-2001	3.11%	2.05%	1.56%	3.53%	51.91%

* Denotes median.

Do Analyst Conflicts Matter? Evidence from Stock Recommendations

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Abstract

We examine whether conflicts of interest with investment banking and brokerage businesses induce sell-side analysts to issue optimistic stock recommendations and, if so, whether investors are misled by such biases. Using quantitative measures of potential conflicts constructed from a novel data set containing revenue breakdowns of analyst employers, we find that recommendation levels are indeed positively related to conflict magnitudes. The optimistic bias stemming from investment banking conflicts was especially pronounced during the late-1990s stock market bubble. However, evidence from the response of stock prices and trading volumes to upgrades and downgrades suggests that the market recognizes analysts' conflicts and properly discounts analysts' opinions. This pattern persists even during the bubble period. Moreover, the 1-year stock performance following revised recommendations is unrelated to the magnitude of conflicts. Overall, our findings do not support the view that conflicted analysts are able to systematically mislead investors with optimistic stock recommendations.

1. Introduction

In April 2003, 10 of the largest Wall Street firms reached a landmark settlement with state and federal securities regulators on the issue of conflicts of interest

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faced by stock analysts.[1] The settlement requires the firms to pay a record $1.4 billion in compensation and penalties in response to government charges that the firms issued optimistic stock research to win favor with potential investment banking (IB) clients. Part of the settlement funds are earmarked for investor education and for provision of research from independent firms. In addition to requiring large monetary payments, the settlement mandates structural changes in the firms' research operations and requires the firms to disclose conflicts of interest in analysts' research reports.

The notion that investors are victims of biased stock research presumes that (1) analysts respond to the conflicts by inflating their stock recommendations and (2) investors take analysts' recommendations at face value. Even if analysts are biased, it is possible that investors understand the conflicts of interest inherent in stock research and rationally discount analysts' opinions. This alternative viewpoint, if accurate, would lead to very different conclusions about the consequences of analysts' research. Indeed, investors' rationality and self-interested behavior imply that stock prices should accurately reflect a consensus about the informational quality of public announcements (Grossman 1976; Grossman and Stiglitz 1980). Rational investors would recognize and adjust for analysts' potential conflicts of interest and thereby largely avoid the adverse consequences of biased stock recommendations.

In this article, we provide evidence on the extent to which analysts and investors respond to conflicts of interest in stock research. We address four questions. First, is the extent of optimism in stock recommendations related to the magnitudes of analysts' conflicts of interest? Second, to what extent do investors discount the opinions of more conflicted analysts? In particular, do stock prices and trading volumes react to recommendation revisions in a manner that rationally reflects the degree of analysts' conflicts? Third, is the medium-term (that is, 3- to 12-month) performance of recommendation revisions related to conflict severity? And, finally, did conflicts of interest affect analysts or investors differently during the late-1990s stock bubble than during the postbubble period? The answers to these questions are clearly of relevance to stock market participants, public policy makers, regulators, and the academic profession.

We use a unique, hand-collected data set that contains the annual revenue breakdown for 232 public and private analyst employers. This information allows us to construct quantitative measures of the magnitude of potential conflicts not only from IB business but also from brokerage business. We analyze a sample of over 110,000 stock recommendations issued by over 4,000 analysts during the 1994–2003 time period. Using univariate tests as well as cross-sectional regressions that control for the size of the company followed and individual analysts' experience, resources, workloads, and reputations, we attempt to shed

[1] Two more securities firms (Deutsche Bank Securities Inc. and Thomas Weisel Partners LLC) were added to the formal settlement in August 2004.

light both on how analysts respond to pressures from IB and brokerage businesses and on how investors compensate for the existence of such conflicts of interest.

A number of studies (for example, Dugar and Nathan 1995; Lin and McNichols 1998; Michaely and Womack 1999; Dechow, Hutton, and Sloan 2000; Bradley, Jordan, and Ritter 2008) focus on conflicts faced by analysts in the context of existing underwriting relationships (see also Malmendier and Shanthikumar 2007; Cliff 2007).[2] Our article complements this literature in several ways. First, we take into account the pressure to generate underwriting business from both current and potential client companies. Even if an analyst's firm does not currently do IB business with a company that the analyst tracks, it might like to do so in the future. Second, we examine the conflict between research and all IB services (including advice on mergers, restructuring, and corporate control), rather than just underwriting. Third, we examine conflicts arising from brokerage business in addition to those from IB.[3]

Fourth, the prior empirical finding that underwriter analysts tend to be more optimistic than other analysts is consistent with two alternative interpretations: (*a*) an optimistic report on a company by an underwriter analyst is a reward for past IB business or an attempt to win future IB business by currying favor with the company or (*b*) a company chooses an underwriter whose analyst already likes the stock. The second interpretation implies that underwriter choice is endogenous and does not necessarily imply a conflict of interest. We sidestep this issue of endogeneity by not focusing on underwriting relations between an analyst's firm and the company followed. Instead, our conflict measures focus on the importance to the analyst's firm of IB and brokerage businesses, as measured by the percentage of its annual revenue derived from IB business and from brokerage commissions. Unlike underwriting relations between an analyst's firm and the company followed, the proportions of the entire firm's revenues from each of these businesses can reasonably be viewed as given, exogenous variables from the viewpoint of an individual analyst. Finally, our approach yields substantially larger sample sizes than those used in prior research, and it therefore leads to greater statistical reliability of the results.

Several articles adopt an approach that is similar in spirit to ours. For example, Barber, Lehavy, and Trueman (2007) find that recommendation upgrades (downgrades) by investment banks—which typically also have brokerage businesses—

[2] Bolton, Freixas, and Shapiro (2007) theoretically analyze a different type of conflict of interest in financial intermediation, one faced by a financial advisor whose firm also produces financial products (such as in-house mutual funds). Mehran and Stulz (2007) provide an excellent review of the literature on conflicts of interest in financial institutions.

[3] Hayes (1998) analyzes how pressure on analysts to generate brokerage commissions affects the availability and accuracy of earnings forecasts. Both Irvine (2004) and Jackson (2005) find that analysts' optimism increases a brokerage firm's share of the trading volume. Ljungqvist et al. (2007) find that analysts employed by larger brokerage houses issue more optimistic recommendations and more accurate earnings forecasts. However, none of these articles examines how investors' responses to analysts' recommendations and the investment performance of recommendations vary with the severity of brokerage conflicts, issues that we investigate here.

underperform (outperform) similar recommendations by non-IB brokerages and independent research firms. Cowen, Groysberg, and Healy (2006) find that full-service securities firms—which have both IB and brokerage businesses—issue less optimistic forecasts and recommendations than do non-IB brokerage houses. Finally, Jacob, Rock, and Weber (2008) find that short-term earnings forecasts made by investment banks are more accurate and less optimistic than those made by independent research firms. We extend this line of research by quantifying the reliance of a securities firm on IB and brokerage businesses. This is an important feature of our article for at least two reasons. First, given that many securities firms operate in multiple lines of business, it is difficult to classify them by business lines. By separately measuring the magnitudes of both IB and brokerage conflicts in each firm, our approach avoids the need to rely on a classification scheme. Second, since the focus of this research is on the consequences of analysts' conflicts, the measurement of those conflicts is important. Our conclusions sometimes differ from those in classification-based studies.

We find that analysts do indeed seem to respond to pressures from IB and brokerage businesses: larger potential conflicts of interest from these businesses are associated with more positive stock recommendations. We also document that the distortive effects of IB conflicts were larger during the late-1990s stock bubble than during the postbubble period. Nonetheless, the empirical analysis yields several pieces of evidence to suggest that investors are sophisticated enough to adjust for these biases. First, the short-term reactions of both stock prices and trading volumes to recommendation upgrades are negatively and statistically significantly related to the magnitudes of potential IB or brokerage conflicts. For downgrades, the corresponding relation is negative for stock prices but positive for trading volumes. Second, the 1-year investment performance after recommendation revisions bears no systematic relation to the magnitude of conflicts. Finally, investors continued to discount conflicted analysts' opinions during the bubble period, even amid the euphoria prevailing in the market at the time. Together these results strongly support the idea that the marginal investor, taking analysts' conflicts into account, rationally discounts optimistic stock recommendations.[4]

The remainder of the article is organized as follows. We discuss the issues in Section 2 and describe our sample and data in Section 3. Section 4 examines the relation between recommendation levels and the degree of IB or brokerage conflict faced by analysts. Section 5 analyzes how conflicts are related to the response of stock prices or trading volumes to recommendation revisions. Section

[4] In a companion paper (Agrawal and Chen 2005), we find that analysts appear to respond to conflicts when making long-term earnings growth projections but not short-term earnings forecasts. This finding is consistent with the idea that, with short-term forecasts, analysts worry about their deception being revealed with the next quarterly earnings release, but they have greater leeway with long-term forecasts. We also find that the frequency of forecast revisions is positively related to the magnitude of brokerage conflicts, and several tests suggest that analysts' trade generation incentives impair the quality of stock research.

6 investigates the relation between conflicts and the investment performance of recommendation revisions. Section 7 presents our results for the late-1990s stock bubble and postbubble periods, and Section 8 concludes.

2. Issues and Hypotheses

Investment banking activity is a potential source of analyst conflict that has received widespread attention in the financial media (for example, Gasparino 2002; Maremont and Bray 2004) as well as the academic literature (for example, Lin and McNichols 1998; Michaely and Womack 1999). When IB business is an important source of revenue for a securities firm, a stock analyst employed by the firm often faces pressure to inflate his or her recommendations. This pressure is due to the fact that the firm would like to sell IB services to a company that the analyst tracks.[5] The company, in turn, would like the analyst to support its stock with a favorable opinion. Thus, we expect that the more critical is IB revenue to an analyst's employer, the greater the incentives an analyst faces to issue optimistic recommendations.[6]

Analysts also face a potential conflict with their employers' brokerage businesses. Here, the pressure on analysts originates not from the companies that they follow but from within their employing firms. Brokerage business generates a large portion of most securities firms' revenues, and analyst compensation schemes are typically related explicitly or implicitly to trading commissions. Thus, analysts have incentives to increase trading volumes in both directions (that is, buys and sells). Given the many institutional constraints that make short sales relatively costly, many more investors participate in stock purchases than in stock sales.[7] Indeed, it is mostly existing shareholders of a stock who sell. This asymmetry between purchases and sales implies that the more important brokerage business is to an analyst's employer, the more pressure the analyst faces to be bullish when issuing recommendations.

Analysts who respond to the conflicts they face by issuing blatantly misleading stock recommendations can develop bad reputations that reduce their labor income and hurt their careers.[8] Stock recommendations, however, are not as easily evaluated as other outputs of analysts' research, such as 12-month price targets or quarterly earnings forecasts, which can be judged against public, near-

[5] Throughout this article, we refer to an analyst's employer as a "firm" and a company followed by an analyst as a "company."

[6] Ljungqvist, Marston, and Wilhelm (2006, forthcoming) find that, while optimistic recommendations do not help the analyst's firm win the lead underwriter or comanager positions in general, they help the firm win the comanager position in deals in which the lead underwriter is a commercial bank.

[7] Numerous regulations in the United States increase the cost of selling shares short (see, for example, Dechow et al. 2001). Therefore, the vast majority of stock sales are regular sales rather than short sales. For example, over the 1994–2001 period, short sales comprised only about 10 percent of the annual New York Stock Exchange trading volume (New York Stock Exchange 2002).

[8] See Jackson (2005) for a theoretical model showing that analysts' concerns about their reputations can reduce optimistic biases arising from brokerage business.

term realizations. So it is not clear whether analysts' career concerns can completely prevent them from responding to pressures to generate IB or brokerage business.

The relation between conflict severity and the short-term (2- or 3-day) stock price impact of a recommendation should depend on whether investors react to the opinion rationally or naively.[9] Under the rational discounting hypothesis, the relation should be asymmetric for upgrades and downgrades. For upgrades, the stock price response should be negatively related to the degree of conflict. This implication arises because analysts who face greater pressure from IB or brokerage business are likely to be more bullish in their recommendations, and rational investors should discount an analyst's optimism more heavily. For downgrades, however, the story is different. When an analyst downgrades a stock despite facing large conflicts, rational investors should find the negative opinion more convincing and should be more likely to revalue the stock accordingly. This implies that the short-term stock price response to a downgrade should be negatively related to the degree of conflict.

The rational discounting hypothesis also predicts cross-sectional relations between conflict severity and the short-term trading volume responses to recommendations. As Kim and Verrecchia (1991) demonstrate in a rational expectations model of trading, the more precise a piece of news, the more individuals will revise their prior beliefs and, hence, the more trading that will result. In the present context, investor rationality implies that an upgrade by a highly conflicted analyst represents less precise news to investors, and so such a revision should be followed by a relatively small abnormal volume. But when an analyst downgrades a stock despite a substantial conflict, the signal is regarded as being more precise, and thus the downgrade should lead to relatively large abnormal trading.

By contrast, under the naive investor hypothesis, investors are largely ignorant of the distortive pressures that analysts face and accept analysts' recommendations at face value. This implies that there should be no relation between conflict severity and the short-term response of either stock prices or trading volume to recommendation revisions. Furthermore, the absence of a systematic relation should hold true for both upgrades and downgrades.

What are the implications of the two hypotheses for the medium-term (3- to 12-month) investment performance of analysts' recommendations? Under the rational discounting hypothesis, there should be no systematic relation between the magnitude of conflicts faced by an analyst and the performance of his or her stock recommendations: the market correctly anticipates the potential distortions up front and accordingly adjusts its response. But the naive investor hypothesis predicts that performance should be negatively related to conflict

[9] This framework follows Kroszner and Rajan (1994) and Gompers and Lerner (1999), who analyze the conflicts that a bank faces in underwriting securities of a company when the bank owns a (debt or equity) stake in it.

severity for both upgrades and downgrades. That is, investors ignore analysts' conflicts up front and pay for their ignorance later.

3. Sample and Data

3.1. Sample

Our sample of stock recommendations comes from the Institutional Brokers Estimate System (I/B/E/S) U.S. Detail Recommendations History file. This file contains data on newly issued recommendations as well as revisions and reiterations of existing recommendations made by individual analysts over the period 1993–2003. Although the exact wording of recommendations can vary considerably across brokerage houses, I/B/E/S classifies all recommendations into five categories ranging from strong buy to strong sell. We rely on the I/B/E/S classification and encode recommendations on a numerical scale from 5 (strong buy) to 1 (strong sell).

Since we are primarily interested in examining how the nature and consequences of analysts' recommendations are related to IB or brokerage business, we require measures of the importance of these business lines to analysts' employers. Under U.S. law, all registered broker-dealer firms must file audited annual financial statements with the Securities and Exchange Commission (SEC) in x-17a-5 filings.[10] These filings contain information on broker-dealer firms' principal sources of revenue, broken down into revenue from IB, brokerage commissions, and all other businesses (such as asset management and proprietary trading). We use these filings to obtain various financial data, including data on our key explanatory variables: the fractions of total brokerage house revenues from IB and from brokerage commissions. Beginning with the names of analyst employers contained in the I/B/E/S Broker Translation file,[11] we search for all available revenue information in x-17a-5 filings from 1994 to 2003.[12] For publicly traded broker-dealer firms, we also use 10-K annual report filings over the sample period to gather information on revenue breakdowns, if necessary. We thus obtain annual data from 1994 to 2003 on IB revenue, brokerage revenue, and other revenue for 188 privately held and 44 publicly traded brokerage houses.[13] For each brokerage house, we match recommendations to the latest broker-year revenue data preceding the recommendation date. Over the sample period, we

[10] The Securities Exchange Act, sections 17(a)–17(e), requires these filings. We accessed them from Thomson Financial's Global Access database and the Securities and Exchange Commission's (SEC's) public reading room in Washington, D.C.

[11] We use the file supplied directly by the Institutional Brokers Estimate System (I/B/E/S) on CD-ROM. This file does not recode the name of an acquired brokerage firm to that of its acquirer for years before the merger.

[12] The electronic availability of x-17a-5 filings is very limited prior to 1994, the year the SEC first mandated electronic form filing. Hence, we do not search for revenue information prior to 1994.

[13] We exclude a small number of firm-years in which the total revenue is negative (for example, because of losses from proprietary trading).

are able to match in this fashion 110,493 I/B/E/S recommendations issued by 4,089 analysts.

All broker-dealer firms are required to publicly disclose their balance sheets as part of their x-17a-5 filings. But a private broker-dealer firm can withhold the public disclosure of its income statement, which contains the revenue breakdown information needed for this study, if the SEC deems that such disclosure would harm the firm's competitive position. Thus, our sample of private securities firms is limited to broker-dealers that disclose their revenue breakdowns in x-17a-5 filings. We examine whether this selection bias affects our main results by separately analyzing the subsample of publicly traded securities firms, for which public disclosure of annual revenue information is mandatory. Our findings do not appear to be affected by this selection bias. All of our results for the subsample of publicly traded securities firms are qualitatively similar to the results for the full sample reported in the article. In the Appendix, we describe the characteristics of disclosing and nondisclosing private securities firms, shed some light on the firms' income statement disclosure decisions, and use a selectivity-corrected probit model to examine whether the resulting selection bias can explain analysts' response to conflicts in these private firms. We find no evidence that selection bias affects our results for these firms.

3.2. Characteristics of Analysts, Their Employers, and Companies Followed

We next measure characteristics of analysts, their employers, and the companies they cover. Prior research (for example, Clement 1999; Jacob, Lys, and Neale 1999) finds that analysts' experience and workloads affect the accuracy and credibility of their research. Using the I/B/E/S Detail History files, we measure an analyst's experience and workloads in terms of all research activity reported in I/B/E/S, including stock recommendations, quarterly and annual earnings-per-share forecasts, and long-term earnings growth forecasts. We measure general research experience as the number of days since an analyst first issued research on any company in the I/B/E/S database and company-specific research experience as the number of days since an analyst first issued research on a particular company. We measure an analyst's workload as the number of different companies or the number of different four-digit I/B/E/S sector industry groups (S/I/Gs)[14] for which the analyst issued research in a given calendar year.

The amount of resources devoted to investment research within brokerage houses also affects the quality of analysts' research (Clement 1999). Larger houses have access to better technology, information, and support staff. Accordingly, we use three measures of brokerage house size: the number of analysts issuing stock recommendations for a brokerage house over the course of a calendar year, book value of total assets, and net sales. All of our subsequent results are qual-

[14] The I/B/E/S sector industry group numbers are six-digit codes that provide information on the industry sectors and subsectors for companies in the I/B/E/S database. We use the first four digits, which correspond to broad industry groupings.

Table 1
Revenue Sources (%) of Analysts' Employers

Recommendation Level	Investment Banking		Brokerage Commission		Sample Size
	Mean	Median	Mean	Median	
5 (Strong buy)	13.94	11.81	29.87	24.09	28,901
4 (Buy)	13.81	11.21	26.68	17.22	37,478
3 (Hold)	12.68	11.13	28.44	24.07	37,883
2 (Sell)	11.61	10.55	23.13	16.12	4,875
1 (Strong sell)	16.27	14.90	33.44	24.95	1,356
p-Value (4 and 5) versus (1 and 2)	.0000	.0000	.0000	.0023	

Note. Shown are the percentages of analyst employer revenues from investment banking and brokerage commissions, by recommendation level. Data are for 110,493 stock recommendations and are drawn from the Institutional Brokers Estimate System U.S. Detail Recommendations History file for 1994–2003.

itatively similar under each of the three size measures. To save space, we report results only of tests based on the first size measure.

To capture the degree to which investors believe that individual analysts have skill in providing timely and accurate research, we use two measures of analysts' reputation. The first is based on *Institutional Investor* (*II*) magazine's All-America Research Team designation. Each year around October 15, *II* mails an issue to subscribers that lists the names of analysts who receive the most votes in a poll of institutional money managers. About 300–400 analysts are identified. We construct a variable that indicates, for each recommendation revision, whether the recommending analyst was named to the first, second, third, or honorable mention team in the latest annual survey. As a complementary, objective measure of analysts' reputation, we use a variable based on the *Wall Street Journal*'s (*WSJ*'s) annual All-Star Analysts Survey. The *WSJ* All-Star Analysts are determined by an explicit set of criteria relating to past stock-picking performance and forecasting accuracy.[15] The survey covers about 50 industries annually and names the top five stock pickers and top five earnings forecasters in each industry.[16]

Tables 1 and 2 report summary data on the characteristics of our sample. In Table 1, both the mean and the median percentages of analyst employer revenues derived from IB decline monotonically over the first four recommendation levels, but these values are the highest for strong sell recommendations. Similarly, it is the brokerage firms issuing strong sell recommendations that generally derive

[15] We recognize that the performance metrics used in the *Wall Street Journal* (*WSJ*) All-Star Analysts Survey are public information and can, in principle, be replicated by investors. However, to the extent that computing and evaluating analysts' performance is a costly activity, being named an All-Star Analyst can still affect an analyst's reputation and credibility.

[16] Since the I/B/E/S Broker Translation File provides only analysts' last names and first initials, in some instances it is not possible to ascertain from the I/B/E/S data alone whether an analyst in our sample was named to the *Institutional Investor* (*II*) or *WSJ* team. For these cases, we determine team membership of analysts from NASD BrokerCheck, an online database (http://www.nasd.com, accessed October 2004) that provides the full names of registered securities professionals as well as their employment and registration histories for the past 10 years. The database also keeps track of analysts' name changes (such as those resulting from marriage).

Table 2
Characteristics of Analysts, Firms, and Companies Followed

Characteristic	Mean	Median	SD	Sample Size
Investment banking revenue (%)	13.60	11.25	11.93	94,892
Brokerage commission revenue (%)	28.74	24.07	24.75	94,892
Analyst's company-specific experience (years)	2.42	1.20	3.29	85,531
Analyst's general experience (years)	6.41	4.90	5.32	85,531
Analysts employed by a firm	86.34	60	79.73	94,618
Companies followed by an analyst	17.24	15	12.93	84,016
Four-digit I/B/E/S S/I/Gs followed by an analyst	3.05	3	1.90	84,014
Institutional Investor All-America stock picker	.005	0	.07	85,531
Institutional Investor All-America Research Team member	.035	0	.18	85,531
Wall Street Journal All-Star stock picker	.018	0	.13	85,531
Wall Street Journal All-Star Analyst	.136	0	.34	85,531
Market capitalization ($ millions)	8,804.46	1,367.22	27,758.81	81,333
Analyst following	9.14	7	6.88	92,869

Note. Data are for 94,892 recommendation revisions and are drawn from the Institutional Brokers Estimate System (I/B/E/S) U.S. Detail Recommendations History file for 1994–2003. Recommendation revisions include recommendation changes as well as initiations, resumptions, and discontinuations of coverage. Analysts' experience is measured from all analyst research activity reported in I/B/E/S, including earnings-per-share forecasts, long-term earnings growth forecasts, and stock recommendations. An analyst is considered to be a top stock picker or team member if he or she appeared in the relevant portion of the most recent analyst survey by *Institutional Investor* or the *Wall Street Journal* at the time of a recommendation revision. Market capitalization is measured 12 months before the end of the current month, and analyst following is measured on the basis of stock recommendation coverage. Market capitalization values are inflation adjusted (with Consumer Price Index numbers and with 2003 as the base year). S/I/G = sector industry group.

the highest percentage of their total revenues from brokerage commissions. Notably, in each of the five categories, the mean percentage of revenue from commissions is about twice as large as the mean percentage of revenue from IB. This fact underscores the importance of trading commissions as a source of revenue for many securities firms. The last column shows that about 95 percent of the recommendations in the sample are at levels 5 (strong buy), 4 (buy), or 3 (hold). Levels 1 (strong sell) and 2 (sell) represent only about 1 percent and 4 percent of all recommendations, respectively.

The data in Table 2 provide a flavor of our sample of analysts and their employers. As noted by Hong, Kubik, and Solomon (2000), careers as analysts tend to be relatively short. The median recommendation is made by an analyst with under 5 years of experience, of which just over a year was spent following a given stock. Stock analysts tend to be highly specialized, following a handful of companies in a few industries. The median recommendation is made by an analyst following 15 companies in three industries who works for a securities firm employing 60 analysts. Being named as an All-America Research Team member by *II* is a rare honor, received by under 5 percent of all analysts in our sample. Finally, the typical company followed is large, with mean (median) market capitalization of about $8.8 billion ($1.4 billion) in inflation-adjusted

2003 dollars. Over the time span of a year, a company is tracked by a mean (median) of 9.1 (7) analysts.

4. Conflicts and the Levels of Analyst Recommendations Net of the Consensus

In this section, we examine whether the level of an analyst's stock recommendation net of the consensus (that is, median) recommendation level is related to the conflicts that he or she faces. We start by ascertaining the level of the outstanding recommendation on each stock by each analyst following it at the end of each quarter (March, June, September, December) from 1995 through 2003. An analyst's recommendation on a stock is included only if it is newly issued, reiterated, or revised in the preceding 12 months.

We estimate a regression explaining individual analysts' net stock recommendation levels at the end of a quarter (which is the recommendation level minus the median recommendation level across all analysts following a stock during the quarter).[17] The regression pools observations across analysts, stocks, and quarters and includes our two main explanatory variables: the percentage of an analyst employer's total revenues from IB and the percentage from brokerage commissions. Following Jegadeesh et al. (2004) and Kadan et al. (forthcoming), who find that momentum is an important determinant of analysts' recommendations, we control for the prior 6-month stock return.

The regression also controls for other factors that can affect the degree of analysts' optimism, such as the size of the company followed and the resources, reputation, experience, and workload of an analyst. As a measure of the resources available to an analyst, a dummy variable is used for a large brokerage house, and it equals one if the firm ranks in the top quartile of all houses in terms of the number of analysts employed during the year. The size of the company followed is measured by the natural logarithm of its market capitalization, measured 12 months before the end of the month. We measure an analyst's reputation by dummy variables that equal one if the recommending analyst was named in the most recent year as an All-America Research Team member by *II* or as an All-Star Analyst by the *WSJ*. An analyst's company-specific research experience is measured by the natural logarithm of one plus the number of days an analyst has been producing research (including earnings-per-share forecasts, long-term growth forecasts, or stock recommendations) on the company. We measure an analyst's workload by the natural logarithm of one plus the number of companies for which he or she produces forecasts or recommendations in the current year.

Finally, we control for industry and time period effects by adding dummy variables for I/B/E/S two-digit S/I/G industries and for each calendar quarter (March 1995, June 1995, and so forth). Since net recommendation levels can

[17] To ensure meaningful variation in the dependent variable, we omit stocks followed by only one analyst in a quarter.

Table 3

Ordered Probit Analysis of Recommendation Levels Net of the Consensus

Explanatory Variable	Coefficient	*z*-Statistic
Investment banking revenue (%)	.4167	17.35
Brokerage commission revenue (%)	.0363	3.00
Prior 6-month stock return	−.0068	−2.89
Large brokerage house dummy	−.0639	−8.60
Company size	.0038	2.89
Institutional Investor All-America Research Team dummy	.0032	.15
Wall Street Journal All-Star Analyst dummy	−.0196	−2.23
Company-specific research experience	.0012	1.42
Number of companies followed	.0070	4.64

Note. The results are from ordered probit regressions explaining individual analysts' stock recommendation levels net of the consensus (that is, median) recommendation level at the end of each quarter (March, June, September, December) for 1995–2003. Observations are excluded if the analyst issued no new or revised recommendation in the preceding 12 months. The regression includes observations pooled across analysts, stocks, and quarters. Data on recommendations are drawn from the Institutional Brokers Estimate System (I/B/E/S) U.S. Detail Recommendations History file for 1994–2003. Investment banking or brokerage commission revenue refer to the percentage of the brokerage firm's total revenues derived from investment banking or brokerage commissions. The large brokerage house dummy is an indicator variable that equals one if a brokerage house is in the top quartile of all houses, based on the number of analysts issuing stock recommendations listed in I/B/E/S in a given calendar year. Company size is the natural logarithm of the market capitalization of the company followed, measured 12 months prior to the end of the current month. The *Institutional Investor* All-America Research Team and *Wall Street Journal* All-Star Analyst dummies are indicator variables that equal one if the recommending analyst was listed as an All-America Research Team member or All-Star Analyst in the most recent analyst ranking. Company-specific research experience is the natural log of one plus the number of days that an analyst has been issuing I/B/E/S research on a company. Number of companies followed equals the natural log of one plus the number of companies followed by an analyst in the current calendar year. The regression includes dummy variables for two-digit I/B/E/S sector industry group industries and for calendar quarters. Test statistics are based on a robust variance estimator. The number of observations is 213,011; the *p*-value of the χ^2 test is <.0001.

take ordered values from −4 (strongly pessimistic) to 4 (strongly optimistic) in increments of .5, we estimate the regression as an ordered probit model.[18] The *Z*-statistics are based on a robust (Huber-White sandwich) variance estimator.

Table 3 shows the regression estimate. The coefficients of IB revenue percentage and commission revenue percentage are both positive. This finding implies that greater conflicts with IB and brokerage businesses lead an analyst to issue a higher recommendation on a stock relative to the consensus. Stocks followed by busier analysts and stocks of larger companies receive higher recommendations relative to the consensus. Stocks that experience a price run-up over the prior 6 months, stocks followed by analysts at large brokerage houses, and stocks followed by *WSJ* All-Star Analysts all receive lower recommendations relative to the consensus. All of these relations are highly statistically significant.

To provide a sense of the magnitude of the main effects of interest, we show in Table 4 the derivatives of the probability of each net recommendation level

[18] Notice that recommendation levels can take integer values from 1 to 5, and the median recommendation can take values from 1 to 5 in increments of .5. See Greene (2003) for a detailed exposition of the ordered probit model.

Table 4

Marginal Effects and Sample Distribution for the Ordered Probit Regression in Table 3

	Recommendation Level Net of the Consensus														
	−4	−3.5	−3	−2.5	−2	−1.5	−1	−.5	0	.5	1	1.5	2	2.5	3
Investment banking revenue (%)	−.00031	−.0002	−.0026	−.0010	−.0199	−.0086	−.0744	−.0321	.0123	.0325	.0671	.0077	.0188	.0002	.0003
Brokerage commission revenue (%)	−.00003	−.00001	−.0002	−.00009	−.0017	−.0008	−.0065	−.0028	.0011	.0028	.0059	.0007	.0016	.00002	.00003
Observed frequency	.0001	.0001	.0016	.0007	.0176	.0094	.1241	.0948	.4940	.0937	.1289	.0111	.0233	.0002	.0003

Note. Shown is the derivative of the probability of each net recommendation level with respect to investment banking or brokerage revenue percentage, estimated from the ordered probit regression in Table 3. Investment banking and brokerage commission revenue refer to the percentage of the brokerage firm's total revenues derived from investment banking and brokerage commissions. The last row shows observed frequency of each net recommendation level as a proportion of the sample of 213,011 observations.

with respect to IB revenue and commission revenue percentages.[19] Thus, for example, a 1-standard-deviation increase in IB revenue percentage increases the probability of an optimistic recommendation (that is, a net recommendation level greater than zero) by $.1193 \times (.0325 + .0671 + \ldots + .0003) = .0151$. Compared to the unconditional probability of an optimistic recommendation by an analyst, this represents an increase of about 5.9 percent (.0151/.2575). The effect of a change in commission revenue percentage is much smaller. A 1-standard-deviation increase in commission revenue percentage increases the probability of an optimistic recommendation by $.2475 \times .01105 = .0027$, or about 1 percent (.0027/.2575) of the unconditional probability. Thus, despite possible concerns about a loss of reputation, analysts seem to respond to conflicts of interest, particularly those stemming from IB.

5. Conflicts and Investor Response to Recommendation Revisions

5.1 Stock Price Response

This section examines whether an analyst's credibility with investors is related to the degree of conflict faced. We interpret the reaction of stock prices to a recommendation revision as an indication of an analyst's credibility. Our analysis focuses on revisions in recommendation levels, rather than on recommendation levels per se, because revisions are discrete events that are likely to be salient for investors, and previous research finds that revisions have significant information content (see, for example, Womack 1996; Jegadeesh et al. 2004). To capture the effects of the most commonly observed and economically important types of revisions, we structure our tests around four basic categories: added to strong buy, added to buy or strong buy, dropped from strong buy, and dropped from buy or strong buy.[20] These four categories are defined to include initiations, resumptions, and discontinuations of coverage because such events also reflect analysts' positive or negative views about a company.[21] Thus, for example, we consider a stock to be added to strong buy under two scenarios: (*a*) the recommendation level is raised to strong buy from a lower level or (*b*) coverage is

[19] Notice that, for each explanatory variable, these derivatives sum to zero across all the net recommendation levels.

[20] Our analysis focuses on these four types of revisions instead of the other four (added to strong sell, and so forth) because, as shown in Table 1, sell and strong sell recommendations are quite rare. But note that dropped-from-buy and dropped-from-buy-or-strong-buy revisions can entail movement to the sell or strong sell category.

[21] We use the I/B/E/S Stopped Recommendations file to determine instances in which a brokerage firm discontinued coverage of a company. This file contains numerous cases in which an analyst stops coverage of a stock only to issue a new recommendation a month or two later. Conversations with I/B/E/S representatives indicate that such events likely represent pauses in coverage due to company quiet periods or analysts' reassignments within a brokerage house. We define a stopped coverage event to be a true stoppage only if the analyst does not issue a recommendation on the stock over the subsequent 6 months.

initiated or resumed at the level of strong buy.[22] Defining revisions in this fashion yields a sample of 94,892 recommendation revisions made over the 1994–2003 period.

5.1.1. Average Response

We compute the abnormal return on an upgraded or downgraded stock over day t as the return (including dividends) on the stock minus the return on the Center for Research in Security Prices equal-weighted market portfolio of New York Stock Exchange (NYSE), American Stock Exchange, and NASDAQ stocks. The cumulative abnormal return (CAR) on the stock over days t_1 to t_2 relative to the revision date (day 0) is measured as the sum of the abnormal returns over those days. Table 5 shows mean and median CARs for three windows: days -1 to 0, -1 to 1, and -5 to 5. The t-statistics for the difference of the mean abnormal returns from zero are computed as in Brown and Warner (1985) and are shown in parentheses. The p-values for the Wilcoxon test are reported in parentheses with the medians.

It is clear from Table 5 that recommendation revisions have large effects on stock prices. For example, when a stock is added to the strong-buy list, it experiences a mean abnormal return of about 2 percent over the 2-day revision period. Downgrades have even larger effects on stock prices than do upgrades. Strikingly, the 2-day mean abnormal return around the dropped-from-strong-buy list is -4 percent. Median values are consistently smaller in magnitude than are means, and this finding indicates that some revisions lead to price reactions of a very large magnitude. Mean and median 2-day abnormal returns are statistically different from zero for all four groups of forecast revisions. The magnitudes of abnormal returns are somewhat larger over the 3-day and 11-day windows than over the 2-day window. Overall, these returns are consistent with those found by prior research that examines the average stock price impact of recommendation revisions (for example, Womack 1996; Jegadeesh et al. 2004).

5.1.2. Cross-Sectional Analysis

Table 6 contains cross-sectional regressions of stock price reactions to recommendation revisions over days -1 to 1. The main explanatory variables of interest in these regressions are our revenue-based measures of the magnitudes of IB and brokerage conflicts. We include controls for the size of an analyst's employer, the size of the company followed, and measures of an analyst's reputation, experience, and workload.[23] We estimate a separate regression for each

[22] Note that the definitions of our four recommendation revision groups imply that stocks can be added to a group more than once on a given day. Nonetheless, excluding days on which a stock experiences multiple revisions does not change any of our qualitative results.

[23] Prior research finds that analysts who have more experience, carry lower workloads, or are employed by larger firms tend to generate more precise research (see, for example, Clement 1999; Jacob, Lys, and Neale 1999; Mikhail, Walther, and Willis 1997). In addition, more reputed analysts tend to generate timelier and more accurate research (see, for example, Stickel 1992; Hong and Kubik 2003). We expect such analysts to be more influential with investors.

Table 5

Cumulative Abnormal Returns surrounding Revisions in Analyst Stock Recommendations

Recommendation Revision	Days −1 to 0			Days −1 to 1			Days −5 to 5		
	Mean (*t*-Statistic)	Median (*p*-Value)	*N*	Mean (*t*-Statistic)	Median (*p*-Value)	*N*	Mean (*t*-Statistic)	Median (*p*-Value)	*N*
Upgrades:									
Added to strong buy	.0207	.0109	24,560	.0240	.0130	24,556	.0263	.0187	24,499
	(49.53)*	(.000)		(46.89)*	(.000)		(26.84)*	(.000)	
Added to buy or strong buy	.0149	.0071	36,879	.0165	.0085	36,875	.0207	.0128	36,780
	(46.47)*	(.000)		(42.01)*	(.000)		(27.53)*	(.000)	
Downgrades:									
Dropped from buy or strong buy	−.0337	−.0126	33,322	−.0358	−.0155	33,262	−.0491	−.0287	33,197
	(−56.21)*	(.000)		(−48.75)*	(.000)		(−34.92)*	(.000)	
Dropped from strong buy	−.0399	−.0153	22,825	−.0427	−.0183	22,795	−.0570	−.0326	22,767
	(−49.88)*	(.000)		(−43.58)*	(.000)		(−30.38)*	(.000)	

Note. The sample of recommendation revisions is drawn from the Institutional Brokers Estimate System (I/B/E/S) U.S. Detail Recommendations History file for 1994–2003. Recommendation revisions include recommendation changes and initiations, resumptions, and discontinuations in coverage. Day 0 is the revision date. Recommendation revisions are classified according to the level of any existing recommendation and whether coverage is being initiated or dropped. For example, a revision by an analyst is classified as added to strong buy if the new recommendation is strong buy and (*a*) the previous recommendation was lower than strong buy or (*b*) analyst coverage by the brokerage house is resumed or initiated. A recommendation is classified as dropped from strong buy if the previous recommendation was strong buy and (*a*) the new recommendation is lower than strong buy or (*b*) research coverage on the company is stopped. The *t*-statistics for the difference from zero are computed as in Brown and Warner (1985). The *p*-values for the difference from zero are from a Wilcoxon test.

* Statistically significant at the 1% level in two-tailed tests.

Table 6

Cross-Sectional Regressions of Cumulative Abnormal Returns over Days −1 to +1 surrounding Recommendation Revisions

Explanatory Variable	Added to Strong Buy	Added to Buy or Strong Buy	Dropped from Buy or Strong Buy	Dropped from Strong Buy
Intercept	.0369	.0412	−.2294	−.2224
	(7.66)**	(11.21)**	(−31.31)**	(−29.25)**
Investment banking revenue (%)	−.0262	−.0139	−.0200	−.0354
	(−5.65)**	(−3.57)**	(−2.74)**	(−3.92)**
Brokerage commission revenue (%)	−.0187	−.0148	−.0089	−.0013
	(−6.51)**	(−6.43)**	(−2.39)*	(−.29)
Large brokerage house dummy	.0116	.0088	−.0242	−.0220
	(7.46)**	(6.88)**	(−12.79)**	(−10.25)**
Company size	−.0056	−.0041	−.0004	.0018
	(−16.13)**	(−15.40)**	(−.97)	(3.77)**
Institutional Investor All-America Research Team dummy	.0159	.0122	−.0148	−.0207
	(4.11)**	(3.82)**	(−2.93)**	(−3.28)**
Wall Street Journal All-Star Analyst dummy	.0015	.0013	−.0011	.0045
	(.81)	(.84)	(−.48)	(1.78)
Company-specific research experience	.0017	.0019	.0039	.0018
	(8.42)**	(12.49)**	(7.37)**	(3.21)**
Number of companies followed	−.0012	−.0016	.0007	.0008
	(−2.97)**	(−5.37)**	(1.49)	(1.31)
Observations	19,440	28,665	28,618	19,632
Adjusted R^2	.038	.0240	.028	.035
P-Value of *F*-test	<.0001	<.0001	<.0001	<.0001

Note. Shown are coefficient estimates and (in parentheses) *t*-statistics from ordinary least squares regressions. Day 0 is the recommendation revision date. Data on recommendations are drawn from the Institutional Brokers Estimate System (I/B/E/S) U.S. Detail Recommendations History file for 1994–2003. Investment banking and brokerage commission revenue refer to the percentages of a brokerage firm's total revenues derived from investment banking and brokerage commissions. The large brokerage house dummy is an indicator variable that equals one if a brokerage house is in the top quartile of all houses, based on the number of analysts issuing stock recommendations listed in I/B/E/S in a given calendar year. Company size is the natural logarithm of the market capitalization of the company followed, measured 12 months prior to the end of the current month. The *Institutional Investor* All-America Research Team and *Wall Street Journal* All-Star Analyst dummies are indicator variables that equal one if the recommending analyst was listed as an All-America Research Team member or All-Star Analyst in the most recent analyst ranking. Company-specific research experience is the natural log of one plus the number of days that an analyst has been issuing I/B/E/S research on a company. Number of companies followed equals the natural log of one plus the number of companies followed by an analyst in the current calendar year. All regressions include dummy variables for calendar-year and two-digit I/B/E/S sector industry group industries (not reported). The *t*-statistics are based on a robust variance estimator.

* Statistically significant at the 5% level in two-tailed tests.

** Statistically significant at the 1% level in two-tailed tests.

of the four groups of recommendation revisions. The *t*-statistics based on a robust variance estimator are reported in parentheses.

The coefficient on IB revenue percentage is statistically significantly negative for both upgrades and downgrades. The coefficient on brokerage commission revenue percentage is also negative in all four regressions; it is statistically significant in all cases, except for the dropped-from-strong-buy revisions.[24] Collectively, these results favor the rational discounting hypothesis over the naive investor hypothesis. The magnitudes of these effects are nontrivial. For instance, a 1-standard-deviation increase in IB revenue percentage leads to a change of about −.31 (−.42) percentage points in the 3-day abnormal return around the move to (from) a strong buy recommendation. Similarly, a 1-standard-deviation increase in brokerage commission revenue percentage leads to a change of about −.37 (−.22) percentage points in the corresponding abnormal return around the move to (from) a buy or strong buy recommendation.[25]

The results for control variables are also noteworthy. The dummy variable for a large analyst employer is positively (negatively) related to the market reaction to upgrades (downgrades). This finding is consistent with the idea that revisions by analysts employed at larger brokerage houses (which tend to be more reputable) have more credibility with investors. The size of the company followed is negatively (positively) related to the market reaction to upgrades (downgrades), which is consistent with the notion that, for larger companies, an analyst's recommendation competes with more alternative sources of information and advice.

Revisions by *II* All-America Research Team analysts are positively (negatively) related to the stock price reaction to upgrades (downgrades), which suggests that they wield more influence with investors. This is a notable finding; we are unaware of previous work documenting a relation between an analyst's reputation and the stock price reaction to both upgrades and downgrades. As the coefficient on the *WSJ* All-Star Analyst dummy indicates, however, being designated as a *WSJ* All-Star Analyst does not seem to enhance the credibility of an analyst's recommendations.[26] The absence of an effect here is somewhat

[24] These and all subsequent regression results in this article are qualitatively similar when we winsorize the dependent variable at the first and ninety-ninth percentiles of its distribution.

[25] For each group of revisions (such as added to strong buy), we also estimate the regression after excluding similar revision events that a stock experiences within 3 days of a given revision event. These results are qualitatively similar to those reported in Tables 6 and 8. We also examine the possibility that investors perceived the conflicts to be more severe, and hence discounted them more, in securities firms that were charged by regulators (that is, the 10 firms that were part of the global analyst settlement) than in other firms. We do this by interacting both investment banking (IB) revenue percentage and brokerage commission revenue percentage variables in the regression with binary (0, 1) dummy variables for securities firms that are part of the global analyst settlement and firms that are not. We find no significant differences between the two groups of firms in their coefficients on IB revenue percentage and commission revenue percentage.

[26] Although *II* All-America Research Team and *WSJ* All-Star Analyst dummies both measure aspects of an analyst's reputation, they are not highly correlated. The correlation coefficient is .14 across all upgrades and .13 across all downgrades.

surprising given that the *WSJ* has a much broader readership base than that of *II*. One explanation is that *II* analyst rankings are based on an opinion poll of money managers, who control substantial assets and therefore directly affect stock prices, while *WSJ* rankings are based on strictly quantitative measures of analysts' past stock-picking or forecasting performance.

The market reaction to upgrades is positively related to an analyst's company-specific research experience. This finding suggests that more experienced analysts tend to be more influential with investors. But the reaction to downgrades is also positively related to analysts' experience. Finally, the stock price reaction to upgrades is negatively related to analysts' workload. This finding suggests that busier analysts' opinions tend to get discounted by the market. All of these relations are statistically significant.

5.2. Response of Trading Volume

In this section, we measure analysts' credibility via changes in the volume of trade around recommendation revisions.[27] Revisions of analysts' recommendations can affect trading volumes by inducing investors to rebalance their portfolios to reflect updated beliefs.

5.2.1. Average Response

We compute the abnormal volume for a trading day t as the mean-adjusted share turnover for stock i:[28]

$$e_{it} = v_{it} - v_i, \quad (1)$$

where v_{it} is the trading volume of stock i over day t divided by common shares outstanding on day t and v_i is the mean of v_{it} over days -35 to -6.

The cumulative abnormal volume (CAV) for stock i over days t_1 to t_2 is measured in the following way:

$$\text{CAV}^i t_1, t_2 = \sum_{t=t_1}^{t_2} e_{it}. \quad (2)$$

Table 7 shows mean and median CAV values over three windows surrounding revisions in analyst stock recommendations. Over the 2-day revision period, the mean abnormal volume is positive for both upgrades and downgrades, but its magnitude is substantially larger for downgrades. The move to (from) the strong-buy list increases a stock's trading volume by a mean of about .9 percent (2.6 percent) of the outstanding shares, compared to a normal day's volume. For longer windows, the mean abnormal volumes are substantially higher for down-

[27] Many prior studies have used trading volume to examine investors' response to informational events (see, for example, Shleifer 1986; Jain 1988; Jarrell and Poulsen 1989; Meulbroek 1992; Sanders and Zdanowicz 1992).

[28] This approach has been used in a number of prior studies (for example, Shleifer 1986; Vijh 1994; Michaely and Vila 1996).

Table 7

Cumulative Abnormal Trading Volumes surrounding Announcements of Revisions in Stock Recommendations by Analysts

	Days −1 to 0			Days −1 to 1			Days −5 to 5		
Recommendation revision	Mean (*t*-Statistic)	Median (*p*-Value)	*N*	Mean (*t*-Statistic)	Median (*p*-Value)	*N*	Mean (*t*-Statistic)	Median (*p*-Value)	*N*
Upgrades:									
Added to strong buy	.0086	.0011		.0097	.0015		.0071	.0030	
	(8.89)*	(.000)	24,506	(8.18)*	(.000)	24,502	(3.13)*	(.000)	24,488
Added to buy or strong buy	.0053	.0002		.0058	.0004		.0020	.0008	
	(5.08)*	(.000)	36,800	(4.54)*	(.000)	36,796	(.818)	(.000)	36,766
Downgrades:									
Dropped from buy or strong buy	.0217	.0010		.0265	.0014		.0381	.0039	
	(114.47)*	(.000)	33,291	(114.14)*	(.000)	33,232	(85.70)*	(.000)	33,175
Dropped from strong buy	.0259	.0017		.0315	.0025		.0453	.0057	
	(128.76)*	(.000)	22,808	(127.86)*	(.000)	22,779	(96.03)*	(.000)	22,756

Note. The abnormal volume for stock i on day t is computed from daily Center for Research in Security Prices data as $e_{it} = v_{it} - v_i$, where v_{it} is the volume on day t and v_i is the average volume over days −35 to −6 relative to the recommendation revision date (day 0). All share volumes are normalized by dividing by common shares outstanding on the same day. The p-values are from a Wilcoxon test.

* Statistically significant at the 1% level in two-tailed tests.

grades. The median values are lower than the mean values. Each mean and median abnormal volume is statistically greater than zero, with a p-value below .01. Clearly, revisions of stock recommendations by analysts generate trading.

5.2.2. Cross-Sectional Analysis

Table 8 presents cross-sectional regressions explaining CAVs over days −1 to 1 surrounding the recommendation revisions. The explanatory variables in the regressions are the same as in regressions of CARs in Section 5.1.2. The results provide strong support for the rational discounting hypothesis. The coefficients on both the IB revenue percentage and commission revenue percentage variables are generally statistically significant and negative (positive) for both groups of upgrades (downgrades). The magnitudes of these effects are nontrivial. For example, a 1-standard-deviation increase in IB revenue percentage leads to a change in the 3-day abnormal volume around the addition (omission) of a stock to (from) the strong-buy list of about −.12 percent (.36 percent) of the outstanding shares; a corresponding change in the commission revenue percentage results in a change in the abnormal volume of about −.15 percent (.22 percent).

Recommendation revisions by larger brokerage houses generate more trading. The abnormal volume is also larger for revisions involving smaller companies. Revisions by *II* All-America Research Team members generate statistically significantly more abnormal volume for the dropped from buy or strong-buy group. Upgrades (downgrades) by more experienced analysts result in larger (smaller) abnormal volumes, and upgrades by busier analysts are less credible.

6. Conflicts and the Performance of Recommendation Revisions

We next consider the investment performance of analysts' recommendation revisions over periods of up to 12 months. Here, the choice of the benchmark used to compute abnormal returns is somewhat more important than it is in Section 5.1, where we measure abnormal returns over a few days around the revision. But the results here are likely to be less sensitive to the benchmark employed than are those in studies of long-run stock performance, where the time period of interest can be as long as 5–10 years (see, for example, Agrawal, Jaffe, and Mandelker 1992; Agrawal and Jaffe 2003).

6.1. Average Performance

We use an approach similar to Barber, Lehavy, and Trueman (2007). To evaluate the performance of stocks over a given window, say, months 1–12 following the month of their inclusion (month 0) in a given group of revisions such as the added-to-strong-buy list, we form a portfolio p that initially invests $1 in each recommendation. Each recommended stock remains in the portfolio until month 12 or the month that the stock is either downgraded or dropped from coverage by the securities firm, whichever is earlier. If multiple securities firms recommend a stock in a given month, the stock appears multiple times in the

Table 8

Cross-Sectional Regressions of Cumulative Abnormal Trading Volumes over Days −1 to +1 surrounding Recommendation Revisions

Explanatory Variable	Added to Strong Buy	Added to Buy or Strong Buy	Dropped from Buy or Strong Buy	Dropped from Strong Buy
Intercept	.0083	.0042	.0946	.0828
	(2.65)**	(1.90)	(13.72)**	(15.01)**
Investment banking revenue (%)	−.0100	−.0085	.0140	.0304
	(−3.31)**	(−2.26)*	(2.18)*	(3.63)**
Brokerage commission revenue (%)	−.0057	−.0059	.0087	.0055
	(−1.76)	(−4.13)**	(2.76)**	(1.45)
Large brokerage house dummy	.0058	.0038	.0168	.0171
	(3.72)**	(4.50)**	(11.12)**	(9.48)**
Company size	−.0031	−.0018	−.0023	−.0041
	(−9.54)**	(−12.30)**	(−7.60)**	(−11.40)**
Institutional Investor All-America Research Team dummy	.0035	.0033	.0084	.0046
	(1.74)	(1.88)	(2.32)*	(1.21)
Wall Street Journal All-Star Analyst dummy	.0008	.0013	.0023	−.0006
	(.74)	(1.42)	(1.36)	(−.29)
Company-specific research experience	.0010	.0010	−.0041	−.0019
	(8.39)**	(11.19)**	(−6.18)**	(−4.11)**
Number of companies followed	−.0009	−.0013	−.0001	−.0005
	(−3.49)**	(−6.23)**	(−.38)	(−.99)
Observations	19,431	28,653	28,594	19,619
Adjusted R^2	.025	.019	.030	.042
p-Value of F-test	<.0001	<.0001	<.0001	<.0001

Note. Shown are coefficient estimates and (in parentheses) *t*-statistics from ordinary least squares regressions. Day 0 is the recommendation revision date. Data on recommendations are drawn from the Institutional Brokers Estimate System (I/B/E/S) U.S. Detail Recommendations History file for 1994–2003. Investment banking and brokerage commission revenue refer to the percentage of brokerage firm's total revenues derived from investment banking and brokerage commissions. The large brokerage house dummy is an indicator variable that equals one if a brokerage house is in the top quartile of all houses, based on the number of analysts issuing stock recommendations listed in I/B/E/S in a given calendar year. Company size is the natural logarithm of the market capitalization of the company followed, measured 12 months prior to the end of the current month. The *Institutional Investor* All-America Research Team and *Wall Street Journal* All-Star Analyst dummies are indicator variables that equal one if the recommending analyst was listed as an All-America Research Team member or All-Star Analyst in the most recent analyst ranking. Company-specific research experience is the natural log of one plus the number of days that an analyst has been issuing I/B/E/S research on a company. Number of companies followed equals the natural log of one plus the number of companies followed by an analyst in the current calendar year. All regressions include dummy variables for calendar-year and two-digit I/B/E/S sector industry group industries (not reported). The *t*-statistics are based on a robust variance estimator.

* Statistically significant at the 5% level in two-tailed tests.

** Statistically significant at the 1% level in two-tailed tests.

portfolio that month, once for each securities firm with a strong buy recommendation. The portfolio return for calendar month t is given by

$$R_{pt} = \sum_{i=1}^{n_t} x_{it} \times R_{it} \Bigg/ \sum_{i=1}^{n_t} x_{it}, \tag{3}$$

where R_{it} is the month t return on recommendation i, x_{it} is one plus the compound return on the recommendation from month 1 to month $t-1$ (that is, x_{it} equals one for a stock that was recommended in month t), and n_t is the number of recommendations in the portfolio. This calculation yields a time series of monthly returns for portfolio p.

We compute the abnormal performance of portfolio p as the estimate of the intercept term α_p from the Fama and French (1993) three-factor model. Accordingly, we estimate the following time-series regression for portfolio p:

$$R_{pt} - R_{ft} = \alpha_p + \beta_{1p}(R_{mt} - R_{ft}) + \beta_{2p}\text{SMB}_t + \beta_{3p}\text{HML}_t + \varepsilon_{pt},$$
$$t = \text{January 1994 to December 2003}, \tag{4}$$

where R_f is the risk-free rate, R_m is the return on the value-weighted market index, SMB equals the monthly return on a portfolio of small firms minus the return on a portfolio of big firms, and HML is the monthly return on a portfolio of firms with high book-to-market ratio minus the return on a portfolio of firms with low book-to-market ratio. The error term in the regression is denoted ε. The time series of monthly returns on $R_m - R_f$, SMB, and HML are obtained from Kenneth French's Web site.[29] We repeat this procedure for each time window of interest, such as months 1–3, and for each group of revisions, such as the dropped-from-strong-buy list.

Table 9 shows the performance of analysts' recommendation revisions. Over the period of 3 months following the month of recommendation revision, the average abnormal returns for upgrades are positive, and the returns for downgrades are negative. The magnitudes of these returns are nontrivial. For example, the addition of a stock to the strong-buy list has an abnormal monthly return of about .875 percent, or about 2.62 percent over the 3-month period. The pattern is generally similar over longer windows. For example, over months 1–12, the abnormal monthly return for the added-to-strong-buy list is .679 percent, or about 8.15 percent over the 12-month period. The abnormal returns are significantly different from zero for upgrades in all cases; they are statistically insignificant for downgrades in all cases except one.

[29] Kenneth R. French, Fama/French Factors (file F-F_Research_Data_Factors.zip at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

Table 9
Medium-Term Investment Performance of Recommendation Revisions

	Months 1–3		Months 1–6		Months 1–12	
Portfolio	Abnormal Monthly Return (%)	*t*-Statistic	Abnormal Monthly Return (%)	*t*-Statistic	Abnormal Monthly Return (%)	*t*-Statistic
Added to strong buy	.875	6.12**	.758	6.12**	.679	5.70**
Added to buy or strong buy	.586	4.49**	.511	4.82**	.503	5.38**
Dropped from buy or strong buy	−.361	−1.60	−.260	−1.28	−.072	−.44
Dropped from strong buy	−.367	−1.58	−.395	−2.00*	−.231	−1.49

Note. Abnormal returns are reported for three event windows relative to the month of revision (month 0) and are computed using an approach similar to that in Barber, Lehavy, and Trueman (2007). The abnormal return is the estimated intercept from a time-series regression of 114 monthly portfolio returns using the Fama and French (1993) three-factor model.
* Statistically significant at the 5% level in two-tailed tests.
** Statistically significant at the 1% level in two-tailed tests.

6.2. Cross-Sectional Analysis

Table 10 shows the results of a regression similar to that in Section 5.1.2, except that the dependent variable here is the average monthly abnormal return for a firm over months 1–12 following the month of a recommendation revision. We compute this abnormal return by estimating a time-series regression similar to that in equation (4) over months 1–12 for each stock in a sample of recommendation revisions. The intercept from this regression is our estimate of the performance of the recommendation revision. Observations involving recommendation revisions on a stock that occur within 12 months of an earlier revision are omitted from each regression.[30]

In each regression result reported in Table 10, the coefficients of IB revenue percentage and commission revenue percentage are not statistically significantly different from zero. These results favor the rational discounting hypothesis, at least for the marginal investor. The performance of both groups of recommendation upgrades is negatively related to company size; the performance of one group of downgrades is positively related to the dummy variable for *WSJ* All-Star Analysts. None of the other variables is statistically significant.

7. Bubble versus Postbubble Periods

We next exploit the fact that our sample spans both the late-1990s U.S. stock bubble and a postbubble period. During the bubble period, initial public offerings, merger activities, and stock prices were near record highs, and media attention was focused on analysts' pronouncements. We therefore examine whether analysts' behavior and investors' responses to analysts' recommendations differed during the bubble and postbubble periods. Given the euphoria on Wall

[30] The results are qualitatively similar when we include these observations.

Table 10

Cross-Sectional Regressions of Average Monthly Abnormal Returns following Recommendation Revisions over Months 1–12

Explanatory Variable	Added to Strong Buy	Added to Buy or Strong Buy	Dropped from Buy or Strong Buy	Dropped from Strong Buy
Intercept	.0523	.0089	−.0646	−.0821
	(1.81)	(.49)	(−6.81)**	(−6.55)**
Investment banking revenue (%)	−.0089	−.0018	.0042	−.0068
	(−1.23)	(−.29)	(.64)	(−.87)
Brokerage commission revenue (%)	.0064	.0059	.0057	.0031
	(1.32)	(1.54)	(1.21)	(.75)
Large brokerage house dummy	.0009	−.0027	.0016	.0015
	(.38)	(−1.32)	(.72)	(.77)
Company size	−.0013	−.0017	−.0007	−.0007
	(−2.74)**	(−4.18)**	(−1.71)	(−1.54)
Institutional Investor All-America analyst dummy	−.0029	.0001	−.0016	−.0009
	(−.58)	(.01)	(−.44)	(−.23)
Wall Street Journal All-Star Analyst dummy	.0031	.0002	−.0029	.0056
	(1.24)	(.12)	(−1.42)	(2.29)*
Company-specific research experience	.0004	.0004	.0004	.0004
	(1.08)	(1.80)	(.76)	(.92)
Number of companies followed	−.0011	−.0008	−.0002	−.0002
	(−1.61)	(−1.79)	(−.45)	(−.47)
Observations	6,411	8,851	10,644	8,368
Adjusted R^2	.026	.023	.019	.020
p-Value of F-test	<.0001	<.0001	<.0001	<.0001

Note. Shown are the coefficient estimates and (in parentheses) t-statistics from ordinary least squares regressions. Month 0 is the month of recommendation revision. The abnormal return is the estimated intercept from a time-series regression of monthly portfolio returns in accordance with the Fama and French (1993) three-factor model. Data on recommendations are drawn from the Institutional Brokers Estimate System (I/B/E/S) U.S. Detail Recommendations History file for 1994–2003. Investment banking and brokerage commission revenue data refer to the percentage of the brokerage firm's total revenues derived from investment banking and brokerage commissions. The large brokerage house dummy is an indicator variable that equals one if a brokerage house is in the top quartile of all houses, based on the number of analysts issuing stock recommendations on I/B/E/S in a given calendar year. Company size is the natural logarithm of the market capitalization of the company followed, measured 12 months prior to the end of the current month. The *Institutional Investor* All-America Research Team and *Wall Street Journal* All-Star Analyst dummies are indicator variables that equal one if the recommending analyst was listed as an All-America Research Team member or All-Star Analyst in the most recent analyst ranking. Company-specific research experience is the natural log of one plus the number of days that an analyst has been issuing I/B/E/S research on a company. Number of companies followed equals the natural log of one plus the number of companies followed by an analyst in the current calendar year. All regressions include dummy variables for calendar-year and two-digit I/B/E/S sector industry group industries (not reported). The t-statistics are based on a robust variance estimator.

*Statistically significant at the 5% level in two-tailed tests.

**Statistically significant at the 1% level in two-tailed tests.

Table 11

Ordered Probit Regression of Recommendation Levels Net of the Consensus for Bubble versus Postbubble Periods

	Bubble	Postbubble	*p*-Value
Investment banking revenue (%)	.5103*	.3089*	<.001
Brokerage revenue (%)	−.1868*	.2286*	<.001

Note. The explanatory variables are as in Table 3, except that (*a*) the investment banking revenue and brokerage commission revenue percentage variables are interacted with dummy variables for the bubble or postbubble period and (*b*) calendar-quarter dummies are replaced with a postregulation indicator (which is equal to one for quarters after May 2002). Shown are the coefficient estimates of investment banking and brokerage revenue percentage variables for the bubble and postbubble periods and the *p*-value for the difference in the coefficient estimate between the two periods. All test statistics use robust variance estimators.
* Statistically significant at the 1% level in two-tailed tests.

Street and among investors during the bubble, analysts appear to have been under acute pressure to generate IB fees and brokerage commissions. As for the response of investors, the rational discounting hypothesis predicts greater discounting of analysts' opinions during this period in response to heightened conflicts, while the naive investor hypothesis predicts less discounting.

We estimate regressions similar to those for relative recommendation levels (Table 3), those for announcement abnormal returns (Table 6), those for announcement abnormal volumes (Table 8), and those for 12-month investment performance of recommendation revisions (Table 10), except that we now interact IB revenue percentage and commission revenue percentage with dummy variables for the bubble (January 1996–March 2000) and postbubble (April 2000–December 2003) periods. Accordingly, we restrict the sample period for these regressions to January 1996–December 2003. For regressions corresponding to those with results shown in Table 3, we also replace the calendar-quarter dummies with a postregulation indicator (equal to one for quarters ending after May 2002). In May 2002, both the NYSE and the National Association of Securities Dealers considerably tightened the regulations on the production and dissemination of sell-side analyst research.[31] The findings of Barber et al. (2006) and Kadan et al. (forthcoming) suggest that these regulations exerted a downward pressure on recommendation levels. The regression results are presented in Tables 11 and 12. To save space, we report only the coefficient estimates for IB revenue percentage and commission revenue percentage.

The results in Table 11 show that analysts appear to have inflated their recommendations in response to IB conflicts during both the bubble and postbubble periods. But the magnitude of this effect is substantially greater during the bubble period than during the postbubble period. This difference is statistically significant. The magnitude of the effect is smaller for brokerage conflicts than for IB conflicts during both periods. In fact, the effect for brokerage conflicts is negative

[31] See NYSE Amended Rule 472, "Communications with the Public," and National Association of Securities Dealers Rule 2711, "Research Analysts and Research Reports."

Table 12

Ordinary Least Squares Regressions of Abnormal Returns, Abnormal Volumes, and Abnormal Stock Performance for Bubble and Postbubble Periods

	Added to Strong Buy			Added to Buy or Strong Buy			Dropped from Buy or Strong Buy			Dropped from Strong Buy		
	Bubble	Postbubble	*p*-Value	Bubble	Postbubble	*p*-Value	Bubble	Postbubble	*p*-Value	Bubble	Postbubble	*p*-Value
CARs, days −1 to 1:												
Investment banking revenue (%)	−.0248**	−.0120	.083	−.0121**	−.0080	.517	−.0125	−.0379**	.027	−.0361**	−.0345**	.908
Brokerage revenue (%)	−.0114**	−.0105**	.827	−.0099**	−.0110**	.720	−.0063	−.0208**	.003	.0017	−.0114*	.024
CAVs, days −1 to 1:												
Investment banking revenue (%)	−.0076	−.0052	.655	−.0065	−.0082*	.699	.0257**	.0130	.214	.0555**	.0153	.002
Brokerage revenue (%)	−.0042	−.0008	.376	−.0054**	−.0031	.179	.0106*	.0139**	.521	.0046	.0141**	.056
Average monthly CARs, months 1–12:												
Investment banking revenue (%)	−.0016	−.0151	.273	.00001	.0083	.420	−.0085	.0223**	.003	−.0123	−.0051	.564
Brokerage revenue (%)	.0069	.0108	.511	.0086	.0096	.842	.0035	.0136	.101	−.0036	.0091	.019

Note. The explanatory variables are as in Tables 6, 8, and 10, except that the investment banking revenue and brokerage commission revenue percentage variables are interacted with dummy variables for the bubble or postbubble period. Shown are the coefficient estimates of the investment banking and brokerage revenue percentage variables for the bubble and postbubble periods and the *p*-value for the difference in the coefficient estimate between the two periods. Day (month) 0 is the recommendation revision date. All test statistics use robust variance estimators. CAR = cumulative abnormal return; CAV = cumulative abnormal volume.

* Statistically significant at the 5% level in two-tailed tests.

** Statistically significant at the 1% level in two-tailed tests.

during the bubble; it is positive and statistically significantly higher during the postbubble period.

Table 12 shows that, in regressions of 3-day abnormal returns, the coefficients of both IB revenue percentage and commission revenue percentage are negative and statistically significant during the bubble period for both groups of upgrades. For the added-to-strong-buy group, the coefficient of IB revenue percentage is significantly lower during the bubble period than during the postbubble period. For downgrades, the coefficients of both variables are generally negative in both periods, and they are statistically significantly lower during the postbubble period.

In regressions of 3-day abnormal volumes, the coefficients of IB revenue percentage and commission revenue percentage are negative for upgrades and positive for downgrades in all cases, both during and after the bubble. These coefficients are not statistically significantly different between the bubble and postbubble periods for both groups of upgrades and one group of downgrades. For the dropped-from-strong-buy group, the coefficient of IB revenue percentage is statistically significantly larger during the bubble period than during the postbubble period, but the coefficient of the commission revenue percentage is statistically significantly smaller. In regressions of 12-month postrecommendation stock performance, the coefficients of both variables are statistically insignificant both during and after the bubble period in nearly all cases, and this finding is consistent with the results shown in Table 10 for the full sample period.

Overall, analysts appear to respond to IB conflicts both during and after the bubble, but the magnitude of their response declines during the postbubble period. Perversely, while analysts do not seem to respond to brokerage conflicts during the bubble, they appear to do so after the bubble. Perhaps the intense regulatory and media focus on IB conflicts has led analysts to look for alternative avenues. Did investors discount conflicted analysts' opinions more during the bubble than in the postbubble period? The answer to this question is unclear. However, our evidence does not support the notion that investors threw caution to the wind during the bubble.

8. Summary and Conclusions

Following the collapse of the late-1990s U.S. stock market bubble, there has been a widespread hue and cry from investors and regulators over the conflicts of interest faced by Wall Street stock analysts. The discovery of e-mail messages, in which analysts were privately disparaging stocks that they were touting publicly, led to the landmark $1.4 billion settlement between a number of leading Wall Street firms and securities regulators in April 2003. The settlement requires the firms to disclose IB conflicts in analyst reports and imposes a variety of restrictions designed to strengthen the firewalls that separate research from IB. Part of the settlement funds are set aside for investor education and for research produced by independent firms. The settlement basically presumes that analysts respond to the conflicts by inflating their stock recommendations and that investors take analysts' recommendations at face value.

Consistent with the view of the media and regulators, we find that optimism in stock recommendations is positively related to the importance of both IB and brokerage businesses to an analyst's employer. This pattern is more pronounced during the late-1990s stock market bubble with respect to IB conflicts. However, we provide several pieces of empirical evidence that suggest that investors are sophisticated enough to adjust for this bias. First, the short-term reactions of both stock prices and trading volumes to recommendation upgrades vary negatively with the magnitude of potential IB or brokerage conflicts faced by analysts. For instance, over the 3 days surrounding an upgrade to strong buy, a 1-standard-deviation increase in the proportion of revenue from IB is associated with a .31 percentage point decrease in abnormal returns and a .12 percentage point decrease in abnormal volume. These results suggest that investors ascribe lower credibility to an analyst's upgrade when the analyst is subject to greater pressures to issue an optimistic view. For downgrades, conflict severity varies negatively with the short-term stock price reaction and positively with the short-term trading volume impact. This pattern is consistent with the idea that investors perceive an analyst to be more credible if he or she is willing to voice an unfavorable opinion on a stock despite greater pressures to be optimistic.

Second, we find no evidence that the 1-year investment performance of recommendation revisions is related to the magnitude of analysts' conflicts, either for upgrades or for downgrades. This finding suggests that, on average, investors properly discount an analyst's opinions for potential conflicts at the time the opinion is issued. Finally, investors discounted conflicted analysts' opinions during the late-1990s stock bubble, even in the face of the prevailing market euphoria. This evidence does not support the popular view that recommendations of sell-side analysts led investors to throw caution to the wind during the bubble period.

Overall, our empirical findings suggest that while analysts do respond to IB and brokerage conflicts by inflating their stock recommendations, the market discounts these recommendations after taking analysts' conflicts into account. These findings are reminiscent of the story of the nail soup told by Brealey and Myers (1991), except that here analysts (rather than accountants) are the ones who put the nail in the soup and investors (rather than analysts) are the ones to take it out. Our finding that the market is not fooled by biases stemming from conflicts of interest echoes similar findings in the literature on conflicts of interest in universal banking (for example, Kroszner and Rajan 1994, 1997; Gompers and Lerner 1999) and on bias in the financial media (for example, Bhattacharya et al., forthcoming; Reuter and Zitzewitz 2006). Finally, while we cannot rule out the possibility that some investors may have been naive, our findings do not support the notion that the marginal investor was systematically misled over the last decade by analysts' recommendations.

Appendix

This Appendix describes the characteristics of disclosing and nondisclosing private securities firms, sheds some light on their decisions to publicly disclose their income statements, and examines whether the resulting selection bias affects our main results in Table 3. Table A1 provides summary statistics of recommendation levels and characteristics of disclosing and nondisclosing private securities firms. Compared with nondisclosing firms, disclosing firms tend to be smaller and more liquid and issue somewhat more optimistic stock recommendations. The mean recommendation level is slightly higher for disclosing firms than for nondisclosing firms. The median disclosing firm is smaller and holds more liquid assets than the median nondisclosing firm. All these differences are statistically significant. The two groups of firms have similar financial leverage ratios and 2-year growth rates in total assets.

We next examine cross-sectional determinants of a private securities firm's decision to disclose its income statement. In an excellent review of the corporate disclosure literature, Healy and Palepu (2001) point out that a firm is more willing to voluntarily disclose financial information when it needs to raise external financing and when it is less concerned that the disclosure would damage its competitive position in product markets. Ceteris paribus, firms with greater growth opportunities, higher financial leverage, and less liquid resources are more likely to need external financing. They are more likely to be open with potential investors by disclosing financial information, including their income statements. Similarly, smaller firms are likely to have greater need for external financing as they try to grow. In addition, given the intense competition in the securities business, smaller private firms are also likely to be more willing to disclose their profits and profitability because they have less business at stake. For both reasons, smaller firms are likely to be more willing to disclose financial information. We control for firm size by the natural logarithm of one plus total assets in millions of dollars, for growth opportunities by the 2-year growth rate of total assets, for financial leverage by the ratio of long-term debt to total assets, and for liquidity by the ratio of cash and equivalents to total assets. We estimate a probit regression of DISCLOSER, which equals one for a disclosing firm and is zero otherwise.

In accordance with the predictions of corporate disclosure theory, the coefficients on firm size and liquidity are negative, and the coefficient on growth is positive. Contrary to the prediction, however, the coefficient on leverage is negative. All of these coefficients are highly statistically significant. The pseudo-R^2-value of this model is .08. To save space, these results are not shown in a table.

Finally, we examine whether the selection bias caused by a private securities firm's disclosure choice (and, consequently, the availability of data on IB revenue percentage and commission revenue percentage) affects our main results in Table 3. While there is no Heckman selectivity correction for the ordered probit model, there is one for the regular probit model. So we define a binary variable to

Table A1

Summary Statistics for Disclosing and Nondisclosing Private Securities Firms

Variable	Mean			Median			Sample Size	
	Disclosers	Nondisclosers	*P*-Value of *t*-Test	Disclosers	Nondisclosers	*p*-Value of Rank Sum Test	Disclosers	Nondisclosers
Recommendation level:								
Level	3.902	3.810	<.001	4	4	<.001	62,417	181,068
Level minus median level	.036	.010	<.001	0	0	<.001	62,417	181,068
Firm size:								
Total assets ($ millions)	383.37	1,863.52	<.001	4.05	28.43	<.001	365	615
Book equity ($ millions)	26.40	68.98	<.001	1.97	10.56	<.001	365	615
Financial leverage:								
Long-term debt to total assets	.0539	.0653	.253	0	.002	.004	365	615
Total debt to total assets	.0685	.1823	.295	0	.018	<.001	365	615
Liquidity: cash and equivalents to total assets	.2392	.1816	.001	.101	.052	.0001	365	615
2-Year growth rate	.0849	.0697	.440	.052	.020	.099	246	541

Note. Disclosers are brokers that publicly disclose their income statements, while nondisclosers are brokers that do not disclose them. The statistics for recommendation level are computed from individual analysts' recommendation levels at the end of each quarter in the sample. The median recommendation level is computed at the end of each quarter and is based on all analysts recommending a stock. The statistics for broker characteristics are computed across broker years. The firm size statistics are inflation adjusted (with Consumer Price Index numbers and with 2003 as the base year). The 2-year growth rate is $(\text{Total assets}_t / \text{Total assets}_{t-2})^{1/2} - 1$.

measure an optimistic recommendation that equals one if an analyst's recommendation level on a stock exceeds the consensus level and equals zero otherwise. We then replace the dependent variable in the regression in Section 4 with this optimistic recommendation dummy. Using the subsample of private securities firms, we estimate the resulting equation in two ways: (*a*) with a regular probit model and (*b*) with a Heckman selectivity-corrected probit model, where we use the equation described in the second paragraph of this Appendix as the selection equation. When we use approach *b,* the coefficient of the selection term (that is, the inverse Mills ratio) is statistically significant in the second-stage probit regression. What is more important for our purposes is that the sign, magnitude, and statistical significance of our main explanatory variables, the IB revenue percentage and the commission revenue percentage, are similar in the regular probit and the Heckman-corrected probit regressions. These results do not support the idea that our main findings are driven by the selection bias caused by a private securities firm's decision to disclose its revenue breakdown. To save space, these results are not shown in a table.

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Appendix A
Monthly and Annual Returns of Basic Series

Appendix A-1

Large-Capitalization Stocks: Total Return
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1926	0.0000	-0.0385	-0.0575	0.0253	0.0179	0.0457	0.0479	0.0248	0.0252	-0.0284	0.0347	0.0196	1926	0.1162
1927	-0.0193	0.0537	0.0087	0.0201	0.0607	-0.0067	0.0670	0.0515	0.0450	-0.0502	0.0721	0.0279	1927	0.3749
1928	-0.0040	-0.0125	0.1101	0.0345	0.0197	-0.0385	0.0141	0.0803	0.0259	0.0168	0.1292	0.0049	1928	0.4361
1929	0.0583	-0.0019	-0.0012	0.0176	-0.0362	0.1140	0.0471	0.1028	-0.0476	-0.1973	-0.1246	0.0282	1929	-0.0842
1930	0.0639	0.0259	0.0812	-0.0080	-0.0096	-0.1625	0.0386	0.0141	-0.1282	-0.0855	-0.0089	-0.0706	1930	-0.2490
1931	0.0502	0.1193	-0.0675	-0.0935	-0.1279	0.1421	-0.0722	0.0182	-0.2973	0.0896	-0.0798	-0.1400	1931	-0.4334
1932	-0.0271	0.0570	-0.1158	-0.1997	-0.2196	-0.0022	0.3815	0.3869	-0.0346	-0.1349	-0.0417	0.0565	1932	-0.0819
1933	0.0087	-0.1772	0.0353	0.4256	0.1683	0.1338	-0.0862	0.1206	-0.1118	-0.0855	0.1127	0.0253	1933	0.5399
1934	0.1069	-0.0322	0.0000	-0.0251	-0.0736	0.0229	-0.1132	0.0611	-0.0033	-0.0286	0.0942	-0.0010	1934	-0.0144
1935	-0.0411	-0.0341	-0.0286	0.0980	0.0409	0.0699	0.0850	0.0280	0.0256	0.0777	0.0474	0.0394	1935	0.4767
1936	0.0670	0.0224	0.0268	-0.0751	0.0545	0.0333	0.0701	0.0151	0.0031	0.0775	0.0134	-0.0029	1936	0.3392
1937	0.0390	0.0191	-0.0077	-0.0809	-0.0024	-0.0504	0.1045	-0.0483	-0.1403	-0.0981	-0.0866	-0.0459	1937	-0.3503
1938	0.0152	0.0674	-0.2487	0.1447	-0.0330	0.2503	0.0744	-0.0226	0.0166	0.0776	-0.0273	0.0401	1938	0.3112
1939	-0.0674	0.0390	-0.1339	-0.0027	0.0733	-0.0612	0.1105	-0.0648	0.1673	-0.0123	-0.0398	0.0270	1939	-0.0041
1940	-0.0336	0.0133	0.0124	-0.0024	-0.2289	0.0809	0.0341	0.0350	0.0123	0.0422	-0.0316	0.0009	1940	-0.0978
1941	-0.0463	-0.0060	0.0071	-0.0612	0.0183	0.0578	0.0579	0.0010	-0.0068	-0.0657	-0.0284	-0.0407	1941	-0.1159
1942	0.0161	-0.0159	-0.0652	-0.0400	0.0796	0.0221	0.0337	0.0164	0.0290	0.0678	-0.0021	0.0549	1942	0.2034
1943	0.0737	0.0583	0.0545	0.0035	0.0552	0.0223	-0.0526	0.0171	0.0263	-0.0108	-0.0654	0.0617	1943	0.2590
1944	0.0171	0.0042	0.0195	-0.0100	0.0505	0.0543	-0.0193	0.0157	-0.0008	0.0023	0.0133	0.0374	1944	0.1975
1945	0.0158	0.0683	-0.0441	0.0902	0.0195	-0.0007	-0.0180	0.0641	0.0438	0.0322	0.0396	0.0116	1945	0.3644
1946	0.0714	-0.0641	0.0480	0.0393	0.0288	-0.0370	-0.0239	-0.0674	-0.0997	-0.0060	-0.0027	0.0457	1946	-0.0807
1947	0.0255	-0.0077	-0.0149	-0.0363	0.0014	0.0554	0.0381	-0.0203	-0.0111	0.0238	-0.0175	0.0233	1947	0.0571
1948	-0.0379	-0.0388	0.0793	0.0292	0.0879	0.0054	-0.0508	0.0158	-0.0276	0.0710	-0.0961	0.0346	1948	0.0550
1949	0.0039	-0.0296	0.0328	-0.0179	-0.0258	0.0014	0.0650	0.0219	0.0263	0.0340	0.0175	0.0486	1949	0.1879
1950	0.0197	0.0199	0.0070	0.0486	0.0509	-0.0548	0.0119	0.0443	0.0592	0.0093	0.0169	0.0513	1950	0.3171
1951	0.0637	0.0157	-0.0156	0.0509	-0.0299	-0.0228	0.0711	0.0478	0.0013	-0.0103	0.0096	0.0424	1951	0.2402
1952	0.0181	-0.0282	0.0503	-0.0402	0.0343	0.0490	0.0196	-0.0071	-0.0176	0.0020	0.0571	0.0382	1952	0.1837
1953	-0.0049	-0.0106	-0.0212	-0.0237	0.0077	-0.0134	0.0273	-0.0501	0.0034	0.0540	0.0204	0.0053	1953	-0.0099
1954	0.0536	0.0111	0.0325	0.0516	0.0418	0.0031	0.0589	-0.0275	0.0851	-0.0167	0.0909	0.0534	1954	0.5262
1955	0.0197	0.0098	-0.0030	0.0396	0.0055	0.0841	0.0622	-0.0025	0.0130	-0.0284	0.0827	0.0015	1955	0.3156

*Compound annual return

Appendix A-1

Large-Capitalization Stocks: Total Return
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1956	-0.0347	0.0413	0.0710	-0.0004	-0.0593	0.0409	0.0530	-0.0328	-0.0440	0.0066	-0.0050	0.0370	1956	0.0656
1957	-0.0401	-0.0264	0.0215	0.0388	0.0437	0.0004	0.0131	-0.0505	-0.0602	-0.0302	0.0231	-0.0395	1957	-0.1078
1958	0.0445	-0.0141	0.0328	0.0337	0.0212	0.0279	0.0449	0.0176	0.0501	0.0270	0.0284	0.0535	1958	0.4336
1959	0.0053	0.0049	0.0020	0.0402	0.0240	-0.0022	0.0363	-0.0102	-0.0443	0.0128	0.0186	0.0292	1959	0.1196
1960	-0.0700	0.0147	-0.0123	-0.0161	0.0326	0.0211	-0.0234	0.0317	-0.0590	-0.0007	0.0465	0.0479	1960	0.0047
1961	0.0645	0.0319	0.0270	0.0051	0.0239	-0.0275	0.0342	0.0243	-0.0184	0.0298	0.0447	0.0046	1961	0.2689
1962	-0.0366	0.0209	-0.0046	-0.0607	-0.0811	-0.0803	0.0652	0.0208	-0.0465	0.0064	0.1086	0.0153	1962	-0.0873
1963	0.0506	-0.0239	0.0370	0.0500	0.0193	-0.0188	-0.0022	0.0535	-0.0097	0.0339	-0.0046	0.0262	1963	0.2280
1964	0.0283	0.0147	0.0165	0.0075	0.0162	0.0178	0.0195	-0.0118	0.0301	0.0096	0.0005	0.0056	1964	0.1648
1965	0.0345	0.0031	-0.0133	0.0356	-0.0030	-0.0473	0.0147	0.0272	0.0334	0.0289	-0.0031	0.0106	1965	0.1245
1966	0.0062	-0.0131	-0.0205	0.0220	-0.0492	-0.0146	-0.0120	-0.0725	-0.0053	0.0494	0.0095	0.0002	1966	-0.1006
1967	0.0798	0.0072	0.0409	0.0437	-0.0477	0.0190	0.0468	-0.0070	0.0342	-0.0276	0.0065	0.0278	1967	0.2398
1968	-0.0425	-0.0261	0.0110	0.0834	0.0161	0.0105	-0.0172	0.0164	0.0400	0.0087	0.0531	-0.0402	1968	0.1106
1969	-0.0068	-0.0426	0.0359	0.0229	0.0026	-0.0542	-0.0587	0.0454	-0.0236	0.0459	-0.0297	-0.0177	1969	-0.0850
1970	-0.0743	0.0557	0.0044	-0.0875	-0.0578	-0.0466	0.0769	0.0478	0.0362	-0.0083	0.0506	0.0598	1970	0.0386
1971	0.0432	0.0117	0.0394	0.0389	-0.0391	0.0033	-0.0387	0.0388	-0.0044	-0.0391	0.0002	0.0888	1971	0.1430
1972	0.0206	0.0277	0.0083	0.0068	0.0197	-0.0194	0.0048	0.0369	-0.0025	0.0119	0.0481	0.0142	1972	0.1899
1973	-0.0149	-0.0352	0.0008	-0.0383	-0.0163	-0.0040	0.0407	-0.0341	0.0427	0.0017	-0.1109	0.0198	1973	-0.1469
1974	-0.0072	-0.0007	-0.0205	-0.0359	-0.0302	-0.0114	-0.0742	-0.0864	-0.1152	0.1681	-0.0489	-0.0156	1974	-0.2647
1975	0.1272	0.0638	0.0254	0.0510	0.0476	0.0477	-0.0644	-0.0176	-0.0312	0.0653	0.0282	-0.0081	1975	0.3723
1976	0.1217	-0.0084	0.0337	-0.0078	-0.0111	0.0443	-0.0048	-0.0018	0.0258	-0.0186	-0.0041	0.0561	1976	0.2393
1977	-0.0473	-0.0182	-0.0105	0.0042	-0.0196	0.0494	-0.0124	-0.0172	0.0015	-0.0389	0.0316	0.0075	1977	-0.0716
1978	-0.0574	-0.0203	0.0294	0.0902	0.0092	-0.0138	0.0583	0.0301	-0.0032	-0.0872	0.0215	0.0196	1978	0.0657
1979	0.0443	-0.0321	0.0596	0.0094	-0.0247	0.0435	0.0134	0.0577	0.0043	-0.0640	0.0475	0.0214	1979	0.1861
1980	0.0622	-0.0001	-0.0972	0.0462	0.0515	0.0316	0.0696	0.0101	0.0294	0.0202	0.1065	-0.0302	1980	0.3250
1981	-0.0418	0.0174	0.0400	-0.0193	0.0026	-0.0063	0.0021	-0.0577	-0.0493	0.0540	0.0413	-0.0256	1981	-0.0492
1982	-0.0131	-0.0559	-0.0052	0.0452	-0.0341	-0.0150	-0.0178	0.1214	0.0125	0.1151	0.0404	0.0193	1982	0.2155
1983	0.0372	0.0229	0.0369	0.0788	-0.0087	0.0389	-0.0295	0.0150	0.0138	-0.0116	0.0211	-0.0052	1983	0.2256
1984	-0.0056	-0.0352	0.0173	0.0095	-0.0554	0.0217	-0.0124	0.1104	0.0002	0.0039	-0.0112	0.0263	1984	0.0627
1985	0.0779	0.0122	0.0007	-0.0009	0.0578	0.0157	-0.0015	-0.0085	-0.0313	0.0462	0.0686	0.0484	1985	0.3173

*Compound annual return

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Appendix A-1

Large-Capitalization Stocks: Total Return
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan-Dec*
1986	0.0056	0.0747	0.0558	-0.0113	0.0532	0.0169	-0.0559	0.0742	-0.0827	0.0577	0.0243	-0.0255	1986	0.1867
1987	0.1347	0.0395	0.0289	-0.0089	0.0087	0.0505	0.0507	0.0373	-0.0219	-0.2154	-0.0824	0.0761	1987	0.0525
1988	0.0421	0.0466	-0.0309	0.0111	0.0086	0.0459	-0.0038	-0.0339	0.0426	0.0278	-0.0143	0.0174	1988	0.1661
1989	0.0732	-0.0249	0.0233	0.0519	0.0405	-0.0057	0.0903	0.0195	-0.0041	-0.0232	0.0204	0.0240	1989	0.3169
1990	-0.0671	0.0129	0.0265	-0.0249	0.0975	-0.0067	-0.0032	-0.0904	-0.0487	-0.0043	0.0646	0.0279	1990	-0.0310
1991	0.0436	0.0715	0.0242	0.0024	0.0431	-0.0458	0.0466	0.0237	-0.0167	0.0134	-0.0403	0.1144	1991	0.3047
1992	-0.0186	0.0130	-0.0194	0.0294	0.0049	-0.0149	0.0409	-0.0205	0.0118	0.0035	0.0341	0.0123	1992	0.0762
1993	0.0084	0.0136	0.0211	-0.0242	0.0268	0.0029	-0.0040	0.0379	-0.0077	0.0207	-0.0095	0.0121	1993	0.1008
1994	0.0340	-0.0271	-0.0436	0.0128	0.0164	-0.0245	0.0328	0.0410	-0.0245	0.0225	-0.0364	0.0148	1994	0.0132
1995	0.0259	0.0390	0.0295	0.0294	0.0400	0.0232	0.0332	0.0025	0.0422	-0.0036	0.0439	0.0193	1995	0.3758
1996	0.0340	0.0093	0.0096	0.0147	0.0258	0.0038	-0.0442	0.0211	0.0563	0.0276	0.0756	-0.0198	1996	0.2296
1997	0.0625	0.0078	-0.0411	0.0597	0.0609	0.0448	0.0796	-0.0560	0.0548	-0.0334	0.0463	0.0172	1997	0.3336
1998	0.0111	0.0721	0.0512	0.0101	-0.0172	0.0406	-0.0106	-0.1446	0.0641	0.0813	0.0606	0.0576	1998	0.2858
1999	0.0418	-0.0311	0.0400	0.0387	-0.0236	0.0555	-0.0312	-0.0049	-0.0274	0.0633	0.0203	0.0589	1999	0.2104
2000	-0.0502	-0.0189	0.0978	-0.0301	-0.0205	0.0247	-0.0156	0.0621	-0.0528	-0.0042	-0.0788	0.0049	2000	-0.0910
2001	0.0355	-0.0912	-0.0634	0.0777	0.0067	-0.0243	-0.0098	-0.0626	-0.0808	0.0191	0.0767	0.0088	2001	-0.1189
2002	-0.0146	-0.0193	0.0376	-0.0606	-0.0074	-0.0712	-0.0780	0.0066	-0.1087	0.0880	0.0589	-0.0587	2002	-0.2210
2003	-0.0262	-0.0150	0.0097	0.0824	0.0527	0.0128	0.0176	0.0195	-0.0106	0.0566	0.0088	0.0524	2003	0.2868
2004	0.0184	0.0139	-0.0151	-0.0157	0.0137	0.0194	-0.0331	0.0040	0.0108	0.0153	0.0405	0.0340	2004	0.1088
2005	-0.0244	0.0210	-0.0177	-0.0190	0.0318	0.0014	0.0372	-0.0091	0.0081	-0.0167	0.0378	0.0003	2005	0.0491
2006	0.0265	0.0027	0.0124	0.0134	-0.0288	0.0014	0.0062	0.0238	0.0258	0.0326	0.0190	0.0140	2006	0.1579
2007	0.0151	-0.0196	0.0112	0.0443	0.0349	-0.0166	-0.0310	0.0150	0.0374	0.0159	-0.0418	-0.0069	2007	0.0549
2008	-0.0600	-0.0325	-0.0043	0.0487	0.0130	-0.0843	-0.0084	0.0145	-0.0891	-0.1679	-0.0718	0.0106	2008	-0.3700
2009	-0.0843	-0.1065	0.0876	0.0957	0.0559	0.0020	0.0756	0.0361	0.0373	-0.0186	0.0600	0.0193	2009	0.2646
2010	-0.0360	0.0310	0.0603	0.0158	-0.0799	-0.0523	0.0701	-0.0451	0.0892	0.0380	0.0001	0.0668	2010	0.1506
2011	0.0237	0.0343	0.0004	0.0296	-0.0113	-0.0167	-0.0203	-0.0543	-0.0703	0.1093	-0.0022	0.0102	2011	0.0211
2012	0.0448	0.0432	0.0329	-0.0063	-0.0601	0.0412	0.0139	0.0225	0.0258	-0.0185	0.0058	0.0091	2012	0.1600
2013	0.0518	0.0136	0.0375	0.0193	0.0234	-0.0134	0.0509	-0.0290	0.0314	0.0460	0.0305	0.0253	2013	0.3239
2014	-0.0346	0.0457	0.0084	0.0074	0.0235	0.0207	-0.0138	0.0400	-0.0140	0.0244	0.0269	-0.0025	2014	0.1369
2015	-0.0300	0.0575	-0.0158	0.0096	0.0129	-0.0194	0.0210	-0.0603	-0.0247	0.0844	0.0030	-0.0158	2015	0.0138
2016	-0.0496	-0.0013	0.0678	0.0039	0.0180	0.0026	0.0369	0.0014	0.0002	-0.0182	0.0370	0.0198	2016	0.1196

*Compound annual return

Appendix A-2

Large-Capitalization Stocks: Income Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1926	0.0016	0.0055	0.0016	0.0026	0.0102	0.0025	0.0024	0.0078	0.0023	0.0030	0.0123	0.0030	1926	0.0541
1927	0.0015	0.0061	0.0022	0.0029	0.0085	0.0027	0.0020	0.0070	0.0018	0.0029	0.0105	0.0029	1927	0.0571
1928	0.0011	0.0051	0.0017	0.0021	0.0071	0.0020	0.0016	0.0062	0.0019	0.0023	0.0092	0.0021	1928	0.0481
1929	0.0012	0.0039	0.0012	0.0016	0.0066	0.0016	0.0014	0.0048	0.0013	0.0020	0.0091	0.0029	1929	0.0398
1930	0.0014	0.0044	0.0013	0.0016	0.0068	0.0020	0.0020	0.0056	0.0019	0.0032	0.0130	0.0036	1930	0.0457
1931	0.0013	0.0050	0.0017	0.0024	0.0093	0.0031	0.0020	0.0087	0.0022	0.0051	0.0180	0.0053	1931	0.0535
1932	0.0012	0.0063	0.0024	0.0027	0.0137	0.0067	0.0045	0.0115	0.0024	0.0037	0.0172	0.0046	1932	0.0616
1933	0.0015	0.0072	0.0018	0.0034	0.0096	0.0021	0.0018	0.0060	0.0018	0.0031	0.0100	0.0030	1933	0.0639
1934	0.0010	0.0045	0.0009	0.0019	0.0076	0.0021	0.0020	0.0069	0.0022	0.0033	0.0114	0.0031	1934	0.0446
1935	0.0011	0.0055	0.0023	0.0024	0.0086	0.0021	0.0020	0.0063	0.0018	0.0026	0.0080	0.0023	1935	0.0495
1936	0.0015	0.0056	0.0014	0.0020	0.0087	0.0028	0.0020	0.0063	0.0019	0.0025	0.0093	0.0029	1936	0.0536
1937	0.0012	0.0045	0.0017	0.0022	0.0079	0.0025	0.0019	0.0071	0.0019	0.0036	0.0146	0.0045	1937	0.0466
1938	0.0019	0.0065	0.0018	0.0035	0.0113	0.0032	0.0017	0.0048	0.0017	0.0016	0.0061	0.0024	1938	0.0483
1939	0.0015	0.0065	0.0016	0.0027	0.0110	0.0026	0.0018	0.0066	0.0027	0.0023	0.0094	0.0033	1939	0.0469
1940	0.0016	0.0066	0.0025	0.0024	0.0107	0.0043	0.0030	0.0087	0.0028	0.0028	0.0108	0.0038	1940	0.0536
1941	0.0019	0.0089	0.0030	0.0040	0.0140	0.0043	0.0030	0.0096	0.0029	0.0029	0.0137	0.0044	1941	0.0671
1942	0.0023	0.0091	0.0023	0.0037	0.0157	0.0037	0.0024	0.0093	0.0023	0.0034	0.0117	0.0032	1942	0.0679
1943	0.0020	0.0076	0.0018	0.0026	0.0104	0.0025	0.0016	0.0068	0.0025	0.0025	0.0101	0.0027	1943	0.0624
1944	0.0017	0.0068	0.0025	0.0025	0.0101	0.0032	0.0015	0.0071	0.0023	0.0023	0.0094	0.0023	1944	0.0548
1945	0.0015	0.0067	0.0021	0.0022	0.0081	0.0027	0.0020	0.0061	0.0019	0.0019	0.0072	0.0017	1945	0.0497
1946	0.0017	0.0054	0.0017	0.0017	0.0064	0.0021	0.0016	0.0056	0.0018	0.0020	0.0088	0.0027	1946	0.0409
1947	0.0020	0.0070	0.0019	0.0026	0.0103	0.0028	0.0020	0.0076	0.0026	0.0026	0.0110	0.0027	1947	0.0549
1948	0.0020	0.0082	0.0021	0.0027	0.0097	0.0024	0.0024	0.0082	0.0025	0.0032	0.0121	0.0041	1948	0.0608
1949	0.0026	0.0099	0.0027	0.0033	0.0115	0.0035	0.0028	0.0100	0.0026	0.0045	0.0162	0.0050	1949	0.0750
1950	0.0024	0.0100	0.0029	0.0035	0.0116	0.0032	0.0034	0.0118	0.0033	0.0051	0.0179	0.0051	1950	0.0877
1951	0.0024	0.0092	0.0028	0.0028	0.0107	0.0033	0.0024	0.0085	0.0021	0.0034	0.0122	0.0035	1951	0.0691
1952	0.0025	0.0083	0.0026	0.0029	0.0111	0.0029	0.0020	0.0075	0.0020	0.0029	0.0106	0.0027	1952	0.0593
1953	0.0023	0.0076	0.0023	0.0028	0.0110	0.0029	0.0021	0.0077	0.0021	0.0030	0.0114	0.0032	1953	0.0546
1954	0.0024	0.0084	0.0023	0.0026	0.0088	0.0024	0.0017	0.0065	0.0020	0.0028	0.0101	0.0026	1954	0.0621
1955	0.0017	0.0063	0.0019	0.0019	0.0068	0.0018	0.0015	0.0053	0.0016	0.0021	0.0078	0.0022	1955	0.0456

*Compound annual return

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Appendix A-2

Large-Capitalization Stocks: Income Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1956	0.0018	0.0066	0.0018	0.0016	0.0064	0.0018	0.0015	0.0053	0.0015	0.0015	0.0059	0.0018	1956	0.0383
1957	0.0017	0.0063	0.0018	0.0018	0.0068	0.0017	0.0017	0.0056	0.0018	0.0019	0.0071	0.0019	1957	0.0384
1958	0.0018	0.0065	0.0020	0.0019	0.0062	0.0018	0.0018	0.0057	0.0017	0.0016	0.0060	0.0015	1958	0.0438
1959	0.0014	0.0051	0.0014	0.0014	0.0050	0.0014	0.0014	0.0048	0.0013	0.0016	0.0054	0.0015	1959	0.0331
1960	0.0015	0.0056	0.0016	0.0014	0.0057	0.0016	0.0014	0.0056	0.0014	0.0017	0.0062	0.0016	1960	0.0326
1961	0.0014	0.0050	0.0014	0.0012	0.0047	0.0014	0.0014	0.0046	0.0013	0.0015	0.0054	0.0014	1961	0.0348
1962	0.0013	0.0046	0.0013	0.0013	0.0049	0.0015	0.0016	0.0055	0.0017	0.0020	0.0071	0.0018	1962	0.0298
1963	0.0014	0.0050	0.0016	0.0015	0.0050	0.0014	0.0013	0.0048	0.0014	0.0017	0.0059	0.0018	1963	0.0361
1964	0.0013	0.0048	0.0013	0.0014	0.0048	0.0014	0.0012	0.0044	0.0013	0.0015	0.0057	0.0017	1964	0.0333
1965	0.0013	0.0046	0.0013	0.0014	0.0047	0.0014	0.0013	0.0047	0.0014	0.0016	0.0056	0.0016	1965	0.0321
1966	0.0013	0.0047	0.0013	0.0015	0.0049	0.0015	0.0014	0.0053	0.0017	0.0018	0.0064	0.0017	1966	0.0311
1967	0.0016	0.0052	0.0015	0.0014	0.0048	0.0015	0.0014	0.0047	0.0014	0.0014	0.0054	0.0015	1967	0.0364
1968	0.0013	0.0051	0.0016	0.0014	0.0049	0.0014	0.0013	0.0049	0.0014	0.0015	0.0051	0.0014	1968	0.0318
1969	0.0013	0.0048	0.0014	0.0014	0.0048	0.0014	0.0014	0.0053	0.0015	0.0016	0.0056	0.0010	1969	0.0298
1970	0.0021	0.0031	0.0029	0.0030	0.0032	0.0034	0.0036	0.0033	0.0032	0.0031	0.0031	0.0030	1970	0.0333
1971	0.0032	0.0022	0.0026	0.0035	0.0017	0.0026	0.0024	0.0029	0.0025	0.0041	0.0013	0.0041	1971	0.0349
1972	0.0011	0.0024	0.0023	0.0024	0.0041	0.0008	0.0024	0.0024	0.0023	0.0025	0.0025	0.0024	1972	0.0295
1973	0.0022	0.0022	0.0022	0.0025	0.0026	0.0026	0.0027	0.0026	0.0041	0.0015	0.0030	0.0032	1973	0.0286
1974	0.0029	0.0029	0.0028	0.0032	0.0033	0.0033	0.0036	0.0039	0.0042	0.0050	0.0043	0.0046	1974	0.0369
1975	0.0044	0.0039	0.0037	0.0037	0.0035	0.0034	0.0033	0.0035	0.0035	0.0037	0.0035	0.0034	1975	0.0537
1976	0.0034	0.0030	0.0030	0.0032	0.0032	0.0034	0.0033	0.0033	0.0032	0.0036	0.0037	0.0036	1976	0.0449
1977	0.0033	0.0034	0.0035	0.0040	0.0040	0.0041	0.0038	0.0038	0.0040	0.0045	0.0047	0.0046	1977	0.0435
1978	0.0041	0.0045	0.0045	0.0048	0.0044	0.0043	0.0044	0.0042	0.0041	0.0044	0.0048	0.0048	1978	0.0533
1979	0.0046	0.0044	0.0045	0.0077	0.0017	0.0048	0.0047	0.0046	0.0043	0.0046	0.0049	0.0046	1979	0.0589
1980	0.0045	0.0043	0.0046	0.0051	0.0049	0.0047	0.0046	0.0043	0.0043	0.0042	0.0042	0.0037	1980	0.0574
1981	0.0039	0.0041	0.0040	0.0041	0.0042	0.0041	0.0043	0.0044	0.0046	0.0048	0.0047	0.0044	1981	0.0488
1982	0.0045	0.0047	0.0049	0.0055	0.0047	0.0054	0.0051	0.0054	0.0049	0.0047	0.0043	0.0041	1982	0.0561
1983	0.0041	0.0039	0.0038	0.0040	0.0035	0.0037	0.0035	0.0037	0.0041	0.0032	0.0037	0.0036	1983	0.0504
1984	0.0036	0.0037	0.0038	0.0040	0.0040	0.0043	0.0040	0.0041	0.0037	0.0039	0.0039	0.0040	1984	0.0457
1985	0.0038	0.0036	0.0035	0.0037	0.0037	0.0035	0.0034	0.0035	0.0036	0.0036	0.0035	0.0033	1985	0.0472

*Compound annual return

Appendix A-2

Large-Capitalization Stocks: Income Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1986	0.0032	0.0033	0.0030	0.0029	0.0030	0.0028	0.0028	0.0030	0.0028	0.0030	0.0027	0.0028	1986	0.0392
1987	0.0029	0.0026	0.0025	0.0026	0.0026	0.0026	0.0024	0.0024	0.0022	0.0023	0.0030	0.0032	1987	0.0364
1988	0.0016	0.0048	0.0025	0.0016	0.0055	0.0026	0.0016	0.0047	0.0029	0.0019	0.0046	0.0028	1988	0.0399
1989	0.0021	0.0040	0.0025	0.0018	0.0053	0.0023	0.0019	0.0040	0.0025	0.0020	0.0039	0.0026	1989	0.0403
1990	0.0017	0.0043	0.0022	0.0019	0.0055	0.0021	0.0020	0.0039	0.0025	0.0024	0.0047	0.0030	1990	0.0343
1991	0.0020	0.0042	0.0020	0.0020	0.0046	0.0021	0.0017	0.0040	0.0024	0.0016	0.0036	0.0028	1991	0.0376
1992	0.0013	0.0034	0.0024	0.0015	0.0039	0.0025	0.0015	0.0035	0.0026	0.0013	0.0038	0.0022	1992	0.0298
1993	0.0013	0.0031	0.0024	0.0012	0.0040	0.0022	0.0013	0.0035	0.0023	0.0013	0.0034	0.0020	1993	0.0291
1994	0.0015	0.0029	0.0021	0.0013	0.0040	0.0023	0.0013	0.0034	0.0025	0.0016	0.0031	0.0025	1994	0.0283
1995	0.0017	0.0029	0.0022	0.0015	0.0036	0.0020	0.0014	0.0028	0.0021	0.0014	0.0028	0.0018	1995	0.0304
1996	0.0014	0.0023	0.0017	0.0013	0.0029	0.0016	0.0016	0.0023	0.0021	0.0014	0.0022	0.0017	1996	0.0243
1997	0.0012	0.0019	0.0015	0.0013	0.0023	0.0013	0.0014	0.0014	0.0016	0.0011	0.0017	0.0014	1997	0.0210
1998	0.0009	0.0017	0.0013	0.0010	0.0016	0.0012	0.0010	0.0012	0.0017	0.0010	0.0015	0.0012	1998	0.0167
1999	0.0008	0.0012	0.0012	0.0008	0.0014	0.0011	0.0008	0.0013	0.0011	0.0007	0.0013	0.0011	1999	0.0136
2000	0.0007	0.0012	0.0011	0.0007	0.0014	0.0007	0.0007	0.0014	0.0007	0.0007	0.0012	0.0008	2000	0.0111
2001	0.0008	0.0011	0.0009	0.0009	0.0016	0.0007	0.0009	0.0015	0.0010	0.0010	0.0015	0.0012	2001	0.0118
2002	0.0010	0.0015	0.0009	0.0008	0.0017	0.0012	0.0010	0.0017	0.0013	0.0016	0.0018	0.0016	2002	0.0139
2003	0.0012	0.0020	0.0014	0.0013	0.0018	0.0014	0.0014	0.0016	0.0013	0.0016	0.0017	0.0017	2003	0.0199
2004	0.0011	0.0017	0.0013	0.0011	0.0016	0.0015	0.0012	0.0018	0.0015	0.0013	0.0019	0.0016	2004	0.0176
2005	0.0009	0.0021	0.0014	0.0011	0.0019	0.0016	0.0012	0.0021	0.0012	0.0011	0.0026	0.0013	2005	0.0184
2006	0.0010	0.0023	0.0014	0.0012	0.0021	0.0013	0.0011	0.0025	0.0012	0.0011	0.0025	0.0014	2006	0.0201
2007	0.0011	0.0023	0.0012	0.0010	0.0023	0.0012	0.0010	0.0021	0.0016	0.0011	0.0022	0.0017	2007	0.0196
2008	0.0012	0.0023	0.0016	0.0012	0.0023	0.0017	0.0015	0.0023	0.0017	0.0015	0.0031	0.0028	2008	0.0192
2009	0.0014	0.0035	0.0022	0.0018	0.0028	0.0018	0.0015	0.0025	0.0016	0.0012	0.0026	0.0015	2009	0.0248
2010	0.0010	0.0025	0.0016	0.0010	0.0021	0.0015	0.0013	0.0023	0.0017	0.0012	0.0024	0.0015	2010	0.0202
2011	0.0011	0.0023	0.0015	0.0011	0.0022	0.0016	0.0011	0.0025	0.0015	0.0016	0.0028	0.0017	2011	0.0213
2012	0.0012	0.0027	0.0016	0.0012	0.0026	0.0016	0.0013	0.0028	0.0016	0.0013	0.0030	0.0020	2012	0.0250
2013	0.0014	0.0025	0.0015	0.0012	0.0026	0.0016	0.0014	0.0023	0.0016	0.0014	0.0024	0.0018	2013	0.0248
2014	0.0010	0.0026	0.0015	0.0012	0.0024	0.0016	0.0013	0.0023	0.0015	0.0012	0.0024	0.0017	2014	0.0216
2015	0.0010	0.0026	0.0016	0.0011	0.0024	0.0017	0.0012	0.0022	0.0017	0.0014	0.0025	0.0018	2015	0.0210
2016	0.0011	0.0028	0.0018	0.0012	0.0026	0.0017	0.0013	0.0026	0.0014	0.0012	0.0029	0.0016	2016	0.0226

*Compound annual return

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Large-Capitalization Stocks: Capital Appreciation Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1926	-0.0016	-0.0440	-0.0591	0.0227	0.0077	0.0432	0.0455	0.0171	0.0229	-0.0313	0.0223	0.0166	1926	0.0572
1927	-0.0208	0.0477	0.0065	0.0172	0.0522	-0.0094	0.0650	0.0445	0.0432	-0.0531	0.0616	0.0250	1927	0.3091
1928	-0.0051	-0.0176	0.1083	0.0324	0.0127	-0.0405	0.0125	0.0741	0.0240	0.0145	0.1199	0.0029	1928	0.3788
1929	0.0571	-0.0058	-0.0023	0.0161	-0.0428	0.1124	0.0456	0.0980	-0.0489	-0.1993	-0.1337	0.0253	1929	-0.1191
1930	0.0625	0.0215	0.0799	-0.0095	-0.0165	-0.1646	0.0367	0.0075	-0.1301	-0.0888	-0.0218	-0.0742	1930	-0.2848
1931	0.0489	0.1144	-0.0692	-0.0959	-0.1372	0.1390	-0.0742	0.0095	-0.2994	0.0844	-0.0978	-0.1453	1931	-0.4707
1932	-0.0283	0.0507	-0.1182	-0.2025	-0.2333	-0.0089	0.3770	0.3754	-0.0369	-0.1386	-0.0589	0.0519	1932	-0.1515
1933	0.0073	-0.1844	0.0336	0.4222	0.1587	0.1317	-0.0880	0.1146	-0.1136	-0.0885	0.1027	0.0223	1933	0.4659
1934	0.1059	-0.0367	-0.0009	-0.0270	-0.0813	0.0208	-0.1152	0.0541	-0.0055	-0.0319	0.0829	-0.0042	1934	-0.0594
1935	-0.0421	-0.0396	-0.0309	0.0956	0.0323	0.0678	0.0831	0.0217	0.0239	0.0751	0.0393	0.0371	1935	0.4137
1936	0.0655	0.0168	0.0254	-0.0771	0.0458	0.0306	0.0681	0.0088	0.0013	0.0750	0.0041	-0.0058	1936	0.2792
1937	0.0378	0.0146	-0.0094	-0.0831	-0.0103	-0.0529	0.1026	-0.0554	-0.1421	-0.1017	-0.1011	-0.0504	1937	-0.3859
1938	0.0133	0.0608	-0.2504	0.1412	-0.0443	0.2470	0.0727	-0.0274	0.0149	0.0760	-0.0334	0.0377	1938	0.2521
1939	-0.0689	0.0325	-0.1354	-0.0055	0.0623	-0.0638	0.1087	-0.0714	0.1646	-0.0146	-0.0491	0.0238	1939	-0.0545
1940	-0.0352	0.0066	0.0099	-0.0049	-0.2395	0.0766	0.0311	0.0262	0.0095	0.0394	-0.0424	-0.0028	1940	-0.1529
1941	-0.0482	-0.0149	0.0040	-0.0653	0.0043	0.0535	0.0548	-0.0087	-0.0097	-0.0686	-0.0421	-0.0451	1941	-0.1786
1942	0.0138	-0.0250	-0.0675	-0.0437	0.0640	0.0184	0.0313	0.0070	0.0267	0.0644	-0.0138	0.0517	1942	0.1243
1943	0.0716	0.0506	0.0527	0.0009	0.0449	0.0198	-0.0543	0.0103	0.0237	-0.0132	-0.0755	0.0590	1943	0.1945
1944	0.0154	-0.0025	0.0169	-0.0125	0.0404	0.0510	-0.0208	0.0087	-0.0031	0.0000	0.0039	0.0351	1944	0.1380
1945	0.0143	0.0616	-0.0462	0.0880	0.0115	-0.0033	-0.0201	0.0580	0.0419	0.0303	0.0324	0.0099	1945	0.3072
1946	0.0697	-0.0695	0.0463	0.0376	0.0224	-0.0391	-0.0255	-0.0729	-0.1015	-0.0080	-0.0115	0.0429	1946	-0.1187
1947	0.0235	-0.0147	-0.0169	-0.0389	-0.0089	0.0526	0.0362	-0.0279	-0.0137	0.0212	-0.0285	0.0207	1947	0.0000
1948	-0.0399	-0.0470	0.0771	0.0265	0.0782	0.0030	-0.0532	0.0076	-0.0301	0.0678	-0.1082	0.0305	1948	-0.0065
1949	0.0013	-0.0394	0.0301	-0.0212	-0.0373	-0.0021	0.0621	0.0120	0.0237	0.0295	0.0012	0.0436	1949	0.1026
1950	0.0173	0.0100	0.0041	0.0451	0.0393	-0.0580	0.0085	0.0325	0.0559	0.0041	-0.0010	0.0461	1950	0.2178
1951	0.0612	0.0065	-0.0183	0.0481	-0.0406	-0.0260	0.0687	0.0393	-0.0009	-0.0138	-0.0026	0.0389	1951	0.1646
1952	0.0156	-0.0365	0.0477	-0.0431	0.0232	0.0461	0.0176	-0.0146	-0.0196	-0.0008	0.0465	0.0355	1952	0.1178
1953	-0.0072	-0.0182	-0.0236	-0.0265	-0.0032	-0.0163	0.0253	-0.0578	0.0013	0.0510	0.0090	0.0020	1953	-0.0662
1954	0.0512	0.0027	0.0302	0.0490	0.0329	0.0007	0.0572	-0.0340	0.0831	-0.0195	0.0808	0.0508	1954	0.4502
1955	0.0181	0.0035	-0.0049	0.0377	-0.0013	0.0823	0.0607	-0.0078	0.0113	-0.0305	0.0749	-0.0007	1955	0.2640

*Compound annual return

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Large-Capitalization Stocks: Capital Appreciation Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1956	-0.0365	0.0347	0.0693	-0.0021	-0.0657	0.0392	0.0515	-0.0381	-0.0455	0.0051	-0.0110	0.0353	1956	0.0262
1957	-0.0418	-0.0326	0.0196	0.0370	0.0369	-0.0013	0.0114	-0.0561	-0.0619	-0.0321	0.0161	-0.0415	1957	-0.1431
1958	0.0428	-0.0206	0.0309	0.0318	0.0150	0.0261	0.0431	0.0119	0.0484	0.0254	0.0224	0.0520	1958	0.3806
1959	0.0038	-0.0002	0.0005	0.0388	0.0189	-0.0036	0.0349	-0.0150	-0.0456	0.0113	0.0132	0.0276	1959	0.0848
1960	-0.0715	0.0092	-0.0139	-0.0175	0.0269	0.0195	-0.0248	0.0261	-0.0604	-0.0024	0.0403	0.0463	1960	-0.0297
1961	0.0632	0.0269	0.0255	0.0038	0.0191	-0.0288	0.0328	0.0196	-0.0197	0.0283	0.0393	0.0032	1961	0.2313
1962	-0.0379	0.0163	-0.0059	-0.0620	-0.0860	-0.0818	0.0636	0.0153	-0.0482	0.0044	0.1016	0.0135	1962	-0.1181
1963	0.0491	-0.0289	0.0355	0.0485	0.0143	-0.0202	-0.0035	0.0487	-0.0110	0.0322	-0.0105	0.0244	1963	0.1889
1964	0.0269	0.0099	0.0152	0.0061	0.0115	0.0164	0.0182	-0.0162	0.0287	0.0081	-0.0052	0.0039	1964	0.1297
1965	0.0332	-0.0015	-0.0145	0.0342	-0.0077	-0.0486	0.0134	0.0225	0.0320	0.0273	-0.0088	0.0090	1965	0.0906
1966	0.0049	-0.0179	-0.0218	0.0205	-0.0541	-0.0161	-0.0135	-0.0778	-0.0070	0.0475	0.0031	-0.0015	1966	-0.1309
1967	0.0782	0.0020	0.0394	0.0422	-0.0524	0.0175	0.0453	-0.0117	0.0328	-0.0291	0.0011	0.0263	1967	0.2009
1968	-0.0438	-0.0312	0.0094	0.0819	0.0112	0.0091	-0.0185	0.0115	0.0385	0.0072	0.0480	-0.0416	1968	0.0766
1969	-0.0082	-0.0474	0.0344	0.0215	-0.0022	-0.0556	-0.0602	0.0401	-0.0250	0.0442	-0.0353	-0.0187	1969	-0.1136
1970	-0.0765	0.0527	0.0015	-0.0905	-0.0610	-0.0500	0.0733	0.0445	0.0330	-0.0114	0.0474	0.0568	1970	0.0010
1971	0.0400	0.0095	0.0368	0.0354	-0.0407	0.0007	-0.0411	0.0359	-0.0070	-0.0432	-0.0011	0.0847	1971	0.1063
1972	0.0195	0.0253	0.0059	0.0044	0.0156	-0.0202	0.0023	0.0345	-0.0049	0.0093	0.0456	0.0118	1972	0.1579
1973	-0.0171	-0.0375	-0.0014	-0.0408	-0.0189	-0.0066	0.0380	-0.0367	0.0401	-0.0013	-0.1139	0.0166	1973	-0.1737
1974	-0.0100	-0.0036	-0.0233	-0.0391	-0.0336	-0.0147	-0.0778	-0.0903	-0.1193	0.1630	-0.0532	-0.0202	1974	-0.2972
1975	0.1228	0.0599	0.0217	0.0473	0.0441	0.0443	-0.0677	-0.0211	-0.0346	0.0616	0.0247	-0.0115	1975	0.3155
1976	0.1183	-0.0114	0.0307	-0.0110	-0.0144	0.0409	-0.0081	-0.0051	0.0226	-0.0222	-0.0078	0.0525	1976	0.1915
1977	-0.0505	-0.0217	-0.0140	0.0002	-0.0236	0.0454	-0.0162	-0.0210	-0.0025	-0.0434	0.0270	0.0028	1977	-0.1150
1978	-0.0615	-0.0248	0.0249	0.0854	0.0048	-0.0181	0.0539	0.0259	-0.0073	-0.0916	0.0166	0.0149	1978	0.0106
1979	0.0397	-0.0365	0.0552	0.0017	-0.0263	0.0387	0.0087	0.0531	0.0000	-0.0686	0.0426	0.0168	1979	0.1231
1980	0.0576	-0.0044	-0.1018	0.0411	0.0466	0.0270	0.0650	0.0058	0.0252	0.0160	0.1024	-0.0339	1980	0.2577
1981	-0.0457	0.0133	0.0360	-0.0235	-0.0017	-0.0104	-0.0022	-0.0621	-0.0538	0.0491	0.0366	-0.0301	1981	-0.0973
1982	-0.0175	-0.0606	-0.0101	0.0397	-0.0388	-0.0204	-0.0229	0.1160	0.0076	0.1104	0.0361	0.0152	1982	0.1476
1983	0.0331	0.0190	0.0331	0.0748	-0.0122	0.0352	-0.0330	0.0113	0.0097	-0.0148	0.0174	-0.0088	1983	0.1727
1984	-0.0092	-0.0389	0.0135	0.0055	-0.0594	0.0175	-0.0165	0.1063	-0.0035	-0.0001	-0.0151	0.0224	1984	0.0140
1985	0.0741	0.0086	-0.0029	-0.0046	0.0541	0.0121	-0.0048	-0.0120	-0.0348	0.0426	0.0651	0.0451	1985	0.2633

*Compound annual return

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Appendix A-3

Large-Capitalization Stocks: Capital Appreciation Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1986	0.0024	0.0715	0.0528	-0.0141	0.0502	0.0141	-0.0587	0.0712	-0.0854	0.0546	0.0216	-0.0283	1986	0.1462
1987	0.1318	0.0369	0.0264	-0.0115	0.0060	0.0479	0.0482	0.0350	-0.0242	-0.2176	-0.0853	0.0729	1987	0.0203
1988	0.0404	0.0418	-0.0333	0.0094	0.0032	0.0433	-0.0054	-0.0386	0.0397	0.0260	-0.0189	0.0147	1988	0.1240
1989	0.0711	-0.0289	0.0208	0.0501	0.0351	-0.0079	0.0884	0.0155	-0.0065	-0.0252	0.0165	0.0214	1989	0.2725
1990	-0.0688	0.0085	0.0243	-0.0269	0.0920	-0.0089	-0.0052	-0.0943	-0.0512	-0.0067	0.0599	0.0248	1990	-0.0656
1991	0.0415	0.0673	0.0222	0.0003	0.0386	-0.0479	0.0449	0.0196	-0.0191	0.0118	-0.0439	0.1116	1991	0.2631
1992	-0.0199	0.0096	-0.0218	0.0279	0.0010	-0.0174	0.0394	-0.0240	0.0091	0.0021	0.0303	0.0101	1992	0.0446
1993	0.0070	0.0105	0.0187	-0.0254	0.0227	0.0008	-0.0053	0.0344	-0.0100	0.0194	-0.0129	0.0101	1993	0.0706
1994	0.0325	-0.0300	-0.0457	0.0115	0.0124	-0.0268	0.0315	0.0376	-0.0269	0.0209	-0.0395	0.0123	1994	-0.0154
1995	0.0243	0.0361	0.0273	0.0280	0.0363	0.0213	0.0318	-0.0003	0.0401	-0.0050	0.0410	0.0174	1995	0.3411
1996	0.0326	0.0069	0.0079	0.0134	0.0229	0.0023	-0.0457	0.0188	0.0542	0.0261	0.0734	-0.0215	1996	0.2026
1997	0.0613	0.0059	-0.0426	0.0584	0.0586	0.0435	0.0781	-0.0574	0.0532	-0.0345	0.0446	0.0157	1997	0.3101
1998	0.0102	0.0704	0.0499	0.0091	-0.0188	0.0394	-0.0116	-0.1458	0.0624	0.0803	0.0591	0.0564	1998	0.2667
1999	0.0410	-0.0323	0.0388	0.0379	-0.0250	0.0544	-0.0320	-0.0063	-0.0286	0.0625	0.0191	0.0578	1999	0.1953
2000	-0.0509	-0.0201	0.0967	-0.0308	-0.0219	0.0239	-0.0163	0.0607	-0.0535	-0.0049	-0.0801	0.0041	2000	-0.1014
2001	0.0346	-0.0923	-0.0642	0.0768	0.0051	-0.0250	-0.0108	-0.0641	-0.0817	0.0181	0.0752	0.0076	2001	-0.1304
2002	-0.0156	-0.0208	0.0367	-0.0614	-0.0091	-0.0725	-0.0790	0.0049	-0.1100	0.0864	0.0571	-0.0603	2002	-0.2337
2003	-0.0274	-0.0170	0.0084	0.0810	0.0509	0.0113	0.0162	0.0179	-0.0119	0.0550	0.0071	0.0508	2003	0.2638
2004	0.0173	0.0122	-0.0164	-0.0168	0.0121	0.0180	-0.0343	0.0023	0.0094	0.0140	0.0386	0.0325	2004	0.0899
2005	-0.0253	0.0189	-0.0191	-0.0201	0.0300	-0.0001	0.0360	-0.0112	0.0069	-0.0177	0.0352	-0.0009	2005	0.0300
2006	0.0255	0.0005	0.0111	0.0122	-0.0309	0.0001	0.0051	0.0213	0.0246	0.0315	0.0165	0.0126	2006	0.1362
2007	0.0141	-0.0218	0.0100	0.0433	0.0326	-0.0178	-0.0320	0.0129	0.0358	0.0148	-0.0440	-0.0086	2007	0.0353
2008	-0.0612	-0.0348	-0.0060	0.0475	0.0107	-0.0860	-0.0099	0.0122	-0.0908	-0.1694	-0.0749	0.0078	2008	-0.3849
2009	-0.0857	-0.1099	0.0854	0.0939	0.0531	0.0002	0.0741	0.0336	0.0357	-0.0198	0.0574	0.0178	2009	0.2345
2010	-0.0370	0.0285	0.0588	0.0148	-0.0820	-0.0539	0.0688	-0.0475	0.0876	0.0369	-0.0023	0.0653	2010	0.1278
2011	0.0226	0.0320	-0.0011	0.0285	-0.0135	-0.0183	-0.0215	-0.0568	-0.0718	0.1077	-0.0051	0.0085	2011	0.0000
2012	0.0436	0.0406	0.0313	-0.0075	-0.0627	0.0396	0.0126	0.0198	0.0242	-0.0198	0.0028	0.0071	2012	0.1341
2013	0.0504	0.0111	0.0360	0.0181	0.0208	-0.0150	0.0495	-0.0313	0.0297	0.0446	0.0280	0.0236	2013	0.2960
2014	-0.0356	0.0431	0.0069	0.0062	0.0210	0.0191	-0.0151	0.0377	-0.0155	0.0232	0.0245	-0.0042	2014	0.1139
2015	-0.0310	0.0549	-0.0174	0.0085	0.0105	-0.0210	0.0197	-0.0626	-0.0264	0.0830	0.0005	-0.0175	2015	-0.0073
2016	-0.0507	-0.0041	0.0660	0.0027	0.0153	0.0009	0.0356	-0.0012	-0.0012	-0.0194	0.0342	0.0182	2016	0.0954

*Compound annual return

Appendix A-4

Small-Capitalization Stocks: Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1926	0.0699	-0.0639	-0.1073	0.0179	-0.0066	0.0378	0.0112	0.0256	-0.0001	-0.0227	0.0207	0.0332	1926	0.0028
1927	0.0296	0.0547	-0.0548	0.0573	0.0734	-0.0303	0.0516	-0.0178	0.0047	-0.0659	0.0808	0.0316	1927	0.2210
1928	0.0482	-0.0236	0.0531	0.0910	0.0438	-0.0842	0.0059	0.0442	0.0890	0.0276	0.1147	-0.0513	1928	0.3969
1929	0.0035	-0.0026	-0.0200	0.0306	-0.1336	0.0533	0.0114	-0.0164	-0.0922	-0.2768	-0.1500	-0.0501	1929	-0.5136
1930	0.1293	0.0643	0.1007	-0.0698	-0.0542	-0.2168	0.0301	-0.0166	-0.1459	-0.1097	-0.0028	-0.1166	1930	-0.3815
1931	0.2103	0.2566	-0.0708	-0.2164	-0.1379	0.1819	-0.0557	-0.0763	-0.3246	0.0770	-0.1008	-0.2195	1931	-0.4975
1932	0.1019	0.0291	-0.1311	-0.2220	-0.1193	0.0033	0.3523	0.7346	-0.1320	-0.1775	-0.1227	-0.0492	1932	-0.0539
1933	-0.0083	-0.1278	0.1118	0.5038	0.6339	0.2617	-0.0550	0.0924	-0.1595	-0.1236	0.0654	0.0055	1933	1.4287
1934	0.3891	0.0166	-0.0012	0.0240	-0.1275	-0.0024	-0.2259	0.1546	-0.0167	0.0097	0.0948	0.0172	1934	0.2422
1935	-0.0328	-0.0592	-0.1189	0.0791	-0.0024	0.0305	0.0855	0.0545	0.0357	0.0994	0.1412	0.0598	1935	0.4019
1936	0.3009	0.0602	0.0066	-0.1795	0.0272	-0.0231	0.0873	0.0210	0.0542	0.0635	0.1400	0.0160	1936	0.6480
1937	0.1267	0.0658	0.0120	-0.1679	-0.0408	-0.1183	0.1235	-0.0736	-0.2539	-0.1093	-0.1453	-0.1694	1937	-0.5801
1938	0.0534	0.0343	-0.3600	0.2776	-0.0849	0.3498	0.1499	-0.1001	-0.0157	0.2136	-0.0689	0.0487	1938	0.3280
1939	-0.0848	0.0107	-0.2466	0.0142	0.1088	-0.1042	0.2535	-0.1590	0.5145	-0.0397	-0.1053	0.0422	1939	0.0035
1940	0.0009	0.0821	0.0632	0.0654	-0.3674	0.1051	0.0231	0.0255	0.0213	0.0545	0.0245	-0.0447	1940	-0.0516
1941	0.0025	-0.0288	0.0319	-0.0669	0.0045	0.0753	0.2165	-0.0060	-0.0469	-0.0672	-0.0495	-0.1204	1941	-0.0900
1942	0.1894	-0.0073	-0.0709	-0.0353	-0.0032	0.0336	0.0737	0.0325	0.0912	0.1087	-0.0511	0.0413	1942	0.4451
1943	0.2132	0.1931	0.1445	0.0933	0.1156	-0.0083	-0.1083	-0.0002	0.0428	0.0123	-0.1113	0.1241	1943	0.8837
1944	0.0641	0.0295	0.0749	-0.0532	0.0740	0.1384	-0.0299	0.0318	-0.0020	-0.0108	0.0499	0.0869	1944	0.5372
1945	0.0482	0.1009	-0.0861	0.1157	0.0500	0.0855	-0.0556	0.0557	0.0679	0.0701	0.1172	0.0171	1945	0.7361
1946	0.1562	-0.0637	0.0273	0.0696	0.0591	-0.0462	-0.0530	-0.0849	-0.1603	-0.0118	-0.0141	0.0373	1946	-0.1163
1947	0.0421	-0.0041	-0.0336	-0.1031	-0.0534	0.0552	0.0789	-0.0037	0.0115	0.0282	-0.0303	0.0359	1947	0.0092
1948	-0.0154	-0.0783	0.0986	0.0368	0.1059	0.0048	-0.0578	0.0006	-0.0526	0.0647	-0.1116	0.0088	1948	-0.0211
1949	0.0182	-0.0481	0.0629	-0.0336	-0.0564	-0.0096	0.0671	0.0256	0.0489	0.0472	0.0016	0.0690	1949	0.1975
1950	0.0492	0.0221	-0.0037	0.0411	0.0255	-0.0777	0.0591	0.0530	0.0521	-0.0059	0.0322	0.0953	1950	0.3875
1951	0.0830	0.0061	-0.0477	0.0367	-0.0331	-0.0529	0.0373	0.0605	0.0215	-0.0222	-0.0083	0.0044	1951	0.0780
1952	0.0191	-0.0300	0.0175	-0.0519	0.0032	0.0272	0.0112	-0.0006	-0.0161	-0.0103	0.0485	0.0160	1952	0.0303
1953	0.0409	0.0269	-0.0067	-0.0287	0.0141	-0.0486	0.0152	-0.0528	-0.0262	0.0292	0.0126	-0.0266	1953	-0.0649
1954	0.0756	0.0094	0.0183	0.0140	0.0451	0.0086	0.0808	0.0014	0.0410	0.0068	0.0779	0.1112	1954	0.6058
1955	0.0201	0.0479	0.0085	0.0150	0.0078	0.0293	0.0064	-0.0028	0.0109	-0.0170	0.0468	0.0163	1955	0.2044

*Compound annual return

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Small-Capitalization Stocks: Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1956	-0.0047	0.0278	0.0431	0.0047	-0.0398	0.0056	0.0283	-0.0134	-0.0260	0.0104	0.0053	0.0038	1956	0.0428
1957	0.0236	-0.0200	0.0167	0.0248	0.0075	0.0073	-0.0060	-0.0386	-0.0452	-0.0832	0.0113	-0.0481	1957	-0.1457
1958	0.1105	-0.0170	0.0471	0.0376	0.0387	0.0324	0.0492	0.0428	0.0518	0.0407	0.0496	0.0313	1958	0.6489
1959	0.0575	0.0295	0.0027	0.0117	0.0014	-0.0042	0.0327	-0.0088	-0.0431	0.0227	0.0222	0.0322	1959	0.1640
1960	-0.0306	0.0050	-0.0315	-0.0187	0.0205	0.0340	-0.0189	0.0525	-0.0738	-0.0401	0.0437	0.0332	1960	-0.0329
1961	0.0915	0.0589	0.0619	0.0127	0.0427	-0.0543	0.0031	0.0130	-0.0339	0.0262	0.0613	0.0079	1961	0.3209
1962	0.0136	0.0187	0.0057	-0.0777	-0.1009	-0.0785	0.0763	0.0289	-0.0659	-0.0373	0.1248	-0.0089	1962	-0.1190
1963	0.0906	0.0034	0.0149	0.0312	0.0436	-0.0118	0.0033	0.0517	-0.0163	0.0236	-0.0106	-0.0048	1963	0.2357
1964	0.0274	0.0365	0.0219	0.0093	0.0157	0.0163	0.0398	-0.0029	0.0402	0.0205	0.0011	-0.0112	1964	0.2352
1965	0.0529	0.0390	0.0238	0.0509	-0.0078	-0.0901	0.0449	0.0595	0.0347	0.0572	0.0371	0.0622	1965	0.4175
1966	0.0756	0.0311	-0.0192	0.0343	-0.0961	-0.0012	-0.0012	-0.1080	-0.0164	-0.0107	0.0491	0.0065	1966	-0.0701
1967	0.1838	0.0450	0.0615	0.0271	-0.0085	0.1017	0.0951	0.0020	0.0565	-0.0311	0.0117	0.0965	1967	0.8357
1968	0.0154	-0.0709	-0.0109	0.1461	0.0999	0.0030	-0.0345	0.0367	0.0599	0.0030	0.0764	0.0062	1968	0.3597
1969	-0.0166	-0.0990	0.0396	0.0395	0.0173	-0.1165	-0.1070	0.0732	-0.0261	0.0610	-0.0557	-0.0687	1969	-0.2505
1970	-0.0608	0.0387	-0.0285	-0.1728	-0.1031	-0.0929	0.0554	0.0949	0.1086	-0.0706	0.0137	0.0726	1970	-0.1743
1971	0.1592	0.0317	0.0564	0.0247	-0.0605	-0.0319	-0.0563	0.0583	-0.0226	-0.0551	-0.0373	0.1144	1971	0.1650
1972	0.1130	0.0296	-0.0143	0.0129	-0.0191	-0.0305	-0.0413	0.0186	-0.0349	-0.0175	0.0592	-0.0214	1972	0.0443
1973	-0.0432	-0.0799	-0.0208	-0.0621	-0.0811	-0.0290	0.1194	-0.0445	0.1064	0.0084	-0.1962	-0.0014	1973	-0.3090
1974	0.1326	-0.0085	-0.0074	-0.0464	-0.0793	-0.0147	-0.0219	-0.0681	-0.0653	0.1063	-0.0438	-0.0788	1974	-0.1995
1975	0.2767	0.0285	0.0618	0.0531	0.0663	0.0750	-0.0254	-0.0574	-0.0182	-0.0050	0.0320	-0.0197	1975	0.5282
1976	0.2684	0.1390	-0.0015	-0.0359	-0.0361	0.0459	0.0045	-0.0290	0.0104	-0.0209	0.0404	0.1180	1976	0.5738
1977	0.0450	-0.0039	0.0131	0.0228	-0.0028	0.0772	0.0030	-0.0107	0.0092	-0.0330	0.1086	0.0081	1977	0.2538
1978	-0.0189	0.0347	0.1032	0.0788	0.0820	-0.0189	0.0684	0.0939	-0.0032	-0.2427	0.0732	0.0168	1978	0.2346
1979	0.1321	-0.0282	0.1120	0.0387	0.0035	0.0472	0.0171	0.0756	-0.0344	-0.1154	0.0858	0.0588	1979	0.4346
1980	0.0836	-0.0284	-0.1778	0.0694	0.0750	0.0452	0.1323	0.0604	0.0418	0.0333	0.0766	-0.0338	1980	0.3988
1981	0.0207	0.0094	0.0943	0.0657	0.0422	0.0076	-0.0316	-0.0684	-0.0733	0.0742	0.0276	-0.0220	1981	0.1388
1982	-0.0196	-0.0296	-0.0086	0.0383	-0.0248	-0.0159	-0.0015	0.0698	0.0327	0.1305	0.0779	0.0132	1982	0.2801
1983	0.0628	0.0712	0.0525	0.0767	0.0870	0.0348	-0.0088	-0.0197	0.0133	-0.0568	0.0516	-0.0145	1983	0.3967
1984	-0.0008	-0.0645	0.0174	-0.0085	-0.0521	0.0300	-0.0420	0.0998	0.0027	-0.0217	-0.0336	0.0150	1984	-0.0667
1985	0.1059	0.0272	-0.0214	-0.0174	0.0276	0.0106	0.0260	-0.0072	-0.0544	0.0261	0.0620	0.0470	1985	0.2466

*Compound annual return

Appendix A-4

Small-Capitalization Stocks: Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1986	0.0112	0.0719	0.0477	0.0064	0.0360	0.0026	-0.0710	0.0218	-0.0559	0.0346	-0.0031	-0.0262	1986	0.0685
1987	0.0943	0.0809	0.0233	-0.0313	-0.0039	0.0266	0.0364	0.0287	-0.0081	-0.2919	-0.0397	0.0520	1987	-0.0930
1988	0.0556	0.0760	0.0408	0.0209	-0.0179	0.0612	-0.0025	-0.0246	0.0227	-0.0123	-0.0437	0.0394	1988	0.2287
1989	0.0404	0.0083	0.0358	0.0279	0.0362	-0.0201	0.0407	0.0122	0.0000	-0.0604	-0.0051	-0.0134	1989	0.1018
1990	-0.0764	0.0187	0.0368	-0.0266	0.0561	0.0144	-0.0382	-0.1296	-0.0829	-0.0572	0.0450	0.0194	1990	-0.2156
1991	0.0841	0.1113	0.0680	0.0034	0.0334	-0.0485	0.0407	0.0261	0.0032	0.0317	-0.0276	0.0601	1991	0.4463
1992	0.1128	0.0452	-0.0249	-0.0403	-0.0014	-0.0519	0.0370	-0.0228	0.0131	0.0259	0.0885	0.0441	1992	0.2335
1993	0.0543	-0.0180	0.0289	-0.0306	0.0342	-0.0036	0.0166	0.0339	0.0316	0.0471	-0.0175	0.0194	1993	0.2098
1994	0.0618	-0.0023	-0.0446	0.0060	-0.0012	-0.0262	0.0184	0.0337	0.0105	0.0115	-0.0326	0.0002	1994	0.0311
1995	0.0283	0.0252	0.0145	0.0352	0.0298	0.0568	0.0645	0.0358	0.0195	-0.0487	0.0192	0.0239	1995	0.3446
1996	0.0028	0.0369	0.0228	0.0848	0.0749	-0.0582	-0.0943	0.0476	0.0291	-0.0175	0.0288	0.0204	1996	0.1762
1997	0.0420	-0.0206	-0.0490	-0.0276	0.1022	0.0498	0.0605	0.0509	0.0844	-0.0386	-0.0155	-0.0171	1997	0.2278
1998	-0.0059	0.0649	0.0481	0.0168	-0.0497	-0.0206	-0.0671	-0.2010	0.0369	0.0356	0.0758	0.0252	1998	-0.0731
1999	0.0279	-0.0687	-0.0379	0.0949	0.0387	0.0568	0.0092	-0.0191	-0.0221	-0.0087	0.0971	0.1137	1999	0.2979
2000	0.0595	0.2358	-0.0751	-0.1251	-0.0808	0.1368	-0.0322	0.0925	-0.0217	-0.0706	-0.1110	0.0189	2000	-0.0359
2001	0.1380	-0.0702	-0.0480	0.0731	0.0960	0.0359	-0.0254	-0.0295	-0.1278	0.0645	0.0674	0.0672	2001	0.2277
2002	0.0110	-0.0277	0.0884	0.0243	-0.0273	-0.0356	-0.1448	-0.0057	-0.0674	0.0257	0.0836	-0.0429	2002	-0.1328
2003	-0.0223	-0.0288	0.0111	0.0928	0.1162	0.0440	0.0738	0.0473	0.0009	0.0894	0.0430	0.0277	2003	0.6070
2004	0.0578	0.0050	0.0014	-0.0409	0.0000	0.0441	-0.0747	-0.0152	0.0501	0.0184	0.0897	0.0458	2004	0.1839
2005	-0.0410	0.0083	-0.0323	-0.0622	0.0603	0.0452	0.0763	-0.0139	0.0061	-0.0281	0.0453	0.0018	2005	0.0569
2006	0.0914	0.0025	0.0455	-0.0041	-0.0589	-0.0089	-0.0345	0.0278	0.0056	0.0545	0.0225	0.0161	2006	0.1617
2007	0.0115	-0.0050	0.0102	0.0150	0.0315	-0.0033	-0.0651	0.0116	0.0148	0.0170	-0.0842	-0.0006	2007	-0.0522
2008	-0.0765	-0.0314	0.0031	0.0207	0.0398	-0.0905	0.0448	0.0338	-0.0737	-0.2071	-0.1284	0.0566	2008	-0.3672
2009	-0.1191	-0.1311	0.0958	0.1739	0.0343	0.0276	0.0982	0.0273	0.0576	-0.0727	0.0178	0.0869	2009	0.2809
2010	-0.0294	0.0439	0.0808	0.0727	-0.0742	-0.0724	0.0714	-0.0798	0.1216	0.0434	0.0424	0.0819	2010	0.3126
2011	-0.0109	0.0587	0.0325	0.0168	-0.0192	-0.0216	-0.0269	-0.0893	-0.1058	0.1543	-0.0060	0.0086	2011	-0.0326
2012	0.0696	0.0191	0.0298	-0.0142	-0.0697	0.0512	-0.0091	0.0312	0.0419	-0.0205	0.0094	0.0380	2012	0.1824
2013	0.0568	0.0110	0.0487	-0.0079	0.0518	0.0046	0.0741	-0.0348	0.0705	0.0348	0.0555	0.0189	2013	0.4507
2014	-0.0443	0.0421	0.0097	-0.0341	0.0010	0.0434	-0.0584	0.0454	-0.0569	0.0652	-0.0075	0.0337	2014	0.0292
2015	-0.0490	0.0603	0.0229	-0.0195	0.0138	0.0163	-0.0244	-0.0443	-0.0431	0.0603	0.0284	-0.0490	2015	-0.0360
2016	-0.0680	0.0110	0.0708	0.0108	0.0107	-0.0017	0.0502	0.0159	0.0072	-0.0353	0.1319	0.0406	2016	0.2565

*Compound annual return

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Appendix A-5

Long-term Corporate Bonds: Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1926	0.0072	0.0045	0.0084	0.0097	0.0044	0.0004	0.0057	0.0044	0.0057	0.0097	0.0057	0.0056	1926	0.0737
1927	0.0056	0.0069	0.0083	0.0055	-0.0011	0.0043	0.0003	0.0083	0.0149	0.0055	0.0068	0.0068	1927	0.0744
1928	0.0027	0.0068	0.0041	0.0014	-0.0078	-0.0024	-0.0010	0.0083	0.0030	0.0083	-0.0036	0.0084	1928	0.0284
1929	0.0043	0.0030	-0.0087	0.0019	0.0045	-0.0046	0.0020	0.0020	0.0034	0.0073	-0.0018	0.0192	1929	0.0327
1930	0.0059	0.0072	0.0138	0.0084	0.0057	0.0110	0.0056	0.0136	0.0108	0.0054	-0.0012	-0.0090	1930	0.0798
1931	0.0203	0.0068	0.0094	0.0067	0.0134	0.0052	0.0052	0.0012	-0.0014	-0.0363	-0.0189	-0.0286	1931	-0.0185
1932	-0.0052	-0.0238	0.0356	-0.0176	0.0107	-0.0009	0.0043	0.0436	0.0301	0.0074	0.0073	0.0139	1932	0.1082
1933	0.0547	-0.0523	0.0047	-0.0095	0.0588	0.0190	0.0161	0.0093	-0.0014	0.0040	-0.0248	0.0257	1933	0.1038
1934	0.0257	0.0146	0.0187	0.0104	0.0090	0.0158	0.0047	0.0047	-0.0061	0.0102	0.0129	0.0101	1934	0.1384
1935	0.0211	0.0141	0.0043	0.0112	0.0042	0.0112	0.0111	-0.0042	0.0000	0.0042	0.0069	0.0083	1935	0.0961
1936	0.0082	0.0054	0.0082	0.0026	0.0040	0.0082	0.0011	0.0067	0.0067	0.0025	0.0109	0.0010	1936	0.0674
1937	0.0024	-0.0046	-0.0114	0.0068	0.0040	0.0053	0.0039	-0.0017	0.0025	0.0067	0.0067	0.0067	1937	0.0275
1938	0.0038	0.0010	-0.0087	0.0138	0.0010	0.0095	0.0066	-0.0019	0.0109	0.0080	0.0037	0.0122	1938	0.0613
1939	0.0022	0.0064	0.0022	0.0064	0.0049	0.0035	-0.0007	-0.0392	0.0151	0.0237	0.0079	0.0078	1939	0.0397
1940	0.0049	0.0021	0.0049	-0.0092	-0.0021	0.0121	0.0021	0.0007	0.0092	0.0049	0.0063	-0.0023	1940	0.0339
1941	0.0006	0.0006	-0.0022	0.0078	0.0049	0.0063	0.0063	0.0034	0.0048	0.0034	-0.0094	0.0006	1941	0.0273
1942	0.0006	-0.0008	0.0063	0.0006	0.0020	0.0034	0.0020	0.0035	0.0020	0.0006	0.0006	0.0049	1942	0.0260
1943	0.0049	0.0006	0.0020	0.0049	0.0048	0.0048	0.0019	0.0019	0.0005	-0.0009	-0.0023	0.0049	1943	0.0283
1944	0.0020	0.0034	0.0048	0.0034	0.0005	0.0020	0.0034	0.0034	0.0019	0.0019	0.0048	0.0149	1944	0.0473
1945	0.0076	0.0046	0.0018	0.0018	-0.0011	0.0032	-0.0011	0.0004	0.0032	0.0032	0.0032	0.0133	1945	0.0408
1946	0.0128	0.0034	0.0034	-0.0043	0.0019	0.0019	-0.0012	-0.0088	-0.0026	0.0020	-0.0025	0.0113	1946	0.0172
1947	0.0005	0.0005	0.0067	0.0020	0.0020	0.0004	0.0020	-0.0071	-0.0131	-0.0099	-0.0098	0.0024	1947	-0.0234
1948	0.0024	0.0039	0.0115	0.0038	0.0008	-0.0083	-0.0052	0.0055	0.0024	0.0024	0.0085	0.0131	1948	0.0414
1949	0.0038	0.0038	0.0007	0.0023	0.0038	0.0084	0.0099	0.0037	0.0021	0.0067	0.0021	-0.0145	1949	0.0331
1950	0.0037	0.0007	0.0022	-0.0008	-0.0008	0.0023	0.0069	0.0038	-0.0039	-0.0008	0.0054	0.0023	1950	0.0212
1951	0.0019	-0.0044	-0.0237	-0.0009	-0.0015	-0.0093	0.0205	0.0114	-0.0057	-0.0145	-0.0061	0.0058	1951	-0.0269
1952	0.0199	-0.0085	0.0076	-0.0004	0.0031	0.0016	0.0016	0.0063	-0.0018	0.0039	0.0108	-0.0091	1952	0.0352
1953	-0.0080	-0.0040	-0.0033	-0.0248	-0.0030	0.0109	0.0177	-0.0085	0.0253	0.0227	-0.0073	0.0172	1953	0.0341
1954	0.0124	0.0198	0.0039	-0.0034	-0.0042	0.0063	0.0040	0.0018	0.0040	0.0040	0.0025	0.0017	1954	0.0539
1955	-0.0097	-0.0063	0.0092	-0.0001	-0.0018	0.0029	-0.0041	-0.0038	0.0076	0.0078	-0.0030	0.0063	1955	0.0048

*Compound annual return

Appendix A-5

Long-term Corporate Bonds: Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1956	0.0104	0.0026	-0.0146	-0.0115	0.0052	-0.0018	-0.0093	-0.0208	0.0012	-0.0105	-0.0126	-0.0082	1956	-0.0681
1957	0.0197	0.0093	0.0050	-0.0066	-0.0075	-0.0322	-0.0110	-0.0009	0.0095	0.0023	0.0311	0.0685	1957	0.0871
1958	0.0099	-0.0008	-0.0046	0.0163	0.0031	-0.0038	-0.0153	-0.0320	-0.0096	0.0107	0.0105	-0.0058	1958	-0.0222
1959	-0.0028	0.0126	-0.0083	-0.0172	-0.0114	0.0044	0.0089	-0.0068	-0.0088	0.0165	0.0135	-0.0096	1959	-0.0097
1960	0.0107	0.0128	0.0191	-0.0022	-0.0021	0.0141	0.0257	0.0117	-0.0063	0.0008	-0.0070	0.0104	1960	0.0907
1961	0.0148	0.0210	-0.0029	-0.0116	0.0049	-0.0080	0.0040	-0.0018	0.0144	0.0127	0.0028	-0.0026	1961	0.0482
1962	0.0080	0.0052	0.0151	0.0142	0.0000	-0.0026	-0.0015	0.0143	0.0089	0.0068	0.0062	0.0023	1962	0.0795
1963	0.0059	0.0023	0.0026	-0.0051	0.0048	0.0043	0.0028	0.0035	-0.0023	0.0049	0.0015	-0.0034	1963	0.0219
1964	0.0087	0.0054	-0.0062	0.0040	0.0057	0.0048	0.0052	0.0037	0.0021	0.0050	-0.0004	0.0088	1964	0.0477
1965	0.0081	0.0009	0.0012	0.0021	-0.0008	0.0003	0.0019	-0.0006	-0.0015	0.0046	-0.0057	-0.0149	1965	-0.0046
1966	0.0022	-0.0113	-0.0059	0.0013	-0.0026	0.0030	-0.0098	-0.0259	0.0078	0.0261	-0.0020	0.0201	1966	0.0020
1967	0.0450	-0.0201	0.0117	-0.0071	-0.0254	-0.0223	0.0041	-0.0007	0.0094	-0.0281	-0.0272	0.0127	1967	-0.0495
1968	0.0361	0.0037	-0.0197	0.0048	0.0032	0.0122	0.0341	0.0206	-0.0053	-0.0160	-0.0226	-0.0233	1968	0.0257
1969	0.0139	-0.0160	-0.0200	0.0335	-0.0227	0.0035	0.0005	-0.0020	-0.0244	0.0127	-0.0471	-0.0134	1969	-0.0809
1970	0.0141	0.0401	-0.0045	-0.0250	-0.0163	0.0001	0.0556	0.0100	0.0139	-0.0096	0.0584	0.0372	1970	0.1837
1971	0.0532	-0.0366	0.0258	-0.0236	-0.0161	0.0107	-0.0025	0.0554	-0.0102	0.0282	0.0029	0.0223	1971	0.1101
1972	-0.0033	0.0107	0.0024	0.0035	0.0163	-0.0068	0.0030	0.0072	0.0031	0.0101	0.0249	-0.0004	1972	0.0726
1973	-0.0054	0.0023	0.0045	0.0061	-0.0039	-0.0056	-0.0476	0.0356	0.0356	-0.0066	0.0078	-0.0089	1973	0.0114
1974	-0.0053	0.0009	-0.0307	-0.0341	0.0105	-0.0285	-0.0211	-0.0268	0.0174	0.0885	0.0117	-0.0075	1974	-0.0306
1975	0.0596	0.0137	-0.0247	-0.0052	0.0106	0.0304	-0.0030	-0.0175	-0.0126	0.0553	-0.0088	0.0442	1975	0.1464
1976	0.0188	0.0061	0.0167	-0.0015	-0.0103	0.0150	0.0149	0.0231	0.0167	0.0070	0.0319	0.0347	1976	0.1865
1977	-0.0303	-0.0020	0.0094	0.0100	0.0106	0.0175	-0.0005	0.0136	-0.0022	-0.0038	0.0061	-0.0105	1977	0.0171
1978	-0.0089	0.0051	0.0042	-0.0023	-0.0108	0.0023	0.0101	0.0257	-0.0048	-0.0205	0.0134	-0.0133	1978	-0.0007
1979	0.0184	-0.0128	0.0107	-0.0052	0.0228	0.0269	-0.0031	0.0006	-0.0179	-0.0890	0.0222	-0.0108	1979	-0.0418
1980	-0.0645	-0.0665	-0.0062	0.1376	0.0560	0.0341	-0.0429	-0.0445	-0.0237	-0.0159	0.0017	0.0248	1980	-0.0276
1981	-0.0130	-0.0269	0.0311	-0.0769	0.0595	0.0023	-0.0372	-0.0345	-0.0199	0.0521	0.1267	-0.0580	1981	-0.0124
1982	-0.0129	0.0312	0.0306	0.0338	0.0245	-0.0468	0.0540	0.0837	0.0623	0.0759	0.0201	0.0108	1982	0.4256
1983	-0.0094	0.0428	0.0072	0.0548	-0.0324	-0.0046	-0.0455	0.0051	0.0392	-0.0025	0.0142	-0.0033	1983	0.0626
1984	0.0270	-0.0172	-0.0235	-0.0073	-0.0483	0.0199	0.0586	0.0307	0.0314	0.0572	0.0212	0.0128	1984	0.1686
1985	0.0325	-0.0373	0.0179	0.0296	0.0820	0.0083	-0.0121	0.0260	0.0071	0.0329	0.0370	0.0469	1985	0.3009

*Compound annual return

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Appendix A-5

Long-term Corporate Bonds: Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1986	0.0045	0.0752	0.0256	0.0016	-0.0164	0.0218	0.0031	0.0275	-0.0114	0.0189	0.0233	0.0117	1986	0.1985
1987	0.0216	0.0058	-0.0087	-0.0502	-0.0052	0.0155	-0.0119	-0.0075	-0.0422	0.0507	0.0125	0.0212	1987	-0.0027
1988	0.0517	0.0138	-0.0188	-0.0149	-0.0057	0.0379	-0.0111	0.0054	0.0326	0.0273	-0.0169	0.0039	1988	0.1070
1989	0.0202	-0.0129	0.0064	0.0213	0.0379	0.0395	0.0178	-0.0163	0.0040	0.0276	0.0070	0.0006	1989	0.1623
1990	-0.0191	-0.0012	-0.0011	-0.0191	0.0385	0.0216	0.0102	-0.0292	0.0091	0.0132	0.0285	0.0167	1990	0.0678
1991	0.0150	0.0121	0.0108	0.0138	0.0039	-0.0018	0.0167	0.0275	0.0271	0.0043	0.0106	0.0436	1991	0.1989
1992	-0.0173	0.0096	-0.0073	0.0016	0.0254	0.0156	0.0308	0.0090	0.0099	-0.0156	0.0069	0.0228	1992	0.0939
1993	0.0250	0.0256	0.0025	0.0052	0.0020	0.0293	0.0100	0.0287	0.0043	0.0051	-0.0188	0.0067	1993	0.1319
1994	0.0202	-0.0286	-0.0383	-0.0097	-0.0062	-0.0081	0.0309	-0.0031	-0.0265	-0.0050	0.0018	0.0157	1994	-0.0576
1995	0.0256	0.0289	0.0095	0.0175	0.0631	0.0079	-0.0101	0.0214	0.0153	0.0185	0.0242	0.0228	1995	0.2720
1996	0.0014	-0.0373	-0.0130	-0.0160	0.0005	0.0172	0.0010	-0.0070	0.0259	0.0361	0.0263	-0.0186	1996	0.0140
1997	-0.0028	0.0028	-0.0221	0.0184	0.0128	0.0187	0.0528	-0.0240	0.0226	0.0191	0.0101	0.0163	1997	0.1295
1998	0.0137	-0.0007	0.0038	0.0053	0.0167	0.0115	-0.0056	0.0089	0.0413	-0.0190	0.0270	0.0010	1998	0.1076
1999	0.0123	-0.0401	0.0002	-0.0024	-0.0176	-0.0160	-0.0113	-0.0026	0.0093	0.0047	-0.0024	-0.0102	1999	-0.0745
2000	-0.0021	0.0092	0.0169	-0.0115	-0.0161	0.0326	0.0179	0.0135	0.0046	0.0045	0.0263	0.0270	2000	0.1287
2001	0.0359	0.0127	-0.0029	-0.0128	0.0132	0.0055	0.0361	0.0157	-0.0152	0.0437	-0.0188	-0.0090	2001	0.1065
2002	0.0175	0.0130	-0.0295	0.0253	0.0113	0.0073	0.0094	0.0452	0.0330	-0.0240	0.0103	0.0361	2002	0.1633
2003	0.0021	0.0264	-0.0080	0.0229	0.0471	-0.0143	-0.0881	0.0219	0.0503	-0.0203	0.0052	0.0139	2003	0.0527
2004	0.0187	0.0176	0.0116	-0.0534	-0.0071	0.0093	0.0184	0.0395	0.0101	0.0164	-0.0200	0.0257	2004	0.0872
2005	0.0277	-0.0112	-0.0125	0.0327	0.0295	0.0141	-0.0244	0.0233	-0.0310	-0.0204	0.0099	0.0225	2005	0.0587
2006	-0.0093	0.0128	-0.0404	-0.0224	-0.0020	0.0039	0.0237	0.0361	0.0183	0.0127	0.0246	-0.0232	2006	0.0324
2007	-0.0051	0.0287	-0.0231	0.0140	-0.0178	-0.0148	-0.0032	0.0152	0.0135	0.0088	0.0079	0.0028	2007	0.0260
2008	0.0017	-0.0071	-0.0059	0.0091	-0.0277	-0.0061	-0.0109	0.0121	-0.0863	-0.0450	0.1174	0.1560	2008	0.0878
2009	-0.0949	-0.0308	-0.0018	-0.0030	0.0489	0.0350	0.0565	0.0235	0.0273	0.0016	0.0044	-0.0275	2009	0.0302
2010	0.0096	0.0039	0.0045	0.0357	-0.0051	0.0519	0.0170	0.0473	-0.0144	-0.0203	-0.0057	-0.0036	2010	0.1244
2011	-0.0198	0.0157	-0.0072	0.0239	0.0257	-0.0210	0.0473	0.0240	0.0575	0.0094	-0.0356	0.0512	2011	0.1795
2012	0.0194	0.0057	-0.0303	0.0251	0.0344	0.0064	0.0612	-0.0093	-0.0126	0.0206	-0.0092	-0.0062	2012	0.1068
2013	-0.0313	0.0093	-0.0018	0.0349	-0.0536	-0.0371	0.0031	-0.0074	0.0014	0.0211	-0.0086	0.0002	2013	-0.0707
2014	0.0331	0.0168	0.0062	0.0160	0.0188	0.0020	0.0024	0.0356	-0.0271	0.0225	0.0173	0.0183	2014	0.1728
2015	0.0599	-0.0320	0.0058	-0.0223	-0.0204	-0.0320	0.0239	-0.0067	0.0133	0.0020	0.0020	0.0000	2015	-0.0102
2016	0.0067	0.0232	0.0423	0.0146	0.0016	0.0377	0.0245	0.0016	-0.0119	-0.0263	-0.0510	0.0059	2016	0.0670

*Compound annual return

Appendix A-6

Long-term Government Bonds: Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1926	0.0138	0.0063	0.0041	0.0076	0.0014	0.0038	0.0004	0.0000	0.0038	0.0102	0.0160	0.0078	1926	0.0777
1927	0.0075	0.0088	0.0253	-0.0005	0.0109	-0.0069	0.0050	0.0076	0.0018	0.0099	0.0097	0.0072	1927	0.0893
1928	-0.0036	0.0061	0.0045	-0.0004	-0.0077	0.0041	-0.0217	0.0076	-0.0041	0.0158	0.0003	0.0004	1928	0.0010
1929	-0.0090	-0.0157	-0.0144	0.0275	-0.0162	0.0110	0.0000	-0.0034	0.0028	0.0382	0.0236	-0.0089	1929	0.0342
1930	-0.0057	0.0129	0.0083	-0.0016	0.0140	0.0051	0.0034	0.0013	0.0074	0.0035	0.0042	-0.0070	1930	0.0466
1931	-0.0121	0.0085	0.0104	0.0086	0.0145	0.0004	-0.0042	0.0012	-0.0281	-0.0330	0.0027	-0.0220	1931	-0.0531
1932	0.0034	0.0413	-0.0018	0.0604	-0.0188	0.0065	0.0481	0.0003	0.0057	-0.0017	0.0032	0.0131	1932	0.1684
1933	0.0148	-0.0258	0.0097	-0.0032	0.0303	0.0050	-0.0017	0.0044	0.0023	-0.0091	-0.0149	-0.0113	1933	-0.0007
1934	0.0257	0.0081	0.0197	0.0126	0.0131	0.0067	0.0040	-0.0118	-0.0146	0.0182	0.0037	0.0112	1934	0.1003
1935	0.0182	0.0092	0.0041	0.0079	-0.0057	0.0092	0.0046	-0.0133	0.0009	0.0061	0.0010	0.0070	1935	0.0498
1936	0.0055	0.0081	0.0106	0.0035	0.0040	0.0021	0.0060	0.0111	-0.0031	0.0006	0.0205	0.0038	1936	0.0752
1937	-0.0013	0.0086	-0.0412	0.0039	0.0053	-0.0018	0.0138	-0.0104	0.0045	0.0042	0.0096	0.0082	1937	0.0023
1938	0.0057	0.0052	-0.0037	0.0210	0.0044	0.0004	0.0043	0.0000	0.0022	0.0087	-0.0022	0.0080	1938	0.0553
1939	0.0059	0.0080	0.0125	0.0118	0.0171	-0.0027	0.0113	-0.0201	-0.0545	0.0410	0.0162	0.0145	1939	0.0594
1940	-0.0017	0.0027	0.0177	-0.0035	-0.0299	0.0258	0.0052	0.0028	0.0110	0.0031	0.0205	0.0067	1940	0.0609
1941	-0.0201	0.0020	0.0096	0.0129	0.0027	0.0066	0.0022	0.0018	-0.0012	0.0140	-0.0029	-0.0177	1941	0.0093
1942	0.0069	0.0011	0.0092	-0.0029	0.0075	0.0003	0.0018	0.0038	0.0003	0.0024	-0.0035	0.0049	1942	0.0322
1943	0.0033	-0.0006	0.0009	0.0048	0.0050	0.0018	-0.0001	0.0021	0.0011	0.0005	-0.0001	0.0018	1943	0.0208
1944	0.0021	0.0032	0.0021	0.0013	0.0028	0.0008	0.0036	0.0027	0.0014	0.0012	0.0024	0.0042	1944	0.0281
1945	0.0127	0.0077	0.0021	0.0160	0.0056	0.0169	-0.0086	0.0026	0.0054	0.0104	0.0125	0.0194	1945	0.1073
1946	0.0025	0.0032	0.0010	-0.0135	-0.0012	0.0070	-0.0040	-0.0112	-0.0009	0.0074	-0.0054	0.0145	1946	-0.0010
1947	-0.0006	0.0021	0.0020	-0.0037	0.0033	0.0010	0.0063	0.0081	-0.0044	-0.0037	-0.0174	-0.0192	1947	-0.0262
1948	0.0020	0.0046	0.0034	0.0045	0.0141	-0.0084	-0.0021	0.0001	0.0014	0.0007	0.0076	0.0056	1948	0.0340
1949	0.0082	0.0049	0.0074	0.0011	0.0019	0.0167	0.0033	0.0111	-0.0011	0.0019	0.0021	0.0052	1949	0.0645
1950	-0.0061	0.0021	0.0008	0.0030	0.0033	-0.0025	0.0055	0.0014	-0.0072	-0.0048	0.0035	0.0016	1950	0.0006
1951	0.0058	-0.0074	-0.0157	-0.0063	-0.0069	-0.0062	0.0138	0.0099	-0.0080	0.0010	-0.0136	-0.0061	1951	-0.0393
1952	0.0028	0.0014	0.0111	0.0171	-0.0034	0.0003	-0.0020	-0.0070	-0.0130	0.0148	-0.0015	-0.0086	1952	0.0116
1953	0.0012	-0.0087	-0.0088	-0.0105	-0.0148	0.0223	0.0039	-0.0008	0.0299	0.0074	-0.0049	0.0206	1953	0.0364
1954	0.0089	0.0240	0.0058	0.0104	-0.0087	0.0163	0.0134	-0.0036	-0.0010	0.0006	-0.0025	0.0064	1954	0.0719
1955	-0.0241	-0.0078	0.0087	0.0001	0.0073	-0.0076	-0.0102	0.0004	0.0073	0.0144	-0.0045	0.0037	1955	-0.0129

*Compound annual return

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Appendix A-6

Long-term Government Bonds: Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1956	0.0083	-0.0002	-0.0149	-0.0113	0.0225	0.0027	-0.0209	-0.0187	0.0050	-0.0054	-0.0057	-0.0179	1956	-0.0559
1957	0.0346	0.0025	-0.0024	-0.0222	-0.0023	-0.0180	-0.0041	0.0002	0.0076	-0.0050	0.0533	0.0307	1957	0.0746
1958	-0.0084	0.0100	0.0102	0.0186	0.0001	-0.0160	-0.0278	-0.0436	-0.0117	0.0139	0.0120	-0.0181	1958	-0.0609
1959	-0.0080	0.0117	0.0017	-0.0117	-0.0006	0.0010	0.0060	-0.0041	-0.0057	0.0150	-0.0119	-0.0159	1959	-0.0226
1960	0.0112	0.0204	0.0282	-0.0170	0.0152	0.0173	0.0368	-0.0057	0.0075	-0.0028	-0.0066	0.0279	1960	0.1378
1961	-0.0107	0.0200	-0.0038	0.0115	-0.0046	-0.0075	0.0035	-0.0038	0.0129	0.0071	-0.0020	-0.0125	1961	0.0097
1962	-0.0014	0.0103	0.0253	0.0082	0.0046	-0.0076	-0.0109	0.0187	0.0061	0.0084	0.0021	0.0035	1962	0.0689
1963	-0.0001	0.0008	0.0009	-0.0012	0.0023	0.0019	0.0031	0.0021	0.0004	-0.0026	0.0051	-0.0006	1963	0.0121
1964	-0.0014	-0.0011	0.0037	0.0047	0.0050	0.0069	0.0008	0.0020	0.0050	0.0043	0.0017	0.0030	1964	0.0351
1965	0.0040	0.0014	0.0054	0.0036	0.0018	0.0047	0.0022	-0.0013	-0.0034	0.0027	-0.0062	-0.0078	1965	0.0071
1966	-0.0104	-0.0250	0.0296	-0.0063	-0.0059	-0.0016	-0.0037	-0.0206	0.0332	0.0228	-0.0149	0.0413	1966	0.0365
1967	0.0154	-0.0221	0.0198	-0.0291	-0.0039	-0.0312	0.0068	-0.0084	-0.0005	-0.0400	-0.0197	0.0192	1967	-0.0918
1968	0.0328	-0.0033	-0.0212	0.0227	0.0043	0.0230	0.0289	-0.0003	-0.0102	-0.0132	-0.0269	-0.0363	1968	-0.0026
1969	-0.0206	0.0042	0.0010	0.0427	-0.0490	0.0214	0.0079	-0.0069	-0.0531	0.0365	-0.0243	-0.0068	1969	-0.0507
1970	-0.0021	0.0587	-0.0068	-0.0413	-0.0468	0.0486	0.0319	-0.0019	0.0228	-0.0109	0.0791	-0.0084	1970	0.1211
1971	0.0506	-0.0163	0.0526	-0.0283	-0.0006	-0.0159	0.0030	0.0471	0.0204	0.0167	-0.0047	0.0044	1971	0.1323
1972	-0.0064	0.0088	-0.0082	0.0027	0.0270	-0.0065	0.0216	0.0029	-0.0083	0.0234	0.0226	-0.0229	1972	0.0569
1973	-0.0321	0.0014	0.0082	0.0046	-0.0105	-0.0021	-0.0433	0.0391	0.0318	0.0215	-0.0183	-0.0082	1973	-0.0111
1974	-0.0083	-0.0024	-0.0292	-0.0253	0.0123	0.0045	-0.0029	-0.0232	0.0247	0.0489	0.0296	0.0171	1974	0.0435
1975	0.0225	0.0131	-0.0267	-0.0182	0.0212	0.0292	-0.0087	-0.0058	-0.0098	0.0475	-0.0109	0.0390	1975	0.0920
1976	0.0090	0.0062	0.0166	0.0018	-0.0158	0.0208	0.0078	0.0211	0.0145	0.0084	0.0339	0.0327	1976	0.1675
1977	-0.0388	-0.0049	0.0091	0.0071	0.0125	0.0164	-0.0070	0.0198	-0.0029	-0.0093	0.0093	-0.0168	1977	-0.0069
1978	-0.0080	0.0004	-0.0021	-0.0005	-0.0058	-0.0062	0.0143	0.0218	-0.0106	-0.0200	0.0189	-0.0130	1978	-0.0118
1979	0.0191	-0.0135	0.0129	-0.0112	0.0261	0.0311	-0.0085	-0.0035	-0.0122	-0.0841	0.0311	0.0057	1979	-0.0123
1980	-0.0741	-0.0467	-0.0315	0.1523	0.0419	0.0359	-0.0476	-0.0432	-0.0262	-0.0263	0.0100	0.0352	1980	-0.0395
1981	-0.0115	-0.0436	0.0384	-0.0518	0.0622	-0.0179	-0.0353	-0.0386	-0.0145	0.0829	0.1410	-0.0713	1981	0.0186
1982	0.0046	0.0182	0.0231	0.0373	0.0034	-0.0223	0.0501	0.0781	0.0618	0.0634	-0.0002	0.0312	1982	0.4036
1983	-0.0309	0.0492	-0.0094	0.0350	-0.0386	0.0039	-0.0486	0.0020	0.0505	-0.0132	0.0183	-0.0059	1983	0.0065
1984	0.0244	-0.0178	-0.0156	-0.0106	-0.0516	0.0150	0.0693	0.0266	0.0343	0.0561	0.0118	0.0091	1984	0.1548
1985	0.0364	-0.0493	0.0307	0.0242	0.0896	0.0142	-0.0180	0.0259	-0.0021	0.0338	0.0401	0.0541	1985	0.3097

*Compound annual return

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Long-term Government Bonds: Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1986	-0.0025	0.1145	0.0770	-0.0080	-0.0505	0.0613	-0.0108	0.0499	-0.0500	0.0289	0.0267	-0.0018	1986	0.2453
1987	0.0161	0.0202	-0.0223	-0.0473	-0.0105	0.0098	-0.0178	-0.0165	-0.0369	0.0623	0.0037	0.0165	1987	-0.0271
1988	0.0666	0.0052	-0.0307	-0.0160	-0.0102	0.0368	-0.0170	0.0058	0.0345	0.0308	-0.0196	0.0110	1988	0.0967
1989	0.0203	-0.0179	0.0122	0.0159	0.0401	0.0550	0.0238	-0.0259	0.0019	0.0379	0.0078	-0.0006	1989	0.1811
1990	-0.0343	-0.0025	-0.0044	-0.0202	0.0415	0.0230	0.0107	-0.0419	0.0117	0.0215	0.0402	0.0187	1990	0.0618
1991	0.0130	0.0030	0.0038	0.0140	0.0000	-0.0063	0.0157	0.0340	0.0303	0.0054	0.0082	0.0581	1991	0.1930
1992	-0.0324	0.0051	-0.0094	0.0016	0.0243	0.0200	0.0398	0.0067	0.0185	-0.0198	0.0010	0.0246	1992	0.0805
1993	0.0280	0.0354	0.0021	0.0072	0.0047	0.0449	0.0191	0.0434	0.0005	0.0096	-0.0259	0.0020	1993	0.1824
1994	0.0257	-0.0450	-0.0395	-0.0150	-0.0082	-0.0100	0.0363	-0.0086	-0.0331	-0.0025	0.0066	0.0161	1994	-0.0777
1995	0.0273	0.0287	0.0091	0.0169	0.0790	0.0139	-0.0168	0.0236	0.0175	0.0294	0.0249	0.0272	1995	0.3167
1996	-0.0011	-0.0483	-0.0210	-0.0165	-0.0054	0.0203	0.0018	-0.0139	0.0290	0.0404	0.0351	-0.0256	1996	-0.0093
1997	-0.0079	0.0005	-0.0252	0.0255	0.0095	0.0197	0.0626	-0.0317	0.0316	0.0341	0.0148	0.0184	1997	0.1585
1998	0.0200	-0.0072	0.0025	0.0026	0.0182	0.0228	-0.0040	0.0455	0.0395	-0.0218	0.0097	-0.0032	1998	0.1306
1999	0.0121	-0.0520	-0.0008	0.0021	-0.0185	-0.0078	-0.0079	-0.0051	0.0084	-0.0012	-0.0061	-0.0155	1999	-0.0896
2000	0.0228	0.0264	0.0367	-0.0076	-0.0054	0.0244	0.0173	0.0240	-0.0157	0.0187	0.0319	0.0243	2000	0.2148
2001	0.0005	0.0191	-0.0074	-0.0313	0.0037	0.0085	0.0376	0.0206	0.0081	0.0464	-0.0471	-0.0183	2001	0.0370
2002	0.0138	0.0115	-0.0436	0.0410	0.0015	0.0187	0.0303	0.0464	0.0417	-0.0294	-0.0122	0.0507	2002	0.1784
2003	-0.0106	0.0329	-0.0135	0.0102	0.0592	-0.0154	-0.0982	0.0166	0.0546	-0.0283	0.0027	0.0139	2003	0.0145
2004	0.0187	0.0230	0.0141	-0.0588	-0.0051	0.0121	0.0155	0.0395	0.0096	0.0154	-0.0234	0.0250	2004	0.0851
2005	0.0300	-0.0128	-0.0072	0.0373	0.0297	0.0167	-0.0288	0.0333	-0.0338	-0.0196	0.0076	0.0267	2005	0.0781
2006	-0.0118	0.0238	-0.0539	-0.0247	0.0010	0.0092	0.0199	0.0299	0.0170	0.0077	0.0207	-0.0236	2006	0.0119
2007	-0.0102	0.0335	-0.0145	0.0085	-0.0200	-0.0091	0.0284	0.0199	0.0012	0.0155	0.0468	-0.0029	2007	0.0988
2008	0.0213	0.0018	0.0106	-0.0288	-0.0164	0.0220	-0.0025	0.0242	0.0112	-0.0383	0.1443	0.0967	2008	0.2587
2009	-0.1124	-0.0056	0.0641	-0.0649	-0.0248	0.0083	0.0019	0.0231	0.0176	-0.0171	0.0208	-0.0584	2009	-0.1490
2010	0.0264	0.0032	-0.0179	0.0304	0.0437	0.0446	0.0024	0.0702	-0.0153	-0.0317	-0.0137	-0.0388	2010	0.1014
2011	-0.0196	0.0113	-0.0006	0.0199	0.0355	-0.0179	0.0422	0.0862	0.0704	-0.0306	0.0251	0.0270	2011	0.2710
2012	0.0002	-0.0196	-0.0302	0.0409	0.0643	-0.0136	0.0247	-0.0068	-0.0146	-0.0014	0.0144	-0.0202	2012	0.0343
2013	-0.0332	0.0114	-0.0062	0.0378	-0.0629	-0.0285	-0.0173	-0.0079	0.0061	0.0128	-0.0236	-0.0207	2013	-0.1278
2014	0.0548	0.0074	0.0063	0.0181	0.0279	-0.0025	0.0057	0.0369	-0.0170	0.0300	0.0286	0.0290	2014	0.2471
2015	0.0709	-0.0523	0.0137	-0.0250	-0.0159	-0.0298	0.0329	0.0012	0.0174	-0.0053	-0.0065	-0.0022	2015	-0.0065
2016	0.0476	0.0294	-0.0003	-0.0053	0.0082	0.0590	0.0081	-0.0140	-0.0124	-0.0314	-0.0599	-0.0057	2016	0.0175

*Compound annual return

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Long-term Government Bonds: Income Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1926	0.0031	0.0028	0.0032	0.0030	0.0028	0.0033	0.0031	0.0031	0.0030	0.0030	0.0031	0.0030	1926	0.0373
1927	0.0030	0.0027	0.0029	0.0027	0.0028	0.0027	0.0027	0.0029	0.0027	0.0028	0.0027	0.0027	1927	0.0341
1928	0.0027	0.0025	0.0027	0.0026	0.0027	0.0027	0.0027	0.0029	0.0027	0.0030	0.0027	0.0029	1928	0.0322
1929	0.0029	0.0027	0.0028	0.0034	0.0030	0.0029	0.0032	0.0030	0.0032	0.0031	0.0026	0.0031	1929	0.0347
1930	0.0029	0.0026	0.0029	0.0027	0.0027	0.0029	0.0028	0.0026	0.0029	0.0027	0.0026	0.0028	1930	0.0332
1931	0.0028	0.0026	0.0029	0.0027	0.0026	0.0028	0.0027	0.0027	0.0027	0.0029	0.0031	0.0032	1931	0.0333
1932	0.0032	0.0032	0.0031	0.0030	0.0028	0.0028	0.0028	0.0028	0.0026	0.0027	0.0026	0.0027	1932	0.0369
1933	0.0027	0.0023	0.0027	0.0025	0.0028	0.0025	0.0026	0.0026	0.0025	0.0026	0.0025	0.0028	1933	0.0312
1934	0.0029	0.0024	0.0027	0.0025	0.0025	0.0024	0.0024	0.0024	0.0023	0.0027	0.0025	0.0025	1934	0.0318
1935	0.0025	0.0021	0.0022	0.0023	0.0023	0.0022	0.0024	0.0023	0.0023	0.0023	0.0024	0.0024	1935	0.0281
1936	0.0024	0.0023	0.0024	0.0022	0.0022	0.0024	0.0023	0.0023	0.0021	0.0023	0.0022	0.0022	1936	0.0277
1937	0.0021	0.0020	0.0022	0.0023	0.0022	0.0025	0.0024	0.0023	0.0023	0.0023	0.0024	0.0023	1937	0.0266
1938	0.0023	0.0021	0.0023	0.0022	0.0022	0.0021	0.0021	0.0022	0.0021	0.0022	0.0021	0.0022	1938	0.0264
1939	0.0021	0.0019	0.0021	0.0019	0.0020	0.0018	0.0019	0.0018	0.0019	0.0023	0.0020	0.0019	1939	0.0240
1940	0.0020	0.0018	0.0019	0.0018	0.0019	0.0019	0.0020	0.0019	0.0018	0.0018	0.0018	0.0017	1940	0.0223
1941	0.0016	0.0016	0.0018	0.0017	0.0017	0.0016	0.0015	0.0016	0.0016	0.0016	0.0014	0.0016	1941	0.0194
1942	0.0021	0.0019	0.0021	0.0020	0.0019	0.0021	0.0021	0.0021	0.0020	0.0021	0.0020	0.0021	1942	0.0246
1943	0.0020	0.0019	0.0021	0.0020	0.0019	0.0021	0.0021	0.0021	0.0020	0.0020	0.0021	0.0021	1943	0.0244
1944	0.0021	0.0020	0.0021	0.0020	0.0022	0.0020	0.0021	0.0021	0.0020	0.0021	0.0020	0.0020	1944	0.0246
1945	0.0021	0.0018	0.0020	0.0019	0.0019	0.0019	0.0018	0.0019	0.0018	0.0019	0.0018	0.0018	1945	0.0234
1946	0.0017	0.0015	0.0016	0.0017	0.0018	0.0016	0.0019	0.0017	0.0018	0.0019	0.0018	0.0019	1946	0.0204
1947	0.0018	0.0016	0.0018	0.0017	0.0017	0.0019	0.0018	0.0017	0.0018	0.0018	0.0017	0.0021	1947	0.0213
1948	0.0020	0.0019	0.0022	0.0020	0.0018	0.0021	0.0019	0.0021	0.0020	0.0019	0.0021	0.0020	1948	0.0240
1949	0.0020	0.0018	0.0019	0.0018	0.0020	0.0019	0.0017	0.0019	0.0017	0.0018	0.0017	0.0017	1949	0.0225
1950	0.0018	0.0016	0.0018	0.0016	0.0019	0.0017	0.0018	0.0018	0.0017	0.0019	0.0018	0.0018	1950	0.0212
1951	0.0020	0.0017	0.0019	0.0020	0.0021	0.0020	0.0023	0.0021	0.0019	0.0023	0.0021	0.0022	1951	0.0238
1952	0.0023	0.0021	0.0023	0.0022	0.0020	0.0022	0.0022	0.0021	0.0023	0.0023	0.0021	0.0024	1952	0.0266
1953	0.0023	0.0021	0.0025	0.0024	0.0024	0.0027	0.0025	0.0025	0.0025	0.0023	0.0024	0.0024	1953	0.0284
1954	0.0023	0.0022	0.0025	0.0022	0.0020	0.0025	0.0022	0.0023	0.0022	0.0021	0.0023	0.0023	1954	0.0279
1955	0.0022	0.0022	0.0024	0.0022	0.0025	0.0023	0.0023	0.0027	0.0024	0.0025	0.0024	0.0024	1955	0.0275

*Compound annual return

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Long-term Government Bonds: Income Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1956	0.0025	0.0023	0.0023	0.0026	0.0026	0.0023	0.0026	0.0026	0.0025	0.0029	0.0027	0.0028	1956	0.0299
1957	0.0029	0.0025	0.0026	0.0029	0.0029	0.0025	0.0033	0.0030	0.0031	0.0031	0.0029	0.0029	1957	0.0344
1958	0.0027	0.0025	0.0027	0.0026	0.0024	0.0027	0.0027	0.0027	0.0032	0.0032	0.0028	0.0033	1958	0.0327
1959	0.0031	0.0031	0.0035	0.0033	0.0033	0.0036	0.0035	0.0035	0.0034	0.0035	0.0035	0.0036	1959	0.0401
1960	0.0035	0.0037	0.0036	0.0032	0.0037	0.0034	0.0032	0.0034	0.0032	0.0033	0.0032	0.0033	1960	0.0426
1961	0.0033	0.0030	0.0031	0.0031	0.0034	0.0032	0.0033	0.0033	0.0032	0.0034	0.0032	0.0031	1961	0.0383
1962	0.0037	0.0032	0.0033	0.0033	0.0032	0.0030	0.0034	0.0034	0.0030	0.0035	0.0031	0.0032	1962	0.0400
1963	0.0032	0.0029	0.0031	0.0034	0.0033	0.0030	0.0036	0.0033	0.0034	0.0034	0.0032	0.0036	1963	0.0389
1964	0.0035	0.0032	0.0037	0.0035	0.0032	0.0038	0.0035	0.0035	0.0034	0.0034	0.0035	0.0035	1964	0.0415
1965	0.0033	0.0032	0.0038	0.0033	0.0033	0.0038	0.0034	0.0037	0.0035	0.0034	0.0037	0.0037	1965	0.0419
1966	0.0038	0.0034	0.0040	0.0036	0.0041	0.0039	0.0038	0.0043	0.0041	0.0040	0.0038	0.0039	1966	0.0449
1967	0.0040	0.0034	0.0039	0.0035	0.0043	0.0039	0.0043	0.0042	0.0040	0.0045	0.0045	0.0044	1967	0.0459
1968	0.0050	0.0042	0.0043	0.0049	0.0046	0.0042	0.0048	0.0042	0.0044	0.0045	0.0043	0.0049	1968	0.0550
1969	0.0050	0.0046	0.0047	0.0055	0.0047	0.0055	0.0052	0.0048	0.0055	0.0057	0.0049	0.0060	1969	0.0595
1970	0.0056	0.0052	0.0056	0.0054	0.0055	0.0064	0.0059	0.0057	0.0056	0.0055	0.0058	0.0053	1970	0.0674
1971	0.0051	0.0046	0.0056	0.0048	0.0047	0.0056	0.0052	0.0055	0.0050	0.0047	0.0051	0.0050	1971	0.0632
1972	0.0050	0.0047	0.0049	0.0048	0.0055	0.0049	0.0051	0.0049	0.0047	0.0052	0.0048	0.0045	1972	0.0587
1973	0.0054	0.0051	0.0056	0.0057	0.0058	0.0055	0.0061	0.0062	0.0055	0.0063	0.0056	0.0060	1973	0.0651
1974	0.0061	0.0055	0.0059	0.0068	0.0068	0.0061	0.0072	0.0065	0.0071	0.0070	0.0062	0.0067	1974	0.0727
1975	0.0068	0.0060	0.0066	0.0067	0.0067	0.0070	0.0068	0.0065	0.0073	0.0072	0.0061	0.0075	1975	0.0799
1976	0.0065	0.0061	0.0071	0.0064	0.0059	0.0073	0.0065	0.0069	0.0064	0.0061	0.0066	0.0063	1976	0.0789
1977	0.0059	0.0057	0.0065	0.0061	0.0067	0.0062	0.0059	0.0067	0.0061	0.0063	0.0063	0.0062	1977	0.0714
1978	0.0069	0.0060	0.0069	0.0063	0.0075	0.0069	0.0073	0.0070	0.0065	0.0073	0.0071	0.0068	1978	0.0790
1979	0.0079	0.0065	0.0074	0.0076	0.0077	0.0071	0.0076	0.0073	0.0068	0.0082	0.0083	0.0083	1979	0.0886
1980	0.0083	0.0084	0.0099	0.0100	0.0087	0.0086	0.0084	0.0081	0.0097	0.0097	0.0091	0.0108	1980	0.0997
1981	0.0094	0.0088	0.0111	0.0101	0.0104	0.0109	0.0109	0.0110	0.0114	0.0117	0.0113	0.0100	1981	0.1155
1982	0.0108	0.0103	0.0124	0.0112	0.0101	0.0120	0.0114	0.0112	0.0100	0.0091	0.0095	0.0093	1982	0.1350
1983	0.0087	0.0081	0.0089	0.0085	0.0091	0.0090	0.0088	0.0103	0.0096	0.0095	0.0094	0.0094	1983	0.1038
1984	0.0103	0.0092	0.0098	0.0104	0.0103	0.0106	0.0116	0.0106	0.0094	0.0108	0.0091	0.0098	1984	0.1174
1985	0.0096	0.0082	0.0094	0.0102	0.0097	0.0080	0.0094	0.0085	0.0088	0.0089	0.0081	0.0086	1985	0.1125

*Compound annual return

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Appendix A-7

Long-term Government Bonds: Income Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1986	0.0079	0.0073	0.0071	0.0063	0.0062	0.0070	0.0066	0.0063	0.0065	0.0069	0.0059	0.0070	1986	0.0898
1987	0.0064	0.0059	0.0066	0.0065	0.0066	0.0075	0.0073	0.0075	0.0075	0.0079	0.0075	0.0078	1987	0.0792
1988	0.0072	0.0071	0.0072	0.0070	0.0078	0.0076	0.0071	0.0083	0.0076	0.0076	0.0070	0.0075	1988	0.0897
1989	0.0080	0.0069	0.0079	0.0070	0.0080	0.0070	0.0068	0.0066	0.0065	0.0072	0.0064	0.0064	1989	0.0881
1990	0.0073	0.0066	0.0071	0.0075	0.0075	0.0068	0.0074	0.0071	0.0069	0.0081	0.0071	0.0072	1990	0.0819
1991	0.0071	0.0064	0.0064	0.0076	0.0068	0.0063	0.0076	0.0068	0.0068	0.0065	0.0060	0.0068	1991	0.0822
1992	0.0061	0.0059	0.0067	0.0065	0.0061	0.0067	0.0063	0.0060	0.0058	0.0057	0.0061	0.0063	1992	0.0726
1993	0.0059	0.0055	0.0063	0.0057	0.0052	0.0062	0.0054	0.0056	0.0050	0.0049	0.0053	0.0055	1993	0.0717
1994	0.0055	0.0049	0.0058	0.0057	0.0063	0.0061	0.0060	0.0066	0.0061	0.0066	0.0064	0.0066	1994	0.0659
1995	0.0070	0.0059	0.0064	0.0058	0.0065	0.0054	0.0056	0.0057	0.0052	0.0057	0.0051	0.0049	1995	0.0760
1996	0.0054	0.0048	0.0052	0.0059	0.0058	0.0054	0.0062	0.0057	0.0060	0.0058	0.0052	0.0056	1996	0.0618
1997	0.0056	0.0051	0.0059	0.0059	0.0058	0.0059	0.0058	0.0049	0.0058	0.0054	0.0047	0.0054	1997	0.0664
1998	0.0048	0.0044	0.0052	0.0049	0.0048	0.0052	0.0049	0.0048	0.0044	0.0042	0.0045	0.0045	1998	0.0583
1999	0.0042	0.0040	0.0053	0.0048	0.0045	0.0055	0.0051	0.0054	0.0052	0.0050	0.0056	0.0055	1999	0.0557
2000	0.0057	0.0051	0.0054	0.0047	0.0056	0.0052	0.0052	0.0050	0.0046	0.0053	0.0048	0.0045	2000	0.0650
2001	0.0049	0.0042	0.0045	0.0047	0.0050	0.0047	0.0052	0.0046	0.0041	0.0048	0.0041	0.0046	2001	0.0553
2002	0.0048	0.0043	0.0043	0.0054	0.0049	0.0044	0.0051	0.0044	0.0042	0.0040	0.0040	0.0045	2002	0.0559
2003	0.0041	0.0038	0.0040	0.0040	0.0039	0.0036	0.0038	0.0042	0.0046	0.0041	0.0039	0.0047	2003	0.0480
2004	0.0042	0.0038	0.0043	0.0039	0.0040	0.0048	0.0043	0.0045	0.0040	0.0038	0.0041	0.0043	2004	0.0502
2005	0.0041	0.0035	0.0041	0.0039	0.0040	0.0036	0.0034	0.0040	0.0035	0.0039	0.0039	0.0039	2005	0.0469
2006	0.0040	0.0036	0.0039	0.0039	0.0048	0.0044	0.0045	0.0043	0.0039	0.0042	0.0039	0.0036	2006	0.0468
2007	0.0043	0.0038	0.0039	0.0042	0.0041	0.0040	0.0046	0.0042	0.0037	0.0043	0.0039	0.0037	2007	0.0486
2008	0.0040	0.0034	0.0037	0.0035	0.0037	0.0040	0.0039	0.0036	0.0039	0.0037	0.0036	0.0033	2008	0.0445
2009	0.0024	0.0030	0.0035	0.0029	0.0033	0.0038	0.0036	0.0036	0.0034	0.0033	0.0035	0.0034	2009	0.0347
2010	0.0036	0.0033	0.0040	0.0038	0.0034	0.0037	0.0031	0.0032	0.0026	0.0027	0.0032	0.0032	2010	0.0425
2011	0.0035	0.0032	0.0036	0.0034	0.0036	0.0032	0.0032	0.0034	0.0026	0.0022	0.0024	0.0022	2011	0.0382
2012	0.0021	0.0020	0.0022	0.0025	0.0023	0.0018	0.0020	0.0018	0.0017	0.0021	0.0019	0.0019	2012	0.0246
2013	0.0022	0.0022	0.0021	0.0026	0.0023	0.0024	0.0030	0.0028	0.0029	0.0029	0.0027	0.0031	2013	0.0288
2014	0.0032	0.0026	0.0029	0.0028	0.0028	0.0025	0.0027	0.0026	0.0023	0.0025	0.0023	0.0022	2014	0.0341
2015	0.0020	0.0015	0.0021	0.0019	0.0020	0.0023	0.0024	0.0022	0.0021	0.0021	0.0022	0.0022	2015	0.0247
2016	0.0021	0.0020	0.0018	0.0017	0.0020	0.0018	0.0014	0.0016	0.0015	0.0016	0.0018	0.0022	2016	0.0230

*Compound annual return

Appendix A-8

Long-term Government Bonds: Capital Appreciation Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1926	0.0106	0.0035	0.0009	0.0046	-0.0014	0.0005	-0.0027	-0.0031	0.0007	0.0072	0.0129	0.0048	1926	0.0391
1927	0.0045	0.0061	0.0224	-0.0032	0.0081	-0.0096	0.0022	0.0047	-0.0009	0.0071	0.0071	0.0045	1927	0.0540
1928	-0.0063	0.0036	0.0019	-0.0029	-0.0104	0.0015	-0.0245	0.0047	-0.0067	0.0128	-0.0024	-0.0024	1928	-0.0312
1929	-0.0119	-0.0184	-0.0171	0.0242	-0.0192	0.0081	-0.0032	-0.0064	-0.0004	0.0351	0.0211	-0.0120	1929	-0.0020
1930	-0.0086	0.0102	0.0055	-0.0043	0.0113	0.0022	0.0007	-0.0013	0.0045	0.0008	0.0017	-0.0098	1930	0.0128
1931	-0.0149	0.0059	0.0076	0.0059	0.0119	-0.0024	-0.0069	-0.0015	-0.0307	-0.0360	-0.0004	-0.0252	1931	-0.0846
1932	0.0002	0.0382	-0.0049	0.0574	-0.0216	0.0037	0.0453	-0.0025	0.0031	-0.0044	0.0006	0.0104	1932	0.1294
1933	0.0122	-0.0282	0.0070	-0.0057	0.0274	0.0025	-0.0043	0.0018	-0.0002	-0.0117	-0.0174	-0.0140	1933	-0.0314
1934	0.0228	0.0057	0.0170	0.0101	0.0106	0.0043	0.0016	-0.0143	-0.0169	0.0155	0.0013	0.0087	1934	0.0676
1935	0.0157	0.0070	0.0019	0.0056	-0.0079	0.0070	0.0022	-0.0156	-0.0014	0.0038	-0.0014	0.0047	1935	0.0214
1936	0.0031	0.0059	0.0083	0.0013	0.0019	-0.0003	0.0037	0.0088	-0.0053	-0.0017	0.0183	0.0017	1936	0.0464
1937	-0.0034	0.0067	-0.0434	0.0016	0.0031	-0.0043	0.0114	-0.0128	0.0022	0.0019	0.0072	0.0059	1937	-0.0248
1938	0.0034	0.0031	-0.0059	0.0187	0.0022	-0.0017	0.0022	-0.0022	0.0001	0.0065	-0.0043	0.0059	1938	0.0283
1939	0.0038	0.0061	0.0105	0.0099	0.0151	-0.0045	0.0095	-0.0219	-0.0564	0.0386	0.0142	0.0125	1939	0.0348
1940	-0.0037	0.0009	0.0158	-0.0053	-0.0318	0.0239	0.0032	0.0009	0.0092	0.0013	0.0187	0.0050	1940	0.0377
1941	-0.0217	0.0004	0.0078	0.0112	0.0011	0.0050	0.0005	0.0002	-0.0028	0.0124	-0.0044	-0.0194	1941	-0.0101
1942	0.0048	-0.0008	0.0071	-0.0049	0.0056	-0.0018	-0.0003	0.0017	-0.0017	0.0004	-0.0055	0.0028	1942	0.0074
1943	0.0013	-0.0024	-0.0012	0.0028	0.0031	-0.0003	-0.0021	0.0000	-0.0009	-0.0015	-0.0021	-0.0003	1943	-0.0037
1944	0.0000	0.0012	0.0000	-0.0006	0.0006	-0.0012	0.0015	0.0006	-0.0006	-0.0009	0.0003	0.0022	1944	0.0032
1945	0.0105	0.0058	0.0001	0.0141	0.0037	0.0150	-0.0104	0.0007	0.0037	0.0085	0.0108	0.0177	1945	0.0827
1946	0.0008	0.0017	-0.0006	-0.0152	-0.0030	0.0054	-0.0058	-0.0129	-0.0028	0.0055	-0.0072	0.0126	1946	-0.0215
1947	-0.0024	0.0005	0.0002	-0.0054	0.0016	-0.0009	0.0044	0.0054	-0.0062	-0.0055	-0.0191	-0.0213	1947	-0.0470
1948	0.0000	0.0028	0.0013	0.0025	0.0123	-0.0105	-0.0041	-0.0020	-0.0006	-0.0012	0.0055	0.0036	1948	0.0096
1949	0.0062	0.0031	0.0055	-0.0006	0.0000	0.0148	0.0016	0.0092	-0.0029	0.0001	0.0004	0.0035	1949	0.0415
1950	-0.0080	0.0005	-0.0010	0.0014	0.0014	-0.0042	0.0037	-0.0004	-0.0089	-0.0067	0.0017	-0.0001	1950	-0.0206
1951	0.0038	-0.0091	-0.0176	-0.0083	-0.0090	-0.0082	0.0116	0.0077	-0.0098	-0.0013	-0.0157	-0.0083	1951	-0.0627
1952	0.0005	-0.0007	0.0088	0.0149	-0.0054	-0.0019	-0.0042	-0.0091	-0.0153	0.0124	-0.0036	-0.0110	1952	-0.0148
1953	-0.0011	-0.0108	-0.0113	-0.0129	-0.0171	0.0195	0.0014	-0.0033	0.0275	0.0051	-0.0073	0.0182	1953	0.0067
1954	0.0066	0.0218	0.0034	0.0081	-0.0107	0.0138	0.0113	-0.0059	-0.0031	-0.0015	-0.0048	0.0042	1954	0.0435
1955	-0.0264	-0.0100	0.0063	-0.0022	0.0048	-0.0099	-0.0125	-0.0023	0.0049	0.0119	-0.0069	0.0013	1955	-0.0407

*Compound annual return

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Long-term Government Bonds: Capital Appreciation Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1956	0.0058	-0.0025	-0.0172	-0.0139	0.0199	0.0004	-0.0234	-0.0213	0.0025	-0.0084	-0.0084	-0.0207	1956	-0.0846
1957	0.0317	0.0000	-0.0050	-0.0250	-0.0052	-0.0206	-0.0074	-0.0028	0.0045	-0.0081	0.0504	0.0277	1957	0.0382
1958	-0.0112	0.0075	0.0075	0.0160	-0.0024	-0.0187	-0.0306	-0.0463	-0.0149	0.0106	0.0092	-0.0213	1958	-0.0923
1959	-0.0111	0.0087	-0.0018	-0.0150	-0.0038	-0.0026	0.0025	-0.0076	-0.0091	0.0115	-0.0154	-0.0195	1959	-0.0620
1960	0.0077	0.0167	0.0246	-0.0202	0.0115	0.0139	0.0335	-0.0101	0.0043	-0.0061	-0.0098	0.0247	1960	0.0929
1961	-0.0140	0.0170	-0.0069	0.0085	-0.0080	-0.0106	0.0001	-0.0071	0.0097	0.0037	-0.0052	-0.0156	1961	-0.0286
1962	-0.0051	0.0071	0.0220	0.0049	0.0014	-0.0106	-0.0143	0.0153	0.0031	0.0049	-0.0010	0.0003	1962	0.0278
1963	-0.0033	-0.0022	-0.0022	-0.0046	-0.0011	-0.0011	-0.0005	-0.0011	-0.0029	-0.0060	0.0019	-0.0042	1963	-0.0270
1964	-0.0048	-0.0043	0.0000	0.0012	0.0018	0.0031	-0.0028	-0.0015	0.0015	0.0009	-0.0018	-0.0005	1964	-0.0072
1965	0.0007	-0.0018	0.0016	0.0003	-0.0015	0.0009	-0.0012	-0.0050	-0.0069	-0.0007	-0.0100	-0.0115	1965	-0.0345
1966	-0.0142	-0.0284	0.0256	-0.0099	-0.0100	-0.0054	-0.0074	-0.0249	0.0292	0.0188	-0.0187	0.0374	1966	-0.0106
1967	0.0115	-0.0255	0.0159	-0.0326	-0.0082	-0.0351	0.0026	-0.0126	-0.0045	-0.0445	-0.0241	0.0148	1967	-0.1355
1968	0.0278	-0.0075	-0.0254	0.0178	-0.0003	0.0188	0.0241	-0.0045	-0.0146	-0.0177	-0.0312	-0.0412	1968	-0.0551
1969	-0.0256	-0.0005	-0.0036	0.0371	-0.0537	0.0159	0.0027	-0.0117	-0.0586	0.0309	-0.0293	-0.0129	1969	-0.1083
1970	-0.0077	0.0535	-0.0124	-0.0467	-0.0523	0.0422	0.0260	-0.0076	0.0172	-0.0164	0.0733	-0.0137	1970	0.0484
1971	0.0455	-0.0209	0.0470	-0.0331	-0.0053	-0.0214	-0.0022	0.0416	0.0154	0.0120	-0.0098	-0.0006	1971	0.0661
1972	-0.0114	0.0041	-0.0131	-0.0021	0.0215	-0.0113	0.0165	-0.0021	-0.0129	0.0182	0.0178	-0.0275	1972	-0.0035
1973	-0.0375	-0.0037	0.0026	-0.0012	-0.0162	-0.0076	-0.0495	0.0329	0.0263	0.0153	-0.0238	-0.0142	1973	-0.0770
1974	-0.0144	-0.0079	-0.0350	-0.0320	0.0055	-0.0016	-0.0101	-0.0298	0.0176	0.0419	0.0233	0.0105	1974	-0.0345
1975	0.0157	0.0071	-0.0333	-0.0249	0.0145	0.0222	-0.0155	-0.0133	-0.0171	0.0403	-0.0170	0.0316	1975	0.0073
1976	0.0025	0.0001	0.0094	-0.0046	-0.0217	0.0135	0.0013	0.0142	0.0081	0.0023	0.0273	0.0265	1976	0.0807
1977	-0.0447	-0.0106	0.0026	0.0010	0.0058	0.0102	-0.0130	0.0131	-0.0089	-0.0156	0.0031	-0.0230	1977	-0.0786
1978	-0.0149	-0.0056	-0.0090	-0.0068	-0.0133	-0.0132	0.0070	0.0148	-0.0171	-0.0273	0.0117	-0.0198	1978	-0.0905
1979	0.0112	-0.0200	0.0056	-0.0188	0.0184	0.0240	-0.0161	-0.0108	-0.0190	-0.0922	0.0229	-0.0026	1979	-0.0984
1980	-0.0824	-0.0551	-0.0413	0.1424	0.0332	0.0272	-0.0560	-0.0513	-0.0359	-0.0360	0.0009	0.0244	1980	-0.1400
1981	-0.0209	-0.0524	0.0274	-0.0618	0.0518	-0.0288	-0.0462	-0.0496	-0.0259	0.0712	0.1297	-0.0813	1981	-0.1033
1982	-0.0062	0.0079	0.0107	0.0262	-0.0067	-0.0343	0.0387	0.0669	0.0519	0.0543	-0.0097	0.0219	1982	0.2395
1983	-0.0396	0.0410	-0.0183	0.0265	-0.0477	-0.0051	-0.0574	-0.0083	0.0408	-0.0227	0.0089	-0.0152	1983	-0.0982
1984	0.0141	-0.0270	-0.0254	-0.0210	-0.0619	0.0044	0.0577	0.0160	0.0248	0.0453	0.0027	-0.0007	1984	0.0232
1985	0.0268	-0.0575	0.0212	0.0140	0.0798	0.0061	-0.0274	0.0174	-0.0109	0.0248	0.0320	0.0455	1985	0.1784

*Compound annual return

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Long-term Government Bonds: Capital Appreciation Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1986	-0.0105	0.1073	0.0699	-0.0142	-0.0567	0.0543	-0.0174	0.0437	-0.0565	0.0220	0.0208	-0.0087	1986	0.1499
1987	0.0096	0.0143	-0.0289	-0.0538	-0.0171	0.0023	-0.0251	-0.0240	-0.0444	0.0544	-0.0038	0.0088	1987	-0.1069
1988	0.0595	-0.0019	-0.0378	-0.0230	-0.0180	0.0292	-0.0241	-0.0025	0.0269	0.0232	-0.0266	0.0035	1988	0.0036
1989	0.0124	-0.0248	0.0044	0.0088	0.0321	0.0480	0.0170	-0.0325	-0.0046	0.0307	0.0014	-0.0070	1989	0.0862
1990	-0.0416	-0.0090	-0.0115	-0.0277	0.0340	0.0162	0.0033	-0.0490	0.0048	0.0135	0.0331	0.0114	1990	-0.0261
1991	0.0059	-0.0033	-0.0026	0.0065	-0.0068	-0.0126	0.0082	0.0272	0.0236	-0.0011	0.0022	0.0513	1991	0.1010
1992	-0.0385	-0.0008	-0.0161	-0.0049	0.0181	0.0133	0.0334	0.0007	0.0127	-0.0255	-0.0051	0.0183	1992	0.0034
1993	0.0222	0.0299	-0.0042	0.0015	-0.0006	0.0387	0.0138	0.0378	-0.0045	0.0048	-0.0312	-0.0035	1993	0.1071
1994	0.0202	-0.0498	-0.0453	-0.0208	-0.0146	-0.0161	0.0303	-0.0152	-0.0392	-0.0091	0.0002	0.0095	1994	-0.1429
1995	0.0203	0.0227	0.0028	0.0112	0.0725	0.0084	-0.0223	0.0179	0.0122	0.0237	0.0198	0.0223	1995	0.2304
1996	-0.0065	-0.0530	-0.0262	-0.0224	-0.0112	0.0149	-0.0045	-0.0196	0.0230	0.0345	0.0299	-0.0312	1996	-0.0737
1997	-0.0135	-0.0046	-0.0311	0.0196	0.0037	0.0138	0.0567	-0.0367	0.0258	0.0287	0.0101	0.0130	1997	0.0851
1998	0.0152	-0.0116	-0.0028	-0.0023	0.0135	0.0176	-0.0088	0.0416	0.0350	-0.0260	0.0052	-0.0077	1998	0.0689
1999	0.0079	-0.0560	-0.0061	-0.0028	-0.0230	-0.0133	-0.0130	-0.0105	0.0032	-0.0062	-0.0117	-0.0210	1999	-0.1435
2000	0.0171	0.0213	0.0312	-0.0123	-0.0111	0.0192	0.0120	0.0190	-0.0203	0.0135	0.0270	0.0198	2000	0.1436
2001	-0.0044	0.0149	-0.0119	-0.0360	-0.0013	0.0038	0.0324	0.0159	0.0040	0.0416	-0.0512	-0.0229	2001	-0.0189
2002	0.0090	0.0072	-0.0479	0.0355	-0.0034	0.0143	0.0252	0.0420	0.0374	-0.0334	-0.0161	0.0462	2002	0.1169
2003	-0.0147	0.0291	-0.0175	0.0062	0.0553	-0.0190	-0.1020	0.0124	0.0501	-0.0324	-0.0012	0.0093	2003	-0.0336
2004	0.0146	0.0192	0.0098	-0.0627	-0.0090	0.0074	0.0113	0.0350	0.0057	0.0115	-0.0275	0.0207	2004	0.0326
2005	0.0260	-0.0163	-0.0112	0.0334	0.0256	0.0131	-0.0322	0.0292	-0.0373	-0.0235	0.0037	0.0228	2005	0.0302
2006	-0.0157	0.0203	-0.0578	-0.0285	-0.0038	0.0048	0.0154	0.0256	0.0132	0.0035	0.0169	-0.0272	2006	-0.0364
2007	-0.0146	0.0297	-0.0184	0.0043	-0.0242	-0.0131	0.0238	0.0157	-0.0025	0.0112	0.0429	-0.0066	2007	0.0469
2008	0.0173	-0.0015	0.0069	-0.0324	-0.0202	0.0180	-0.0064	0.0206	0.0074	-0.0420	0.1407	0.0934	2008	0.2050
2009	-0.1149	-0.0086	0.0606	-0.0679	-0.0281	0.0046	-0.0018	0.0195	0.0142	-0.0203	0.0173	-0.0618	2009	-0.1825
2010	0.0228	-0.0002	-0.0219	0.0266	0.0403	0.0409	-0.0007	0.0570	-0.0180	-0.0344	-0.0169	-0.0420	2010	0.0589
2011	-0.0231	0.0081	-0.0042	0.0165	0.0318	-0.0212	0.0389	0.0829	0.0679	-0.0328	0.0228	0.0248	2011	0.2262
2012	-0.0020	-0.0216	-0.0324	0.0384	0.0620	-0.0153	0.0227	-0.0087	-0.0163	-0.0035	0.0124	-0.0221	2012	0.0095
2013	-0.0354	0.0092	-0.0083	0.0352	-0.0651	-0.0309	-0.0203	-0.0107	0.0032	0.0099	-0.0262	-0.0262	2013	-0.1570
2014	0.0516	0.0048	0.0034	0.0154	0.0251	-0.0051	0.0030	0.0343	-0.0194	0.0274	0.0263	0.0268	2014	0.2093
2015	0.0689	-0.0538	0.0116	-0.0269	-0.0179	-0.0321	0.0305	-0.0010	0.0153	-0.0074	-0.0086	-0.0044	2015	-0.0311
2016	0.0455	0.0275	-0.0022	-0.0070	0.0063	0.0572	0.0066	-0.0156	-0.0140	-0.0331	-0.0616	-0.0079	2016	-0.0040

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Long-term Government Bonds: Yields
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1926	0.0374	0.0372	0.0371	0.0368	0.0369	0.0368	0.0370	0.0373	0.0372	0.0367	0.0358	0.0354	1926	0.0354
1927	0.0351	0.0347	0.0331	0.0333	0.0327	0.0334	0.0333	0.0329	0.0330	0.0325	0.0320	0.0317	1927	0.0317
1928	0.0321	0.0318	0.0317	0.0319	0.0327	0.0326	0.0344	0.0341	0.0346	0.0336	0.0338	0.0340	1928	0.0340
1929	0.0349	0.0363	0.0377	0.0358	0.0373	0.0367	0.0369	0.0375	0.0375	0.0347	0.0331	0.0340	1929	0.0340
1930	0.0347	0.0339	0.0335	0.0338	0.0329	0.0328	0.0327	0.0328	0.0324	0.0324	0.0322	0.0330	1930	0.0330
1931	0.0343	0.0338	0.0332	0.0327	0.0317	0.0319	0.0325	0.0326	0.0353	0.0385	0.0385	0.0407	1931	0.0407
1932	0.0390	0.0367	0.0370	0.0336	0.0349	0.0347	0.0320	0.0321	0.0319	0.0322	0.0322	0.0315	1932	0.0315
1933	0.0308	0.0326	0.0321	0.0325	0.0308	0.0306	0.0309	0.0308	0.0308	0.0315	0.0327	0.0336	1933	0.0336
1934	0.0321	0.0317	0.0307	0.0300	0.0292	0.0289	0.0288	0.0299	0.0310	0.0300	0.0299	0.0293	1934	0.0293
1935	0.0281	0.0275	0.0274	0.0269	0.0276	0.0270	0.0268	0.0281	0.0282	0.0279	0.0280	0.0276	1935	0.0276
1936	0.0285	0.0281	0.0275	0.0274	0.0273	0.0273	0.0271	0.0264	0.0268	0.0269	0.0257	0.0255	1936	0.0255
1937	0.0258	0.0253	0.0285	0.0284	0.0282	0.0285	0.0277	0.0286	0.0284	0.0283	0.0278	0.0273	1937	0.0273
1938	0.0271	0.0268	0.0273	0.0259	0.0257	0.0259	0.0257	0.0259	0.0259	0.0254	0.0257	0.0252	1938	0.0252
1939	0.0249	0.0245	0.0237	0.0229	0.0217	0.0221	0.0213	0.0231	0.0278	0.0247	0.0236	0.0226	1939	0.0226
1940	0.0229	0.0228	0.0215	0.0220	0.0246	0.0227	0.0224	0.0223	0.0215	0.0214	0.0199	0.0194	1940	0.0194
1941	0.0213	0.0213	0.0206	0.0196	0.0195	0.0191	0.0191	0.0190	0.0193	0.0182	0.0186	0.0204	1941	0.0204
1942	0.0247	0.0247	0.0244	0.0246	0.0243	0.0244	0.0244	0.0244	0.0244	0.0244	0.0247	0.0246	1942	0.0246
1943	0.0245	0.0246	0.0247	0.0246	0.0244	0.0244	0.0245	0.0245	0.0246	0.0247	0.0248	0.0248	1943	0.0248
1944	0.0248	0.0247	0.0247	0.0248	0.0247	0.0248	0.0247	0.0247	0.0247	0.0247	0.0247	0.0246	1944	0.0246
1945	0.0240	0.0237	0.0236	0.0228	0.0226	0.0217	0.0224	0.0223	0.0221	0.0216	0.0210	0.0199	1945	0.0199
1946	0.0199	0.0198	0.0198	0.0207	0.0209	0.0206	0.0209	0.0217	0.0219	0.0216	0.0220	0.0212	1946	0.0212
1947	0.0214	0.0214	0.0213	0.0217	0.0216	0.0216	0.0214	0.0210	0.0213	0.0217	0.0229	0.0243	1947	0.0243
1948	0.0243	0.0241	0.0241	0.0239	0.0231	0.0238	0.0241	0.0242	0.0242	0.0243	0.0239	0.0237	1948	0.0237
1949	0.0233	0.0231	0.0227	0.0227	0.0227	0.0217	0.0216	0.0210	0.0212	0.0212	0.0212	0.0209	1949	0.0209
1950	0.0215	0.0214	0.0215	0.0214	0.0213	0.0216	0.0214	0.0214	0.0220	0.0225	0.0224	0.0224	1950	0.0224
1951	0.0221	0.0228	0.0241	0.0248	0.0254	0.0259	0.0252	0.0246	0.0253	0.0254	0.0264	0.0269	1951	0.0269
1952	0.0268	0.0269	0.0263	0.0254	0.0257	0.0259	0.0261	0.0267	0.0277	0.0269	0.0272	0.0279	1952	0.0279
1953	0.0279	0.0287	0.0294	0.0303	0.0314	0.0301	0.0301	0.0303	0.0284	0.0281	0.0286	0.0274	1953	0.0274
1954	0.0291	0.0279	0.0278	0.0273	0.0279	0.0272	0.0266	0.0269	0.0271	0.0272	0.0274	0.0272	1954	0.0272
1955	0.0286	0.0292	0.0288	0.0290	0.0287	0.0293	0.0300	0.0301	0.0298	0.0292	0.0295	0.0295	1955	0.0295

*Compound annual return

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Long-term Government Bonds: Yields
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan-Dec*
1956	0.0292	0.0293	0.0303	0.0311	0.0299	0.0299	0.0313	0.0325	0.0324	0.0329	0.0333	0.0345	1956	0.0345
1957	0.0328	0.0328	0.0331	0.0345	0.0348	0.0361	0.0365	0.0367	0.0364	0.0369	0.0340	0.0323	1957	0.0323
1958	0.0330	0.0326	0.0321	0.0311	0.0313	0.0324	0.0343	0.0371	0.0380	0.0374	0.0368	0.0382	1958	0.0382
1959	0.0408	0.0402	0.0403	0.0414	0.0417	0.0419	0.0417	0.0423	0.0429	0.0421	0.0432	0.0447	1959	0.0447
1960	0.0441	0.0429	0.0411	0.0426	0.0417	0.0407	0.0382	0.0390	0.0387	0.0391	0.0399	0.0380	1960	0.0380
1961	0.0404	0.0392	0.0397	0.0391	0.0397	0.0404	0.0404	0.0410	0.0403	0.0400	0.0404	0.0415	1961	0.0415
1962	0.0419	0.0414	0.0398	0.0394	0.0393	0.0401	0.0412	0.0401	0.0398	0.0395	0.0396	0.0395	1962	0.0395
1963	0.0398	0.0400	0.0401	0.0405	0.0406	0.0407	0.0407	0.0408	0.0410	0.0415	0.0414	0.0417	1963	0.0417
1964	0.0421	0.0424	0.0424	0.0423	0.0422	0.0419	0.0421	0.0423	0.0421	0.0421	0.0422	0.0423	1964	0.0423
1965	0.0422	0.0424	0.0422	0.0422	0.0423	0.0423	0.0424	0.0428	0.0433	0.0433	0.0441	0.0450	1965	0.0450
1966	0.0458	0.0477	0.0460	0.0467	0.0473	0.0477	0.0482	0.0499	0.0480	0.0467	0.0480	0.0455	1966	0.0455
1967	0.0448	0.0465	0.0455	0.0477	0.0482	0.0507	0.0505	0.0514	0.0517	0.0549	0.0567	0.0556	1967	0.0556
1968	0.0536	0.0542	0.0560	0.0547	0.0548	0.0534	0.0517	0.0520	0.0531	0.0543	0.0566	0.0598	1968	0.0598
1969	0.0617	0.0618	0.0620	0.0593	0.0635	0.0623	0.0621	0.0630	0.0677	0.0653	0.0676	0.0687	1969	0.0687
1970	0.0693	0.0651	0.0661	0.0699	0.0743	0.0709	0.0687	0.0694	0.0680	0.0693	0.0637	0.0648	1970	0.0648
1971	0.0612	0.0629	0.0593	0.0619	0.0624	0.0641	0.0643	0.0610	0.0598	0.0588	0.0596	0.0597	1971	0.0597
1972	0.0606	0.0602	0.0613	0.0615	0.0597	0.0607	0.0593	0.0595	0.0606	0.0591	0.0577	0.0599	1972	0.0599
1973	0.0685	0.0688	0.0686	0.0687	0.0703	0.0710	0.0760	0.0728	0.0703	0.0689	0.0712	0.0726	1973	0.0726
1974	0.0740	0.0748	0.0783	0.0816	0.0810	0.0812	0.0823	0.0855	0.0837	0.0795	0.0771	0.0760	1974	0.0760
1975	0.0796	0.0788	0.0824	0.0852	0.0836	0.0813	0.0829	0.0844	0.0862	0.0819	0.0838	0.0805	1975	0.0805
1976	0.0802	0.0802	0.0792	0.0797	0.0821	0.0807	0.0805	0.0790	0.0781	0.0779	0.0749	0.0721	1976	0.0721
1977	0.0764	0.0775	0.0772	0.0771	0.0765	0.0754	0.0768	0.0754	0.0764	0.0781	0.0777	0.0803	1977	0.0803
1978	0.0816	0.0822	0.0831	0.0838	0.0852	0.0865	0.0858	0.0843	0.0860	0.0889	0.0877	0.0898	1978	0.0898
1979	0.0886	0.0908	0.0902	0.0922	0.0903	0.0877	0.0895	0.0907	0.0927	0.1034	0.1009	0.1012	1979	0.1012
1980	0.1114	0.1186	0.1239	0.1076	0.1037	0.1006	0.1074	0.1140	0.1185	0.1231	0.1230	0.1199	1980	0.1199
1981	0.1211	0.1283	0.1248	0.1332	0.1265	0.1304	0.1370	0.1445	0.1482	0.1384	0.1220	0.1334	1981	0.1334
1982	0.1415	0.1402	0.1387	0.1348	0.1358	0.1412	0.1352	0.1254	0.1183	0.1112	0.1125	0.1095	1982	0.1095
1983	0.1113	0.1060	0.1083	0.1051	0.1112	0.1119	0.1198	0.1210	0.1157	0.1188	0.1176	0.1197	1983	0.1197
1984	0.1180	0.1217	0.1253	0.1284	0.1381	0.1374	0.1293	0.1270	0.1235	0.1173	0.1169	0.1170	1984	0.1170
1985	0.1127	0.1209	0.1181	0.1162	0.1062	0.1055	0.1091	0.1068	0.1082	0.1051	0.1011	0.0956	1985	0.0956

*Compound annual return

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Long-term Government Bonds: Yields
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1986	0.0958	0.0841	0.0766	0.0782	0.0848	0.0790	0.0809	0.0763	0.0827	0.0803	0.0779	0.0789	1986	0.0789
1987	0.0778	0.0763	0.0795	0.0859	0.0880	0.0877	0.0907	0.0936	0.0992	0.0926	0.0931	0.0920	1987	0.0920
1988	0.0852	0.0854	0.0901	0.0929	0.0952	0.0917	0.0947	0.0950	0.0917	0.0889	0.0923	0.0919	1988	0.0919
1989	0.0903	0.0935	0.0929	0.0918	0.0878	0.0822	0.0801	0.0841	0.0847	0.0810	0.0808	0.0816	1989	0.0816
1990	0.0865	0.0876	0.0889	0.0924	0.0883	0.0864	0.0860	0.0920	0.0914	0.0898	0.0858	0.0844	1990	0.0844
1991	0.0837	0.0841	0.0844	0.0837	0.0845	0.0860	0.0850	0.0818	0.0790	0.0791	0.0789	0.0730	1991	0.0730
1992	0.0776	0.0777	0.0797	0.0803	0.0781	0.0765	0.0726	0.0725	0.0710	0.0741	0.0748	0.0726	1992	0.0726
1993	0.0725	0.0698	0.0702	0.0701	0.0701	0.0668	0.0656	0.0623	0.0627	0.0623	0.0651	0.0654	1993	0.0654
1994	0.0637	0.0682	0.0725	0.0745	0.0759	0.0774	0.0746	0.0761	0.0800	0.0809	0.0808	0.0799	1994	0.0799
1995	0.0780	0.0758	0.0755	0.0745	0.0677	0.0670	0.0691	0.0674	0.0663	0.0641	0.0623	0.0603	1995	0.0603
1996	0.0609	0.0659	0.0684	0.0706	0.0717	0.0703	0.0707	0.0726	0.0704	0.0671	0.0643	0.0673	1996	0.0673
1997	0.0689	0.0694	0.0723	0.0705	0.0701	0.0688	0.0637	0.0672	0.0649	0.0623	0.0614	0.0602	1997	0.0602
1998	0.0589	0.0599	0.0602	0.0604	0.0592	0.0576	0.0584	0.0547	0.0517	0.0540	0.0535	0.0542	1998	0.0542
1999	0.0536	0.0587	0.0592	0.0594	0.0615	0.0627	0.0639	0.0649	0.0646	0.0651	0.0662	0.0682	1999	0.0682
2000	0.0666	0.0646	0.0618	0.0630	0.0640	0.0622	0.0611	0.0594	0.0612	0.0600	0.0576	0.0558	2000	0.0558
2001	0.0562	0.0549	0.0559	0.0593	0.0594	0.0590	0.0561	0.0546	0.0542	0.0506	0.0553	0.0575	2001	0.0575
2002	0.0569	0.0563	0.0604	0.0575	0.0578	0.0566	0.0544	0.0510	0.0480	0.0508	0.0521	0.0484	2002	0.0484
2003	0.0495	0.0472	0.0486	0.0481	0.0436	0.0452	0.0542	0.0532	0.0490	0.0518	0.0519	0.0511	2003	0.0511
2004	0.0499	0.0483	0.0474	0.0531	0.0539	0.0532	0.0523	0.0493	0.0488	0.0478	0.0502	0.0484	2004	0.0484
2005	0.0465	0.0479	0.0488	0.0461	0.0440	0.0429	0.0456	0.0432	0.0464	0.0484	0.0481	0.0461	2005	0.0461
2006	0.0474	0.0457	0.0507	0.0532	0.0535	0.0531	0.0518	0.0496	0.0484	0.0481	0.0467	0.0491	2006	0.0491
2007	0.0502	0.0477	0.0493	0.0489	0.0510	0.0521	0.0501	0.0487	0.0489	0.0480	0.0445	0.0450	2007	0.0450
2008	0.0436	0.0438	0.0432	0.0458	0.0475	0.0460	0.0465	0.0449	0.0443	0.0478	0.0372	0.0303	2008	0.0303
2009	0.0394	0.0401	0.0355	0.0410	0.0432	0.0429	0.0430	0.0415	0.0403	0.0420	0.0406	0.0458	2009	0.0458
2010	0.0441	0.0441	0.0458	0.0437	0.0407	0.0376	0.0377	0.0327	0.0341	0.0367	0.0380	0.0414	2010	0.0414
2011	0.0432	0.0426	0.0429	0.0416	0.0391	0.0409	0.0378	0.0315	0.0265	0.0291	0.0273	0.0255	2011	0.0255
2012	0.0249	0.0272	0.0297	0.0268	0.0221	0.0233	0.0216	0.0223	0.0235	0.0238	0.0228	0.0246	2012	0.0246
2013	0.0291	0.0285	0.0287	0.0264	0.0309	0.0330	0.0344	0.0351	0.0349	0.0342	0.0361	0.0378	2013	0.0378
2014	0.0342	0.0339	0.0337	0.0326	0.0309	0.0313	0.0310	0.0287	0.0300	0.0282	0.0264	0.0246	2014	0.0246
2015	0.0200	0.0238	0.0230	0.0249	0.0262	0.0285	0.0263	0.0264	0.0253	0.0259	0.0265	0.0268	2015	0.0268
2016	0.0236	0.0217	0.0218	0.0223	0.0219	0.0179	0.0175	0.0186	0.0196	0.0220	0.0267	0.0272	2016	0.0272

*Compound annual return

Appendix A-10

Intermediate-term Government Bonds: Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1926	0.0068	0.0032	0.0041	0.0090	0.0008	0.0027	0.0013	0.0009	0.0050	0.0054	0.0045	0.0089	1926	0.0538
1927	0.0057	0.0038	0.0038	0.0016	0.0020	0.0029	0.0043	0.0056	0.0060	-0.0035	0.0083	0.0037	1927	0.0452
1928	0.0046	-0.0004	0.0010	-0.0003	-0.0006	0.0017	-0.0090	0.0050	0.0028	0.0032	0.0019	-0.0007	1928	0.0092
1929	-0.0029	-0.0018	0.0005	0.0089	-0.0061	0.0107	0.0066	0.0052	-0.0014	0.0168	0.0180	0.0044	1929	0.0601
1930	-0.0041	0.0094	0.0161	-0.0071	0.0061	0.0142	0.0054	0.0022	0.0063	0.0076	0.0070	0.0024	1930	0.0672
1931	-0.0071	0.0099	0.0052	0.0083	0.0119	-0.0214	0.0016	0.0017	-0.0113	-0.0105	0.0049	-0.0159	1931	-0.0232
1932	-0.0032	0.0128	0.0078	0.0194	-0.0090	0.0108	0.0120	0.0124	0.0027	0.0045	0.0031	0.0118	1932	0.0881
1933	-0.0016	-0.0001	0.0099	0.0057	0.0199	0.0008	-0.0006	0.0073	0.0026	-0.0025	0.0027	-0.0253	1933	0.0183
1934	0.0130	0.0052	0.0189	0.0182	0.0120	0.0091	-0.0024	-0.0092	-0.0138	0.0190	0.0046	0.0125	1934	0.0900
1935	0.0114	0.0105	0.0125	0.0107	-0.0035	0.0113	0.0038	-0.0071	-0.0057	0.0109	0.0014	0.0120	1935	0.0701
1936	-0.0004	0.0069	0.0031	0.0024	0.0038	0.0012	0.0022	0.0050	0.0010	0.0025	0.0081	-0.0057	1936	0.0306
1937	-0.0031	0.0007	-0.0164	0.0047	0.0080	-0.0013	0.0059	-0.0043	0.0081	0.0032	0.0042	0.0062	1937	0.0156
1938	0.0085	0.0052	-0.0013	0.0230	0.0023	0.0075	0.0010	0.0015	-0.0013	0.0093	-0.0001	0.0052	1938	0.0623
1939	0.0029	0.0082	0.0081	0.0038	0.0095	0.0002	0.0040	-0.0147	-0.0263	0.0315	0.0074	0.0108	1939	0.0452
1940	-0.0014	0.0035	0.0088	0.0002	-0.0214	0.0187	0.0003	0.0043	0.0047	0.0036	0.0056	0.0028	1940	0.0296
1941	0.0001	-0.0047	0.0069	0.0033	0.0012	0.0056	0.0000	0.0011	0.0000	0.0023	-0.0092	-0.0016	1941	0.0050
1942	0.0074	0.0015	0.0023	0.0022	0.0016	0.0013	0.0000	0.0017	-0.0023	0.0017	0.0017	0.0000	1942	0.0194
1943	0.0039	0.0013	0.0021	0.0024	0.0057	0.0033	0.0021	0.0002	0.0014	0.0017	0.0015	0.0021	1943	0.0281
1944	0.0011	0.0016	0.0020	0.0028	0.0005	0.0007	0.0029	0.0024	0.0011	0.0011	0.0009	0.0010	1944	0.0180
1945	0.0052	0.0038	0.0004	0.0014	0.0012	0.0019	0.0000	0.0016	0.0017	0.0016	0.0010	0.0021	1945	0.0222
1946	0.0039	0.0048	-0.0038	-0.0020	0.0006	0.0033	-0.0010	0.0004	-0.0011	0.0026	-0.0008	0.0032	1946	0.0100
1947	0.0023	0.0006	0.0024	-0.0013	0.0008	0.0008	0.0006	0.0026	0.0000	-0.0023	0.0006	0.0021	1947	0.0091
1948	0.0015	0.0018	0.0018	0.0019	0.0053	-0.0008	-0.0002	-0.0004	0.0010	0.0013	0.0021	0.0032	1948	0.0185
1949	0.0028	0.0011	0.0025	0.0015	0.0023	0.0050	0.0020	0.0031	0.0008	0.0006	0.0002	0.0012	1949	0.0232
1950	-0.0005	0.0008	0.0000	0.0008	0.0020	0.0003	0.0020	-0.0007	-0.0004	0.0001	0.0018	0.0008	1950	0.0070
1951	0.0022	0.0007	-0.0127	0.0057	-0.0040	0.0050	0.0058	0.0036	-0.0057	0.0016	0.0032	-0.0016	1951	0.0036
1952	0.0038	-0.0020	0.0067	0.0054	0.0019	-0.0035	-0.0034	-0.0024	0.0019	0.0066	-0.0006	0.0019	1952	0.0163
1953	-0.0002	0.0003	-0.0017	-0.0096	-0.0117	0.0155	0.0056	-0.0008	0.0194	0.0038	0.0014	0.0103	1953	0.0323
1954	0.0065	0.0100	0.0027	0.0043	-0.0073	0.0125	-0.0005	0.0011	-0.0020	-0.0009	-0.0001	0.0005	1954	0.0268
1955	-0.0032	-0.0052	0.0024	0.0004	0.0001	-0.0036	-0.0071	0.0007	0.0082	0.0072	-0.0053	-0.0011	1955	-0.0065

*Compound annual return

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Intermediate-term Government Bonds: Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1956	0.0105	0.0003	-0.0100	-0.0001	0.0112	0.0003	-0.0095	-0.0103	0.0092	-0.0019	-0.0047	0.0011	1956	-0.0042
1957	0.0237	-0.0013	0.0018	-0.0101	-0.0017	-0.0106	-0.0015	0.0109	0.0002	0.0043	0.0396	0.0216	1957	0.0784
1958	0.0034	0.0139	0.0053	0.0052	0.0060	-0.0068	-0.0091	-0.0356	-0.0017	0.0002	0.0132	-0.0061	1958	-0.0129
1959	-0.0013	0.0107	-0.0037	-0.0052	-0.0001	-0.0077	0.0034	-0.0078	0.0020	0.0174	-0.0092	-0.0020	1959	-0.0039
1960	0.0154	0.0072	0.0292	-0.0064	0.0031	0.0217	0.0267	-0.0005	0.0029	0.0016	-0.0094	0.0210	1960	0.1176
1961	-0.0059	0.0090	0.0037	0.0054	-0.0028	-0.0025	0.0007	0.0019	0.0079	0.0014	-0.0019	0.0018	1961	0.0185
1962	-0.0045	0.0155	0.0089	0.0025	0.0049	-0.0028	-0.0012	0.0125	0.0021	0.0051	0.0060	0.0056	1962	0.0556
1963	-0.0029	0.0017	0.0027	0.0030	0.0014	0.0014	0.0003	0.0019	0.0014	0.0011	0.0040	0.0003	1963	0.0164
1964	0.0033	0.0012	0.0016	0.0033	0.0081	0.0036	0.0027	0.0027	0.0045	0.0032	-0.0004	0.0058	1964	0.0404
1965	0.0042	0.0018	0.0043	0.0026	0.0035	0.0049	0.0017	0.0019	-0.0005	0.0000	0.0007	-0.0149	1965	0.0102
1966	0.0003	-0.0084	0.0187	-0.0019	0.0011	-0.0024	-0.0025	-0.0125	0.0216	0.0075	0.0028	0.0223	1966	0.0469
1967	0.0118	-0.0013	0.0183	-0.0089	0.0044	-0.0227	0.0133	-0.0036	0.0007	-0.0049	0.0028	0.0007	1967	0.0101
1968	0.0145	0.0040	-0.0026	-0.0016	0.0064	0.0167	0.0176	0.0021	0.0055	0.0009	-0.0013	-0.0173	1968	0.0454
1969	0.0086	-0.0013	0.0097	0.0079	-0.0082	-0.0084	0.0082	-0.0018	-0.0300	0.0333	-0.0047	-0.0193	1969	-0.0074
1970	0.0030	0.0439	0.0087	-0.0207	0.0110	0.0061	0.0152	0.0116	0.0196	0.0095	0.0451	0.0054	1970	0.1686
1971	0.0168	0.0224	0.0186	-0.0327	0.0011	-0.0187	0.0027	0.0350	0.0026	0.0220	0.0052	0.0110	1971	0.0872
1972	0.0106	0.0014	0.0015	0.0014	0.0016	0.0045	0.0015	0.0015	0.0014	0.0016	0.0045	0.0192	1972	0.0516
1973	-0.0006	-0.0075	0.0046	0.0064	0.0057	-0.0006	-0.0276	0.0254	0.0250	0.0050	0.0064	0.0040	1973	0.0461
1974	0.0009	0.0035	-0.0212	-0.0152	0.0130	-0.0087	0.0007	-0.0012	0.0319	0.0109	0.0236	0.0185	1974	0.0569
1975	0.0053	0.0148	-0.0059	-0.0186	0.0260	0.0027	-0.0030	-0.0009	0.0010	0.0366	-0.0010	0.0198	1975	0.0783
1976	0.0057	0.0084	0.0075	0.0116	-0.0145	0.0159	0.0119	0.0189	0.0076	0.0147	0.0321	0.0026	1976	0.1287
1977	-0.0190	0.0048	0.0055	0.0051	0.0056	0.0102	0.0001	0.0008	0.0015	-0.0060	0.0079	-0.0023	1977	0.0141
1978	0.0013	0.0017	0.0037	0.0024	-0.0002	-0.0021	0.0098	0.0079	0.0057	-0.0112	0.0092	0.0063	1978	0.0349
1979	0.0055	-0.0059	0.0112	0.0033	0.0193	0.0205	-0.0011	-0.0091	0.0006	-0.0468	0.0363	0.0087	1979	0.0409
1980	-0.0135	-0.0641	0.0143	0.1198	0.0490	-0.0077	-0.0106	-0.0387	-0.0038	-0.0152	0.0029	0.0171	1980	0.0391
1981	0.0032	-0.0235	0.0263	-0.0216	0.0245	0.0060	-0.0270	-0.0178	0.0164	0.0611	0.0624	-0.0142	1981	0.0945
1982	0.0050	0.0148	0.0042	0.0299	0.0146	-0.0135	0.0464	0.0469	0.0325	0.0531	0.0080	0.0185	1982	0.2910
1983	0.0007	0.0252	-0.0049	0.0259	-0.0122	0.0016	-0.0198	0.0081	0.0315	0.0019	0.0103	0.0047	1983	0.0741
1984	0.0177	-0.0064	-0.0035	-0.0003	-0.0250	0.0099	0.0393	0.0101	0.0202	0.0383	0.0192	0.0143	1984	0.1402
1985	0.0206	-0.0179	0.0166	0.0264	0.0485	0.0108	-0.0045	0.0148	0.0113	0.0162	0.0195	0.0257	1985	0.2033

*Compound annual return

Appendix A-10

Intermediate-term Government Bonds: Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1986	0.0082	0.0275	0.0338	0.0081	-0.0215	0.0276	0.0157	0.0266	-0.0110	0.0162	0.0113	0.0007	1986	0.1514
1987	0.0107	0.0059	-0.0031	-0.0244	-0.0038	0.0122	0.0025	-0.0038	-0.0141	0.0299	0.0083	0.0093	1987	0.0290
1988	0.0316	0.0123	-0.0086	-0.0044	-0.0049	0.0181	-0.0047	-0.0009	0.0196	0.0148	-0.0115	-0.0010	1988	0.0610
1989	0.0121	-0.0051	0.0049	0.0220	0.0212	0.0324	0.0235	-0.0246	0.0069	0.0237	0.0084	0.0012	1989	0.1329
1990	-0.0105	0.0007	0.0002	-0.0077	0.0261	0.0151	0.0174	-0.0092	0.0094	0.0171	0.0193	0.0161	1990	0.0973
1991	0.0107	0.0048	0.0023	0.0117	0.0059	-0.0023	0.0129	0.0247	0.0216	0.0134	0.0128	0.0265	1991	0.1546
1992	-0.0195	0.0022	-0.0079	0.0098	0.0222	0.0177	0.0242	0.0150	0.0194	-0.0182	-0.0084	0.0146	1992	0.0719
1993	0.0270	0.0243	0.0043	0.0088	-0.0009	0.0201	0.0005	0.0223	0.0056	0.0018	-0.0093	0.0032	1993	0.1124
1994	0.0138	-0.0258	-0.0257	-0.0105	-0.0002	-0.0028	0.0169	0.0026	-0.0158	-0.0023	-0.0070	0.0053	1994	-0.0514
1995	0.0182	0.0234	0.0063	0.0143	0.0369	0.0079	-0.0016	0.0086	0.0064	0.0121	0.0149	0.0095	1995	0.1680
1996	0.0006	-0.0138	-0.0118	-0.0050	-0.0032	0.0117	0.0025	-0.0005	0.0155	0.0183	0.0149	-0.0078	1996	0.0210
1997	0.0025	0.0002	-0.0114	0.0148	0.0077	0.0103	0.0264	-0.0098	0.0151	0.0150	-0.0001	0.0106	1997	0.0838
1998	0.0180	-0.0039	0.0026	0.0061	0.0070	0.0079	0.0027	0.0271	0.0330	0.0041	-0.0098	0.0037	1998	0.1021
1999	0.0055	-0.0262	0.0086	0.0021	-0.0147	0.0032	-0.0005	0.0015	0.0097	-0.0008	-0.0008	-0.0048	1999	-0.0177
2000	-0.0053	0.0078	0.0203	-0.0043	0.0052	0.0191	0.0072	0.0134	0.0096	0.0079	0.0174	0.0214	2000	0.1259
2001	0.0098	0.0105	0.0076	-0.0114	-0.0007	0.0066	0.0247	0.0095	0.0253	0.0180	-0.0171	-0.0082	2001	0.0762
2002	0.0036	0.0108	-0.0242	0.0239	0.0118	0.0169	0.0272	0.0167	0.0288	-0.0024	-0.0169	0.0279	2002	0.1293
2003	-0.0089	0.0179	-0.0007	0.0013	0.0273	-0.0035	-0.0319	-0.0027	0.0307	-0.0136	-0.0014	0.0109	2003	0.0240
2004	0.0052	0.0124	0.0100	-0.0334	-0.0049	0.0049	0.0082	0.0195	0.0012	0.0064	-0.0127	0.0067	2004	0.0225
2005	0.0026	-0.0111	-0.0038	0.0167	0.0103	0.0043	-0.0144	0.0161	-0.0124	-0.0063	0.0059	0.0061	2005	0.0136
2006	-0.0036	-0.0017	-0.0056	-0.0008	-0.0004	0.0022	0.0125	0.0135	0.0079	0.0052	0.0088	-0.0067	2006	0.0314
2007	-0.0020	0.0170	0.0024	0.0047	-0.0102	0.0011	0.0175	0.0185	0.0057	-0.0048	0.0425	0.0048	2007	0.1005
2008	0.0263	0.0234	0.0073	-0.0293	-0.0084	0.0075	0.0064	0.0104	0.0085	0.0146	0.0430	0.0160	2008	0.1311
2009	-0.0163	-0.0082	0.0186	-0.0166	-0.0132	-0.0076	0.0056	0.0097	0.0075	0.0030	0.0184	-0.0241	2009	-0.0240
2010	0.0194	0.0071	-0.0088	0.0094	0.0151	0.0129	0.0158	0.0128	0.0049	0.0064	-0.0082	-0.0171	2010	0.0712
2011	0.0062	-0.0053	-0.0005	0.0154	0.0179	-0.0001	0.0202	0.0210	0.0008	0.0009	0.0036	0.0049	2011	0.0881
2012	0.0008	-0.0052	-0.0070	0.0129	0.0083	-0.0020	0.0077	0.0015	0.0004	-0.0024	0.0048	-0.0031	2012	0.0166
2013	-0.0061	0.0064	-0.0143	0.0060	-0.0165	-0.0139	0.0026	-0.0074	0.0121	0.0051	0.0011	-0.0122	2013	-0.0368
2014	0.0131	0.0024	-0.0068	0.0048	0.0094	-0.0017	-0.0047	0.0075	-0.0049	0.0089	0.0078	-0.0060	2014	0.0300
2015	0.0241	-0.0123	0.0074	-0.0014	0.0005	-0.0053	0.0051	0.0011	0.0099	-0.0052	-0.0040	-0.0017	2015	0.0179
2016	0.0233	0.0057	0.0045	-0.0010	-0.0021	0.0175	0.0004	-0.0064	0.0027	-0.0052	-0.0189	-0.0007	2016	0.0192

*Compound annual return

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Intermediate-term Government Bonds: Income Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan-Dec*
1926	0.0032	0.0032	0.0032	0.0031	0.0031	0.0031	0.0032	0.0032	0.0032	0.0031	0.0031	0.0030	1926	0.0378
1927	0.0029	0.0029	0.0029	0.0029	0.0029	0.0029	0.0029	0.0029	0.0028	0.0029	0.0028	0.0028	1927	0.0349
1928	0.0028	0.0028	0.0029	0.0029	0.0030	0.0030	0.0032	0.0032	0.0032	0.0032	0.0032	0.0033	1928	0.0364
1929	0.0034	0.0035	0.0036	0.0035	0.0037	0.0035	0.0035	0.0034	0.0035	0.0033	0.0030	0.0030	1929	0.0407
1930	0.0031	0.0030	0.0028	0.0030	0.0029	0.0027	0.0026	0.0026	0.0026	0.0025	0.0024	0.0024	1930	0.0330
1931	0.0026	0.0025	0.0024	0.0023	0.0021	0.0026	0.0026	0.0026	0.0028	0.0031	0.0031	0.0034	1931	0.0316
1932	0.0035	0.0034	0.0033	0.0030	0.0032	0.0031	0.0029	0.0027	0.0027	0.0027	0.0027	0.0025	1932	0.0363
1933	0.0026	0.0026	0.0025	0.0025	0.0021	0.0022	0.0022	0.0021	0.0021	0.0022	0.0022	0.0027	1933	0.0283
1934	0.0030	0.0024	0.0027	0.0024	0.0023	0.0021	0.0021	0.0021	0.0021	0.0026	0.0022	0.0023	1934	0.0293
1935	0.0021	0.0018	0.0018	0.0017	0.0016	0.0015	0.0015	0.0014	0.0015	0.0016	0.0015	0.0016	1935	0.0202
1936	0.0014	0.0013	0.0013	0.0012	0.0012	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	1936	0.0144
1937	0.0010	0.0010	0.0012	0.0015	0.0013	0.0014	0.0014	0.0013	0.0014	0.0012	0.0012	0.0011	1937	0.0148
1938	0.0018	0.0016	0.0017	0.0017	0.0015	0.0014	0.0013	0.0014	0.0013	0.0014	0.0013	0.0013	1938	0.0182
1939	0.0013	0.0011	0.0012	0.0010	0.0011	0.0009	0.0009	0.0009	0.0011	0.0015	0.0010	0.0009	1939	0.0131
1940	0.0009	0.0008	0.0008	0.0007	0.0007	0.0010	0.0008	0.0008	0.0007	0.0007	0.0006	0.0005	1940	0.0090
1941	0.0006	0.0006	0.0008	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005	0.0004	0.0007	1941	0.0067
1942	0.0008	0.0006	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	1942	0.0076
1943	0.0014	0.0013	0.0014	0.0013	0.0013	0.0013	0.0013	0.0012	0.0012	0.0012	0.0012	0.0012	1943	0.0156
1944	0.0013	0.0012	0.0013	0.0012	0.0013	0.0012	0.0012	0.0012	0.0011	0.0012	0.0011	0.0011	1944	0.0144
1945	0.0012	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0009	0.0010	0.0009	0.0009	1945	0.0119
1946	0.0009	0.0008	0.0007	0.0009	0.0009	0.0009	0.0009	0.0009	0.0010	0.0010	0.0009	0.0010	1946	0.0108
1947	0.0010	0.0009	0.0010	0.0009	0.0010	0.0011	0.0010	0.0010	0.0010	0.0010	0.0010	0.0012	1947	0.0121
1948	0.0013	0.0012	0.0014	0.0013	0.0012	0.0013	0.0012	0.0013	0.0013	0.0013	0.0014	0.0013	1948	0.0156
1949	0.0013	0.0012	0.0013	0.0012	0.0013	0.0012	0.0010	0.0011	0.0010	0.0010	0.0010	0.0010	1949	0.0136
1950	0.0011	0.0010	0.0011	0.0010	0.0012	0.0011	0.0012	0.0011	0.0011	0.0013	0.0013	0.0013	1950	0.0139
1951	0.0016	0.0014	0.0015	0.0018	0.0017	0.0017	0.0018	0.0017	0.0015	0.0019	0.0017	0.0018	1951	0.0198
1952	0.0018	0.0017	0.0019	0.0017	0.0016	0.0017	0.0018	0.0018	0.0021	0.0020	0.0017	0.0021	1952	0.0219
1953	0.0019	0.0018	0.0021	0.0021	0.0022	0.0027	0.0024	0.0023	0.0023	0.0020	0.0020	0.0020	1953	0.0255
1954	0.0016	0.0014	0.0014	0.0013	0.0011	0.0016	0.0011	0.0012	0.0011	0.0012	0.0014	0.0014	1954	0.0160
1955	0.0018	0.0017	0.0020	0.0019	0.0021	0.0020	0.0020	0.0025	0.0023	0.0023	0.0021	0.0022	1955	0.0245

*Compound annual return

Appendix A-11

Intermediate-term Government Bonds: Income Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1956	0.0025	0.0021	0.0022	0.0026	0.0026	0.0023	0.0025	0.0027	0.0026	0.0030	0.0028	0.0030	1956	0.0305
1957	0.0030	0.0025	0.0026	0.0029	0.0030	0.0027	0.0036	0.0032	0.0032	0.0033	0.0031	0.0028	1957	0.0359
1958	0.0024	0.0021	0.0022	0.0021	0.0019	0.0021	0.0021	0.0022	0.0032	0.0032	0.0029	0.0032	1958	0.0293
1959	0.0031	0.0030	0.0033	0.0032	0.0033	0.0037	0.0038	0.0037	0.0039	0.0039	0.0038	0.0041	1959	0.0418
1960	0.0039	0.0039	0.0039	0.0032	0.0037	0.0035	0.0031	0.0030	0.0028	0.0029	0.0028	0.0031	1960	0.0415
1961	0.0030	0.0028	0.0029	0.0027	0.0030	0.0029	0.0031	0.0031	0.0030	0.0032	0.0030	0.0030	1961	0.0354
1962	0.0035	0.0031	0.0031	0.0031	0.0031	0.0029	0.0033	0.0032	0.0028	0.0033	0.0029	0.0030	1962	0.0373
1963	0.0030	0.0028	0.0029	0.0032	0.0031	0.0029	0.0034	0.0031	0.0033	0.0033	0.0031	0.0034	1963	0.0371
1964	0.0034	0.0030	0.0035	0.0033	0.0031	0.0036	0.0034	0.0033	0.0033	0.0033	0.0034	0.0034	1964	0.0400
1965	0.0033	0.0031	0.0037	0.0033	0.0033	0.0037	0.0034	0.0036	0.0034	0.0034	0.0038	0.0037	1965	0.0415
1966	0.0040	0.0036	0.0043	0.0038	0.0042	0.0040	0.0040	0.0047	0.0046	0.0044	0.0042	0.0042	1966	0.0493
1967	0.0041	0.0035	0.0040	0.0033	0.0042	0.0038	0.0045	0.0042	0.0042	0.0047	0.0046	0.0044	1967	0.0488
1968	0.0051	0.0043	0.0043	0.0049	0.0048	0.0043	0.0049	0.0042	0.0044	0.0044	0.0042	0.0047	1968	0.0549
1969	0.0054	0.0048	0.0049	0.0057	0.0050	0.0058	0.0059	0.0054	0.0061	0.0067	0.0056	0.0068	1969	0.0665
1970	0.0066	0.0061	0.0063	0.0059	0.0062	0.0067	0.0065	0.0062	0.0060	0.0057	0.0058	0.0050	1970	0.0749
1971	0.0047	0.0043	0.0047	0.0040	0.0044	0.0053	0.0053	0.0056	0.0048	0.0046	0.0047	0.0046	1971	0.0575
1972	0.0048	0.0044	0.0046	0.0044	0.0052	0.0048	0.0049	0.0050	0.0047	0.0053	0.0051	0.0049	1972	0.0575
1973	0.0056	0.0048	0.0054	0.0056	0.0056	0.0053	0.0059	0.0064	0.0055	0.0060	0.0055	0.0056	1973	0.0658
1974	0.0057	0.0051	0.0054	0.0065	0.0067	0.0059	0.0073	0.0067	0.0072	0.0067	0.0061	0.0064	1974	0.0724
1975	0.0061	0.0055	0.0059	0.0060	0.0063	0.0063	0.0063	0.0061	0.0069	0.0068	0.0065	0.0067	1975	0.0735
1976	0.0060	0.0055	0.0066	0.0059	0.0054	0.0069	0.0060	0.0062	0.0056	0.0054	0.0058	0.0050	1976	0.0710
1977	0.0051	0.0050	0.0056	0.0053	0.0058	0.0055	0.0052	0.0059	0.0056	0.0059	0.0059	0.0059	1977	0.0649
1978	0.0066	0.0057	0.0066	0.0060	0.0071	0.0066	0.0070	0.0068	0.0065	0.0072	0.0072	0.0069	1978	0.0783
1979	0.0079	0.0066	0.0075	0.0077	0.0077	0.0070	0.0074	0.0073	0.0070	0.0084	0.0089	0.0086	1979	0.0904
1980	0.0086	0.0083	0.0107	0.0103	0.0081	0.0075	0.0079	0.0076	0.0097	0.0094	0.0096	0.0111	1980	0.1055
1981	0.0101	0.0095	0.0117	0.0106	0.0110	0.0118	0.0116	0.0120	0.0130	0.0129	0.0121	0.0108	1981	0.1297
1982	0.0107	0.0102	0.0122	0.0112	0.0101	0.0118	0.0113	0.0109	0.0097	0.0089	0.0087	0.0085	1982	0.1281
1983	0.0084	0.0079	0.0084	0.0081	0.0086	0.0085	0.0082	0.0103	0.0094	0.0092	0.0091	0.0091	1983	0.1035
1984	0.0096	0.0088	0.0095	0.0101	0.0104	0.0105	0.0113	0.0105	0.0095	0.0110	0.0093	0.0093	1984	0.1168
1985	0.0090	0.0081	0.0089	0.0097	0.0090	0.0073	0.0083	0.0081	0.0082	0.0081	0.0074	0.0078	1985	0.1029

*Compound annual return

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Intermediate-term Government Bonds: Income Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1986	0.0071	0.0066	0.0068	0.0060	0.0060	0.0068	0.0062	0.0057	0.0058	0.0060	0.0052	0.0060	1986	0.0772
1987	0.0055	0.0052	0.0060	0.0058	0.0062	0.0071	0.0065	0.0068	0.0068	0.0073	0.0070	0.0070	1987	0.0747
1988	0.0065	0.0066	0.0064	0.0063	0.0072	0.0070	0.0064	0.0077	0.0072	0.0071	0.0067	0.0071	1988	0.0824
1989	0.0077	0.0066	0.0078	0.0071	0.0080	0.0070	0.0067	0.0061	0.0065	0.0071	0.0063	0.0060	1989	0.0846
1990	0.0071	0.0064	0.0069	0.0071	0.0075	0.0067	0.0072	0.0068	0.0065	0.0074	0.0067	0.0067	1990	0.0815
1991	0.0064	0.0059	0.0059	0.0070	0.0065	0.0059	0.0069	0.0052	0.0061	0.0058	0.0052	0.0056	1991	0.0743
1992	0.0052	0.0052	0.0060	0.0058	0.0056	0.0058	0.0053	0.0050	0.0047	0.0044	0.0050	0.0053	1992	0.0627
1993	0.0049	0.0045	0.0049	0.0045	0.0041	0.0050	0.0041	0.0044	0.0041	0.0038	0.0042	0.0043	1993	0.0553
1994	0.0045	0.0039	0.0048	0.0049	0.0058	0.0055	0.0055	0.0060	0.0055	0.0060	0.0061	0.0063	1994	0.0607
1995	0.0067	0.0056	0.0060	0.0054	0.0062	0.0050	0.0051	0.0051	0.0047	0.0052	0.0047	0.0043	1995	0.0669
1996	0.0046	0.0041	0.0045	0.0053	0.0054	0.0050	0.0058	0.0052	0.0056	0.0053	0.0047	0.0050	1996	0.0582
1997	0.0052	0.0047	0.0054	0.0055	0.0054	0.0055	0.0054	0.0046	0.0054	0.0050	0.0043	0.0052	1997	0.0614
1998	0.0046	0.0041	0.0049	0.0046	0.0045	0.0049	0.0047	0.0046	0.0041	0.0035	0.0036	0.0039	1998	0.0529
1999	0.0037	0.0035	0.0048	0.0043	0.0041	0.0052	0.0048	0.0051	0.0048	0.0046	0.0052	0.0052	1999	0.0530
2000	0.0054	0.0052	0.0056	0.0048	0.0059	0.0054	0.0053	0.0051	0.0047	0.0051	0.0047	0.0043	2000	0.0619
2001	0.0032	0.0026	0.0027	0.0033	0.0042	0.0040	0.0044	0.0039	0.0034	0.0035	0.0030	0.0035	2001	0.0427
2002	0.0038	0.0034	0.0034	0.0045	0.0039	0.0034	0.0037	0.0029	0.0027	0.0022	0.0022	0.0028	2002	0.0398
2003	0.0024	0.0024	0.0024	0.0023	0.0023	0.0019	0.0020	0.0025	0.0029	0.0022	0.0023	0.0029	2003	0.0285
2004	0.0026	0.0024	0.0026	0.0023	0.0027	0.0034	0.0030	0.0031	0.0026	0.0026	0.0027	0.0030	2004	0.0328
2005	0.0031	0.0028	0.0034	0.0033	0.0034	0.0031	0.0030	0.0037	0.0031	0.0035	0.0036	0.0036	2005	0.0392
2006	0.0037	0.0034	0.0039	0.0037	0.0044	0.0041	0.0043	0.0040	0.0036	0.0039	0.0036	0.0034	2006	0.0454
2007	0.0041	0.0036	0.0037	0.0038	0.0038	0.0038	0.0043	0.0038	0.0032	0.0037	0.0035	0.0028	2007	0.0444
2008	0.0031	0.0024	0.0022	0.0020	0.0025	0.0028	0.0028	0.0025	0.0026	0.0024	0.0020	0.0015	2008	0.0296
2009	0.0012	0.0014	0.0018	0.0014	0.0016	0.0021	0.0022	0.0021	0.0019	0.0018	0.0018	0.0015	2009	0.0201
2010	0.0022	0.0019	0.0021	0.0021	0.0018	0.0019	0.0015	0.0013	0.0010	0.0009	0.0009	0.0011	2010	0.0192
2011	0.0019	0.0017	0.0019	0.0019	0.0018	0.0014	0.0014	0.0012	0.0007	0.0008	0.0007	0.0007	2011	0.0164
2012	0.0007	0.0006	0.0008	0.0009	0.0007	0.0005	0.0006	0.0005	0.0004	0.0005	0.0005	0.0004	2012	0.0073
2013	0.0007	0.0007	0.0004	0.0007	0.0006	0.0008	0.0012	0.0011	0.0013	0.0011	0.0009	0.0010	2013	0.0102
2014	0.0016	0.0012	0.0013	0.0014	0.0014	0.0012	0.0013	0.0013	0.0014	0.0014	0.0012	0.0012	2014	0.0163
2015	0.0014	0.0010	0.0014	0.0012	0.0011	0.0013	0.0014	0.0013	0.0013	0.0011	0.0012	0.0014	2015	0.0151
2016	0.0015	0.0012	0.0011	0.0010	0.0011	0.0011	0.0008	0.0009	0.0010	0.0010	0.0011	0.0015	2016	0.0136

*Compound annual return

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Intermediate-term Government Bonds: Capital Appreciation Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1926	0.0036	0.0000	0.0009	0.0059	-0.0023	-0.0005	-0.0018	-0.0023	0.0018	0.0023	0.0014	0.0059	1926	0.0151
1927	0.0027	0.0009	0.0009	-0.0014	-0.0009	0.0000	0.0014	0.0027	0.0032	-0.0064	0.0055	0.0009	1927	0.0096
1928	0.0018	-0.0032	-0.0018	-0.0032	-0.0036	-0.0014	-0.0122	0.0018	-0.0005	0.0000	-0.0014	-0.0041	1928	-0.0273
1929	-0.0063	-0.0054	-0.0031	0.0054	-0.0098	0.0072	0.0031	0.0018	-0.0049	0.0135	0.0150	0.0014	1929	0.0177
1930	-0.0073	0.0064	0.0133	-0.0100	0.0032	0.0115	0.0028	-0.0005	0.0037	0.0051	0.0046	0.0000	1930	0.0330
1931	-0.0097	0.0074	0.0028	0.0060	0.0098	-0.0240	-0.0009	-0.0009	-0.0142	-0.0136	0.0018	-0.0193	1931	-0.0540
1932	-0.0067	0.0094	0.0045	0.0164	-0.0122	0.0077	0.0091	0.0096	0.0000	0.0018	0.0005	0.0092	1932	0.0502
1933	-0.0041	-0.0028	0.0074	0.0032	0.0178	-0.0014	-0.0028	0.0051	0.0005	-0.0047	0.0005	-0.0280	1933	-0.0099
1934	0.0100	0.0028	0.0162	0.0158	0.0097	0.0070	-0.0045	-0.0113	-0.0160	0.0164	0.0024	0.0102	1934	0.0597
1935	0.0093	0.0088	0.0107	0.0090	-0.0050	0.0098	0.0022	-0.0086	-0.0072	0.0093	-0.0002	0.0105	1935	0.0494
1936	-0.0017	0.0056	0.0018	0.0012	0.0026	-0.0001	0.0010	0.0038	-0.0001	0.0014	0.0070	-0.0067	1936	0.0160
1937	-0.0042	-0.0004	-0.0176	0.0032	0.0067	-0.0027	0.0045	-0.0056	0.0068	0.0020	0.0030	0.0051	1937	0.0005
1938	0.0067	0.0036	-0.0030	0.0214	0.0008	0.0061	-0.0003	0.0000	-0.0026	0.0079	-0.0014	0.0039	1938	0.0437
1939	0.0016	0.0071	0.0069	0.0028	0.0084	-0.0007	0.0030	-0.0155	-0.0273	0.0300	0.0063	0.0098	1939	0.0318
1940	-0.0023	0.0027	0.0080	-0.0005	-0.0221	0.0177	-0.0005	0.0035	0.0040	0.0030	0.0050	0.0023	1940	0.0204
1941	-0.0006	-0.0052	0.0061	0.0027	0.0006	0.0051	-0.0004	0.0006	-0.0004	0.0018	-0.0096	-0.0023	1941	-0.0017
1942	0.0066	0.0009	0.0016	0.0016	0.0010	0.0006	-0.0006	0.0011	-0.0029	0.0011	0.0011	-0.0006	1942	0.0117
1943	0.0025	0.0001	0.0007	0.0010	0.0044	0.0020	0.0008	-0.0010	0.0002	0.0005	0.0002	0.0008	1943	0.0123
1944	-0.0002	0.0004	0.0007	0.0016	-0.0008	-0.0005	0.0016	0.0012	0.0000	-0.0001	-0.0003	-0.0001	1944	0.0035
1945	0.0040	0.0028	-0.0006	0.0005	0.0002	0.0009	-0.0010	0.0006	0.0008	0.0006	0.0001	0.0012	1945	0.0102
1946	0.0030	0.0040	-0.0045	-0.0028	-0.0003	0.0024	-0.0019	-0.0005	-0.0020	0.0015	-0.0018	0.0022	1946	-0.0008
1947	0.0012	-0.0003	0.0014	-0.0022	-0.0002	-0.0003	-0.0005	0.0016	-0.0010	-0.0033	-0.0004	0.0008	1947	-0.0030
1948	0.0002	0.0006	0.0003	0.0006	0.0042	-0.0021	-0.0015	-0.0018	-0.0003	0.0000	0.0006	0.0019	1948	0.0027
1949	0.0015	-0.0001	0.0012	0.0003	0.0010	0.0038	0.0010	0.0019	-0.0002	-0.0004	-0.0008	0.0002	1949	0.0095
1950	-0.0016	-0.0002	-0.0011	-0.0003	0.0007	-0.0008	0.0009	-0.0019	-0.0015	-0.0013	0.0005	-0.0004	1950	-0.0069
1951	0.0006	-0.0007	-0.0142	0.0040	-0.0058	0.0033	0.0040	0.0019	-0.0072	-0.0003	0.0015	-0.0034	1951	-0.0163
1952	0.0019	-0.0037	0.0048	0.0037	0.0004	-0.0053	-0.0052	-0.0042	-0.0002	0.0046	-0.0023	-0.0002	1952	-0.0057
1953	-0.0022	-0.0016	-0.0038	-0.0117	-0.0139	0.0129	0.0032	-0.0031	0.0171	0.0018	-0.0006	0.0083	1953	0.0061
1954	0.0049	0.0086	0.0013	0.0031	-0.0084	0.0109	-0.0016	-0.0001	-0.0032	-0.0021	-0.0015	-0.0010	1954	0.0108
1955	-0.0050	-0.0070	0.0004	-0.0015	-0.0020	-0.0057	-0.0091	-0.0018	0.0059	0.0050	-0.0074	-0.0033	1955	-0.0310

*Compound annual return

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Appendix A-12

Intermediate-term Government Bonds: Capital Appreciation Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1956	0.0080	-0.0018	-0.0122	-0.0027	0.0086	-0.0020	-0.0120	-0.0130	0.0066	-0.0049	-0.0075	-0.0019	1956	-0.0345
1957	0.0207	-0.0037	-0.0009	-0.0130	-0.0047	-0.0133	-0.0051	0.0077	-0.0030	0.0010	0.0365	0.0188	1957	0.0405
1958	0.0010	0.0117	0.0031	0.0031	0.0041	-0.0088	-0.0112	-0.0378	-0.0048	-0.0029	0.0103	-0.0093	1958	-0.0417
1959	-0.0045	0.0078	-0.0070	-0.0084	-0.0034	-0.0113	-0.0004	-0.0116	-0.0019	0.0134	-0.0130	-0.0060	1959	-0.0456
1960	0.0115	0.0032	0.0253	-0.0096	-0.0006	0.0182	0.0236	-0.0034	0.0001	-0.0013	-0.0122	0.0180	1960	0.0742
1961	-0.0089	0.0063	0.0008	0.0026	-0.0058	-0.0054	-0.0024	-0.0013	0.0049	-0.0018	-0.0049	-0.0012	1961	-0.0172
1962	-0.0080	0.0124	0.0058	-0.0006	0.0018	-0.0056	-0.0045	0.0092	-0.0007	0.0018	0.0031	0.0026	1962	0.0173
1963	-0.0059	-0.0011	-0.0002	-0.0002	-0.0017	-0.0015	-0.0030	-0.0012	-0.0019	-0.0022	0.0008	-0.0032	1963	-0.0210
1964	-0.0001	-0.0019	-0.0019	-0.0001	0.0049	0.0000	-0.0006	-0.0006	0.0012	0.0000	-0.0037	0.0024	1964	-0.0003
1965	0.0009	-0.0013	0.0006	-0.0007	0.0002	0.0012	-0.0016	-0.0017	-0.0039	-0.0034	-0.0031	-0.0186	1965	-0.0310
1966	-0.0037	-0.0120	0.0145	-0.0056	-0.0032	-0.0064	-0.0065	-0.0171	0.0170	0.0031	-0.0015	0.0180	1966	-0.0041
1967	0.0077	-0.0048	0.0144	-0.0122	0.0002	-0.0265	0.0089	-0.0078	-0.0035	-0.0095	-0.0018	-0.0038	1967	-0.0385
1968	0.0095	-0.0003	-0.0069	-0.0065	0.0015	0.0123	0.0128	-0.0021	0.0011	-0.0035	-0.0054	-0.0220	1968	-0.0099
1969	0.0032	-0.0061	0.0048	0.0021	-0.0131	-0.0142	0.0024	-0.0072	-0.0361	0.0266	-0.0103	-0.0260	1969	-0.0727
1970	-0.0035	0.0378	0.0024	-0.0266	0.0049	-0.0006	0.0087	0.0054	0.0136	0.0037	0.0393	0.0005	1970	0.0871
1971	0.0121	0.0181	0.0139	-0.0367	-0.0034	-0.0240	-0.0027	0.0294	-0.0022	0.0173	0.0005	0.0064	1971	0.0272
1972	0.0058	-0.0030	-0.0031	-0.0030	-0.0035	-0.0003	-0.0034	-0.0035	-0.0033	-0.0037	-0.0006	0.0143	1972	-0.0075
1973	-0.0062	-0.0123	-0.0008	0.0007	0.0001	-0.0059	-0.0336	0.0190	0.0195	-0.0010	0.0009	-0.0016	1973	-0.0219
1974	-0.0048	-0.0016	-0.0266	-0.0217	0.0063	-0.0147	-0.0066	-0.0078	0.0247	0.0043	0.0175	0.0120	1974	-0.0199
1975	-0.0008	0.0092	-0.0119	-0.0246	0.0197	-0.0035	-0.0094	-0.0070	-0.0059	0.0298	-0.0065	0.0131	1975	0.0012
1976	-0.0003	0.0028	0.0010	0.0057	-0.0200	0.0090	0.0059	0.0127	0.0019	0.0093	0.0264	-0.0024	1976	0.0525
1977	-0.0241	-0.0002	-0.0001	-0.0001	-0.0002	0.0048	-0.0051	-0.0052	-0.0041	-0.0118	0.0019	-0.0082	1977	-0.0515
1978	-0.0053	-0.0041	-0.0029	-0.0036	-0.0073	-0.0087	0.0028	0.0010	-0.0008	-0.0184	0.0020	-0.0005	1978	-0.0449
1979	-0.0024	-0.0125	0.0038	-0.0044	0.0116	0.0135	-0.0086	-0.0163	-0.0065	-0.0553	0.0274	0.0001	1979	-0.0507
1980	-0.0221	-0.0724	0.0036	0.1095	0.0409	-0.0152	-0.0185	-0.0463	-0.0135	-0.0246	-0.0067	0.0060	1980	-0.0681
1981	-0.0069	-0.0331	0.0146	-0.0322	0.0135	-0.0059	-0.0386	-0.0298	0.0034	0.0482	0.0502	-0.0250	1981	-0.0455
1982	-0.0057	0.0046	-0.0080	0.0186	0.0045	-0.0253	0.0351	0.0359	0.0228	0.0442	-0.0007	0.0100	1982	0.1423
1983	-0.0076	0.0173	-0.0133	0.0177	-0.0208	-0.0069	-0.0280	-0.0023	0.0220	-0.0073	0.0012	-0.0043	1983	-0.0330
1984	0.0081	-0.0153	-0.0129	-0.0104	-0.0353	-0.0007	0.0280	-0.0005	0.0106	0.0274	0.0099	0.0050	1984	0.0122
1985	0.0116	-0.0261	0.0077	0.0167	0.0395	0.0035	-0.0129	0.0067	0.0031	0.0081	0.0121	0.0178	1985	0.0901

*Compound annual return

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Intermediate-term Government Bonds: Capital Appreciation Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1986	0.0011	0.0210	0.0270	0.0021	-0.0274	0.0208	0.0095	0.0209	-0.0168	0.0102	0.0061	-0.0053	1986	0.0699
1987	0.0051	0.0007	-0.0091	-0.0302	-0.0100	0.0051	-0.0040	-0.0106	-0.0209	0.0226	0.0013	0.0023	1987	-0.0475
1988	0.0251	0.0057	-0.0151	-0.0107	-0.0122	0.0111	-0.0111	-0.0086	0.0124	0.0077	-0.0182	-0.0081	1988	-0.0226
1989	0.0044	-0.0117	-0.0029	0.0149	0.0132	0.0254	0.0168	-0.0307	0.0004	0.0166	0.0021	-0.0048	1989	0.0434
1990	-0.0176	-0.0057	-0.0067	-0.0148	0.0186	0.0084	0.0102	-0.0160	0.0030	0.0096	0.0126	0.0095	1990	0.0102
1991	0.0042	-0.0011	-0.0036	0.0046	-0.0006	-0.0081	0.0060	0.0184	0.0155	0.0077	0.0076	0.0209	1991	0.0736
1992	-0.0247	-0.0030	-0.0139	0.0039	0.0166	0.0118	0.0189	0.0100	0.0147	-0.0226	-0.0134	0.0093	1992	0.0064
1993	0.0221	0.0198	-0.0006	0.0043	-0.0051	0.0152	-0.0035	0.0179	0.0015	-0.0020	-0.0135	-0.0011	1993	0.0556
1994	0.0093	-0.0297	-0.0306	-0.0154	-0.0060	-0.0084	0.0115	-0.0034	-0.0213	-0.0084	-0.0131	-0.0010	1994	-0.1114
1995	0.0115	0.0178	0.0003	0.0090	0.0307	0.0030	-0.0066	0.0035	0.0017	0.0069	0.0102	0.0052	1995	0.0966
1996	-0.0040	-0.0178	-0.0164	-0.0103	-0.0086	0.0067	-0.0033	-0.0057	0.0100	0.0129	0.0102	-0.0128	1996	-0.0390
1997	-0.0027	-0.0045	-0.0168	0.0093	0.0024	0.0048	0.0210	-0.0143	0.0098	0.0100	-0.0045	0.0054	1997	0.0195
1998	0.0134	-0.0080	-0.0024	0.0015	0.0025	0.0030	-0.0020	0.0225	0.0289	0.0006	-0.0134	-0.0002	1998	0.0466
1999	0.0018	-0.0297	0.0038	-0.0023	-0.0188	-0.0020	-0.0053	-0.0035	0.0049	-0.0054	-0.0060	-0.0100	1999	-0.0706
2000	-0.0107	0.0026	0.0147	-0.0091	-0.0007	0.0138	0.0019	0.0083	0.0049	0.0028	0.0127	0.0171	2000	0.0594
2001	0.0066	0.0079	0.0049	-0.0146	-0.0049	0.0025	0.0203	0.0056	0.0219	0.0145	-0.0201	-0.0117	2001	0.0323
2002	-0.0003	0.0073	-0.0276	0.0193	0.0079	0.0135	0.0234	0.0138	0.0261	-0.0046	-0.0191	0.0251	2002	0.0865
2003	-0.0113	0.0155	-0.0031	-0.0010	0.0250	-0.0054	-0.0339	-0.0053	0.0279	-0.0158	-0.0038	0.0080	2003	-0.0048
2004	0.0025	0.0100	0.0074	-0.0357	-0.0076	0.0015	0.0051	0.0164	-0.0014	0.0039	-0.0154	0.0036	2004	-0.0107
2005	-0.0005	-0.0139	-0.0073	0.0134	0.0069	0.0012	-0.0173	0.0124	-0.0155	-0.0098	0.0023	0.0026	2005	-0.0258
2006	-0.0073	-0.0051	-0.0095	-0.0045	-0.0049	-0.0019	0.0082	0.0095	0.0042	0.0012	0.0052	-0.0102	2006	-0.0151
2007	-0.0061	0.0134	-0.0012	0.0009	-0.0141	-0.0028	0.0132	0.0147	0.0026	-0.0085	0.0390	0.0021	2007	0.0533
2008	0.0231	0.0210	0.0051	-0.0314	-0.0110	0.0047	0.0036	0.0079	0.0058	0.0122	0.0410	0.0145	2008	0.0992
2009	-0.0175	-0.0096	0.0168	-0.0179	-0.0148	-0.0097	0.0034	0.0076	0.0056	0.0012	0.0166	-0.0256	2009	-0.0442
2010	0.0172	0.0052	-0.0109	0.0073	0.0133	0.0111	0.0143	0.0115	0.0039	0.0055	-0.0091	-0.0182	2010	0.0516
2011	0.0044	-0.0070	-0.0024	0.0136	0.0161	-0.0015	0.0188	0.0198	0.0001	0.0002	0.0028	0.0043	2011	0.0709
2012	0.0002	-0.0059	-0.0078	0.0120	0.0075	-0.0025	0.0071	0.0010	0.0000	-0.0029	0.0043	-0.0036	2012	0.0093
2013	-0.0068	0.0057	-0.0146	0.0053	-0.0170	-0.0147	0.0014	-0.0084	0.0108	0.0040	0.0002	-0.0132	2013	-0.0468
2014	0.0115	0.0011	-0.0082	0.0033	0.0080	-0.0030	-0.0060	0.0061	-0.0062	0.0075	0.0066	-0.0072	2014	0.0135
2015	0.0227	-0.0132	0.0061	-0.0025	-0.0007	-0.0065	0.0037	-0.0002	0.0087	-0.0063	-0.0053	-0.0031	2015	0.0029
2016	0.0218	0.0045	0.0034	-0.0020	-0.0033	0.0164	-0.0004	-0.0074	0.0017	-0.0062	-0.0199	-0.0022	2016	0.0058

*Compound annual return

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Intermediate-term Government Bonds: Yields
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1926	0.0386	0.0386	0.0384	0.0371	0.0376	0.0377	0.0381	0.0386	0.0382	0.0377	0.0374	0.0361	1926	0.0361
1927	0.0355	0.0353	0.0351	0.0354	0.0356	0.0356	0.0353	0.0347	0.0340	0.0354	0.0342	0.0340	1927	0.0340
1928	0.0336	0.0343	0.0347	0.0354	0.0362	0.0365	0.0392	0.0388	0.0389	0.0389	0.0392	0.0401	1928	0.0401
1929	0.0415	0.0427	0.0434	0.0422	0.0444	0.0428	0.0421	0.0417	0.0428	0.0398	0.0365	0.0362	1929	0.0362
1930	0.0378	0.0364	0.0335	0.0357	0.0350	0.0325	0.0319	0.0320	0.0312	0.0301	0.0291	0.0291	1930	0.0291
1931	0.0312	0.0296	0.0290	0.0277	0.0256	0.0308	0.0310	0.0312	0.0343	0.0373	0.0369	0.0412	1931	0.0412
1932	0.0427	0.0406	0.0396	0.0360	0.0387	0.0370	0.0350	0.0329	0.0329	0.0325	0.0324	0.0304	1932	0.0304
1933	0.0313	0.0319	0.0303	0.0296	0.0258	0.0261	0.0267	0.0256	0.0255	0.0265	0.0264	0.0325	1933	0.0325
1934	0.0325	0.0321	0.0296	0.0272	0.0257	0.0246	0.0253	0.0271	0.0298	0.0271	0.0267	0.0249	1934	0.0249
1935	0.0233	0.0218	0.0199	0.0184	0.0193	0.0175	0.0171	0.0187	0.0201	0.0183	0.0183	0.0163	1935	0.0163
1936	0.0166	0.0155	0.0151	0.0149	0.0143	0.0143	0.0141	0.0133	0.0133	0.0130	0.0114	0.0129	1936	0.0129
1937	0.0134	0.0135	0.0184	0.0175	0.0156	0.0164	0.0151	0.0168	0.0147	0.0141	0.0131	0.0114	1937	0.0114
1938	0.0205	0.0200	0.0204	0.0174	0.0173	0.0164	0.0164	0.0164	0.0168	0.0156	0.0158	0.0152	1938	0.0152
1939	0.0149	0.0138	0.0127	0.0122	0.0108	0.0110	0.0105	0.0131	0.0180	0.0127	0.0116	0.0098	1939	0.0098
1940	0.0103	0.0098	0.0083	0.0084	0.0127	0.0092	0.0093	0.0086	0.0078	0.0072	0.0061	0.0057	1940	0.0057
1941	0.0077	0.0089	0.0075	0.0069	0.0067	0.0055	0.0056	0.0055	0.0056	0.0051	0.0076	0.0082	1941	0.0082
1942	0.0083	0.0081	0.0077	0.0074	0.0071	0.0070	0.0071	0.0069	0.0076	0.0073	0.0070	0.0072	1942	0.0072
1943	0.0166	0.0166	0.0164	0.0162	0.0153	0.0149	0.0147	0.0149	0.0149	0.0147	0.0147	0.0145	1943	0.0145
1944	0.0150	0.0150	0.0148	0.0143	0.0146	0.0147	0.0142	0.0139	0.0139	0.0139	0.0140	0.0140	1944	0.0140
1945	0.0127	0.0118	0.0120	0.0118	0.0117	0.0114	0.0118	0.0115	0.0112	0.0109	0.0109	0.0103	1945	0.0103
1946	0.0099	0.0087	0.0101	0.0111	0.0112	0.0103	0.0110	0.0112	0.0120	0.0114	0.0121	0.0112	1946	0.0112
1947	0.0116	0.0117	0.0112	0.0120	0.0121	0.0122	0.0124	0.0117	0.0121	0.0136	0.0138	0.0134	1947	0.0134
1948	0.0160	0.0158	0.0157	0.0155	0.0142	0.0149	0.0154	0.0160	0.0161	0.0161	0.0158	0.0151	1948	0.0151
1949	0.0153	0.0153	0.0148	0.0147	0.0144	0.0129	0.0125	0.0117	0.0118	0.0120	0.0124	0.0123	1949	0.0123
1950	0.0131	0.0132	0.0137	0.0138	0.0134	0.0139	0.0134	0.0145	0.0154	0.0162	0.0159	0.0162	1950	0.0162
1951	0.0179	0.0180	0.0211	0.0202	0.0215	0.0208	0.0199	0.0194	0.0212	0.0212	0.0209	0.0217	1951	0.0217
1952	0.0212	0.0222	0.0209	0.0199	0.0198	0.0213	0.0228	0.0241	0.0242	0.0227	0.0235	0.0235	1952	0.0235
1953	0.0242	0.0245	0.0253	0.0277	0.0307	0.0279	0.0272	0.0279	0.0241	0.0237	0.0238	0.0218	1953	0.0218
1954	0.0187	0.0157	0.0153	0.0142	0.0173	0.0131	0.0138	0.0138	0.0152	0.0161	0.0168	0.0172	1954	0.0172
1955	0.0227	0.0240	0.0240	0.0242	0.0246	0.0257	0.0276	0.0280	0.0267	0.0257	0.0273	0.0280	1955	0.0280

*Compound annual return

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Intermediate-term Government Bonds: Yields

From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1956	0.0271	0.0275	0.0300	0.0305	0.0287	0.0292	0.0317	0.0346	0.0331	0.0342	0.0359	0.0363	1956	0.0363
1957	0.0326	0.0333	0.0334	0.0357	0.0366	0.0390	0.0399	0.0385	0.0390	0.0388	0.0320	0.0284	1957	0.0284
1958	0.0282	0.0259	0.0253	0.0246	0.0238	0.0250	0.0281	0.0365	0.0376	0.0382	0.0359	0.0381	1958	0.0381
1959	0.0395	0.0378	0.0393	0.0413	0.0420	0.0447	0.0448	0.0477	0.0482	0.0448	0.0482	0.0498	1959	0.0498
1960	0.0471	0.0464	0.0409	0.0431	0.0432	0.0390	0.0334	0.0343	0.0343	0.0346	0.0377	0.0331	1960	0.0331
1961	0.0363	0.0350	0.0348	0.0342	0.0355	0.0368	0.0373	0.0376	0.0365	0.0369	0.0381	0.0384	1961	0.0384
1962	0.0402	0.0377	0.0366	0.0367	0.0363	0.0375	0.0384	0.0365	0.0366	0.0362	0.0355	0.0350	1962	0.0350
1963	0.0368	0.0370	0.0370	0.0371	0.0374	0.0378	0.0385	0.0388	0.0392	0.0398	0.0396	0.0404	1963	0.0404
1964	0.0402	0.0407	0.0411	0.0411	0.0399	0.0399	0.0401	0.0402	0.0399	0.0399	0.0409	0.0403	1964	0.0403
1965	0.0413	0.0416	0.0414	0.0416	0.0415	0.0413	0.0416	0.0420	0.0429	0.0437	0.0444	0.0490	1965	0.0490
1966	0.0482	0.0507	0.0477	0.0489	0.0496	0.0510	0.0525	0.0565	0.0526	0.0519	0.0522	0.0479	1966	0.0479
1967	0.0459	0.0470	0.0437	0.0466	0.0465	0.0530	0.0508	0.0528	0.0537	0.0562	0.0566	0.0577	1967	0.0577
1968	0.0548	0.0549	0.0563	0.0577	0.0574	0.0547	0.0518	0.0523	0.0520	0.0528	0.0541	0.0596	1968	0.0596
1969	0.0637	0.0651	0.0640	0.0636	0.0666	0.0699	0.0693	0.0711	0.0799	0.0735	0.0761	0.0829	1969	0.0829
1970	0.0820	0.0730	0.0724	0.0790	0.0778	0.0780	0.0757	0.0743	0.0707	0.0697	0.0591	0.0590	1970	0.0590
1971	0.0570	0.0526	0.0493	0.0585	0.0593	0.0656	0.0663	0.0585	0.0591	0.0545	0.0543	0.0525	1971	0.0525
1972	0.0556	0.0563	0.0570	0.0577	0.0586	0.0587	0.0595	0.0604	0.0613	0.0623	0.0625	0.0585	1972	0.0585
1973	0.0641	0.0671	0.0673	0.0671	0.0671	0.0686	0.0776	0.0725	0.0674	0.0677	0.0674	0.0679	1973	0.0679
1974	0.0687	0.0691	0.0751	0.0801	0.0786	0.0822	0.0838	0.0857	0.0797	0.0787	0.0743	0.0712	1974	0.0712
1975	0.0730	0.0709	0.0737	0.0798	0.0749	0.0758	0.0782	0.0800	0.0815	0.0736	0.0754	0.0719	1975	0.0719
1976	0.0743	0.0736	0.0733	0.0719	0.0771	0.0747	0.0732	0.0697	0.0692	0.0667	0.0594	0.0600	1976	0.0600
1977	0.0673	0.0673	0.0673	0.0674	0.0674	0.0662	0.0675	0.0689	0.0700	0.0733	0.0727	0.0751	1977	0.0751
1978	0.0773	0.0784	0.0791	0.0800	0.0820	0.0843	0.0836	0.0833	0.0835	0.0887	0.0882	0.0883	1978	0.0883
1979	0.0895	0.0928	0.0918	0.0929	0.0899	0.0864	0.0887	0.0933	0.0951	0.1112	0.1033	0.1033	1979	0.1033
1980	0.1093	0.1294	0.1285	0.1009	0.0903	0.0944	0.0996	0.1133	0.1171	0.1244	0.1264	0.1245	1980	0.1245
1981	0.1275	0.1371	0.1328	0.1427	0.1385	0.1404	0.1533	0.1636	0.1625	0.1472	0.1311	0.1396	1981	0.1396
1982	0.1397	0.1385	0.1406	0.1355	0.1343	0.1417	0.1315	0.1209	0.1144	0.1018	0.1020	0.0990	1982	0.0990
1983	0.1057	0.1010	0.1048	0.0997	0.1059	0.1080	0.1168	0.1175	0.1108	0.1131	0.1127	0.1141	1983	0.1141
1984	0.1137	0.1181	0.1219	0.1251	0.1363	0.1365	0.1274	0.1276	0.1242	0.1154	0.1121	0.1104	1984	0.1104
1985	0.1081	0.1152	0.1131	0.1084	0.0974	0.0964	0.1002	0.0982	0.0973	0.0949	0.0911	0.0855	1985	0.0855

*Compound annual return

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Intermediate-term Government Bonds: Yields
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1986	0.0870	0.0815	0.0743	0.0737	0.0816	0.0756	0.0728	0.0668	0.0718	0.0687	0.0669	0.0685	1986	0.0685
1987	0.0685	0.0683	0.0708	0.0793	0.0821	0.0806	0.0818	0.0849	0.0912	0.0844	0.0840	0.0832	1987	0.0832
1988	0.0782	0.0768	0.0807	0.0836	0.0870	0.0839	0.0871	0.0895	0.0859	0.0837	0.0892	0.0917	1988	0.0917
1989	0.0896	0.0927	0.0934	0.0895	0.0860	0.0791	0.0745	0.0834	0.0833	0.0786	0.0779	0.0794	1989	0.0794
1990	0.0842	0.0855	0.0871	0.0907	0.0864	0.0843	0.0819	0.0859	0.0851	0.0826	0.0795	0.0770	1990	0.0770
1991	0.0772	0.0774	0.0783	0.0772	0.0773	0.0793	0.0778	0.0732	0.0693	0.0673	0.0653	0.0597	1991	0.0597
1992	0.0683	0.0690	0.0720	0.0711	0.0674	0.0647	0.0604	0.0581	0.0547	0.0601	0.0634	0.0611	1992	0.0611
1993	0.0588	0.0547	0.0549	0.0540	0.0551	0.0517	0.0526	0.0486	0.0483	0.0488	0.0519	0.0522	1993	0.0522
1994	0.0515	0.0575	0.0638	0.0670	0.0682	0.0699	0.0675	0.0683	0.0730	0.0749	0.0778	0.0780	1994	0.0780
1995	0.0754	0.0708	0.0707	0.0685	0.0606	0.0598	0.0616	0.0606	0.0601	0.0582	0.0553	0.0538	1995	0.0538
1996	0.0528	0.0573	0.0614	0.0640	0.0663	0.0645	0.0654	0.0670	0.0643	0.0607	0.0578	0.0616	1996	0.0616
1997	0.0629	0.0639	0.0677	0.0656	0.0650	0.0639	0.0589	0.0624	0.0601	0.0576	0.0587	0.0573	1997	0.0573
1998	0.0545	0.0562	0.0567	0.0564	0.0558	0.0551	0.0556	0.0503	0.0435	0.0434	0.0467	0.0468	1998	0.0468
1999	0.0467	0.0535	0.0526	0.0532	0.0576	0.0581	0.0594	0.0602	0.0590	0.0604	0.0619	0.0645	1999	0.0645
2000	0.0675	0.0669	0.0636	0.0657	0.0658	0.0626	0.0621	0.0601	0.0589	0.0582	0.0551	0.0507	2000	0.0507
2001	0.0499	0.0482	0.0471	0.0504	0.0515	0.0510	0.0464	0.0450	0.0399	0.0365	0.0413	0.0442	2001	0.0442
2002	0.0459	0.0442	0.0504	0.0461	0.0443	0.0412	0.0358	0.0325	0.0265	0.0276	0.0323	0.0261	2002	0.0261
2003	0.0310	0.0276	0.0283	0.0285	0.0228	0.0240	0.0322	0.0335	0.0267	0.0307	0.0317	0.0297	2003	0.0297
2004	0.0315	0.0293	0.0276	0.0360	0.0378	0.0374	0.0362	0.0322	0.0326	0.0316	0.0356	0.0347	2004	0.0347
2005	0.0375	0.0405	0.0421	0.0392	0.0377	0.0374	0.0415	0.0385	0.0422	0.0446	0.0440	0.0434	2005	0.0434
2006	0.0449	0.0460	0.0481	0.0491	0.0502	0.0506	0.0487	0.0465	0.0455	0.0452	0.0439	0.0465	2006	0.0465
2007	0.0479	0.0448	0.0451	0.0449	0.0483	0.0490	0.0457	0.0420	0.0413	0.0435	0.0333	0.0328	2007	0.0328
2008	0.0301	0.0256	0.0245	0.0316	0.0340	0.0330	0.0322	0.0303	0.0289	0.0260	0.0161	0.0126	2008	0.0126
2009	0.0180	0.0202	0.0168	0.0206	0.0238	0.0259	0.0251	0.0234	0.0222	0.0219	0.0180	0.0242	2009	0.0242
2010	0.0242	0.0231	0.0254	0.0238	0.0208	0.0184	0.0151	0.0124	0.0115	0.0102	0.0124	0.0170	2010	0.0170
2011	0.0215	0.0229	0.0234	0.0206	0.0172	0.0175	0.0134	0.0091	0.0090	0.0090	0.0084	0.0074	2011	0.0074
2012	0.0078	0.0093	0.0109	0.0084	0.0068	0.0074	0.0058	0.0056	0.0056	0.0062	0.0052	0.0061	2012	0.0061
2013	0.0094	0.0083	0.0079	0.0067	0.0104	0.0136	0.0133	0.0152	0.0127	0.0118	0.0117	0.0149	2013	0.0149
2014	0.0160	0.0158	0.0175	0.0168	0.0150	0.0157	0.0171	0.0157	0.0171	0.0153	0.0137	0.0155	2014	0.0155
2015	0.0125	0.0153	0.0140	0.0146	0.0147	0.0162	0.0154	0.0154	0.0134	0.0149	0.0162	0.0169	2015	0.0169
2016	0.0142	0.0133	0.0126	0.0130	0.0137	0.0102	0.0103	0.0120	0.0116	0.0130	0.0179	0.0185	2016	0.0185

*Compound annual return

Appendix A-14

U.S. Treasury Bills: Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1926	0.0034	0.0027	0.0030	0.0034	0.0001	0.0035	0.0022	0.0025	0.0023	0.0032	0.0031	0.0028	1926	0.0327
1927	0.0025	0.0026	0.0030	0.0025	0.0030	0.0026	0.0030	0.0028	0.0021	0.0025	0.0021	0.0022	1927	0.0312
1928	0.0025	0.0033	0.0029	0.0022	0.0032	0.0031	0.0032	0.0032	0.0027	0.0041	0.0038	0.0006	1928	0.0356
1929	0.0034	0.0036	0.0034	0.0036	0.0044	0.0052	0.0033	0.0040	0.0035	0.0046	0.0037	0.0037	1929	0.0475
1930	0.0014	0.0030	0.0035	0.0021	0.0026	0.0027	0.0020	0.0009	0.0022	0.0009	0.0013	0.0014	1930	0.0241
1931	0.0015	0.0004	0.0013	0.0008	0.0009	0.0008	0.0006	0.0003	0.0003	0.0010	0.0017	0.0012	1931	0.0107
1932	0.0023	0.0023	0.0016	0.0011	0.0006	0.0002	0.0003	0.0003	0.0003	0.0002	0.0002	0.0001	1932	0.0096
1933	0.0001	-0.0003	0.0004	0.0010	0.0004	0.0002	0.0002	0.0003	0.0002	0.0001	0.0002	0.0002	1933	0.0030
1934	0.0005	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	1934	0.0016
1935	0.0001	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0001	1935	0.0017
1936	0.0001	0.0001	0.0002	0.0002	0.0002	0.0003	0.0001	0.0002	0.0001	0.0002	0.0001	0.0000	1936	0.0018
1937	0.0001	0.0002	0.0001	0.0003	0.0006	0.0003	0.0003	0.0002	0.0004	0.0002	0.0002	0.0000	1937	0.0031
1938	0.0000	0.0000	-0.0001	0.0001	0.0000	0.0000	-0.0001	0.0000	0.0002	0.0001	-0.0006	0.0000	1938	-0.0002
1939	-0.0001	0.0001	-0.0001	0.0000	0.0001	0.0001	0.0000	-0.0001	0.0001	0.0000	0.0000	0.0000	1939	0.0002
1940	0.0000	0.0000	0.0000	0.0000	-0.0002	0.0000	0.0001	-0.0001	0.0000	0.0000	0.0000	0.0000	1940	0.0000
1941	-0.0001	-0.0001	0.0001	-0.0001	0.0000	0.0000	0.0003	0.0001	0.0001	0.0000	0.0000	0.0001	1941	0.0006
1942	0.0002	0.0001	0.0001	0.0001	0.0003	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	1942	0.0027
1943	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	1943	0.0035
1944	0.0003	0.0003	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0003	0.0003	0.0002	1944	0.0033
1945	0.0003	0.0002	0.0002	0.0003	0.0003	0.0002	0.0003	0.0003	0.0003	0.0003	0.0002	0.0003	1945	0.0033
1946	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	1946	0.0035
1947	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0006	0.0006	0.0006	0.0008	1947	0.0050
1948	0.0007	0.0007	0.0009	0.0008	0.0008	0.0009	0.0008	0.0009	0.0004	0.0004	0.0004	0.0004	1948	0.0081
1949	0.0010	0.0009	0.0010	0.0009	0.0010	0.0010	0.0009	0.0009	0.0009	0.0009	0.0008	0.0009	1949	0.0110
1950	0.0009	0.0009	0.0010	0.0009	0.0010	0.0010	0.0010	0.0010	0.0010	0.0012	0.0011	0.0011	1950	0.0120
1951	0.0013	0.0010	0.0011	0.0013	0.0012	0.0012	0.0013	0.0013	0.0012	0.0016	0.0011	0.0012	1951	0.0149
1952	0.0015	0.0012	0.0011	0.0012	0.0013	0.0015	0.0015	0.0015	0.0016	0.0014	0.0010	0.0016	1952	0.0166
1953	0.0016	0.0014	0.0018	0.0016	0.0017	0.0018	0.0015	0.0017	0.0016	0.0013	0.0008	0.0013	1953	0.0182
1954	0.0011	0.0007	0.0008	0.0009	0.0005	0.0006	0.0005	0.0005	0.0009	0.0007	0.0006	0.0008	1954	0.0086
1955	0.0008	0.0009	0.0010	0.0010	0.0014	0.0010	0.0010	0.0016	0.0016	0.0018	0.0017	0.0018	1955	0.0157

*Compound annual return

Appendix A-14

U.S. Treasury Bills: Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1956	0.0022	0.0019	0.0015	0.0019	0.0023	0.0020	0.0022	0.0017	0.0018	0.0025	0.0020	0.0024	1956	0.0246
1957	0.0027	0.0024	0.0023	0.0025	0.0026	0.0024	0.0030	0.0025	0.0026	0.0029	0.0028	0.0024	1957	0.0314
1958	0.0028	0.0012	0.0009	0.0008	0.0011	0.0003	0.0007	0.0004	0.0019	0.0018	0.0011	0.0022	1958	0.0154
1959	0.0021	0.0019	0.0022	0.0020	0.0022	0.0025	0.0025	0.0019	0.0031	0.0030	0.0026	0.0034	1959	0.0295
1960	0.0033	0.0029	0.0035	0.0019	0.0027	0.0024	0.0013	0.0017	0.0016	0.0022	0.0013	0.0016	1960	0.0266
1961	0.0019	0.0014	0.0020	0.0017	0.0018	0.0020	0.0018	0.0014	0.0017	0.0019	0.0015	0.0019	1961	0.0213
1962	0.0024	0.0020	0.0020	0.0022	0.0024	0.0020	0.0027	0.0023	0.0021	0.0026	0.0020	0.0023	1962	0.0273
1963	0.0025	0.0023	0.0023	0.0025	0.0024	0.0023	0.0027	0.0025	0.0027	0.0029	0.0027	0.0029	1963	0.0312
1964	0.0030	0.0026	0.0031	0.0029	0.0026	0.0030	0.0030	0.0028	0.0028	0.0029	0.0029	0.0031	1964	0.0354
1965	0.0028	0.0030	0.0036	0.0031	0.0031	0.0035	0.0031	0.0033	0.0031	0.0031	0.0035	0.0033	1965	0.0393
1966	0.0038	0.0035	0.0038	0.0034	0.0041	0.0038	0.0035	0.0041	0.0040	0.0045	0.0040	0.0040	1966	0.0476
1967	0.0043	0.0036	0.0039	0.0032	0.0033	0.0027	0.0032	0.0031	0.0032	0.0039	0.0036	0.0033	1967	0.0421
1968	0.0040	0.0039	0.0038	0.0043	0.0045	0.0043	0.0048	0.0042	0.0043	0.0044	0.0042	0.0043	1968	0.0521
1969	0.0053	0.0046	0.0046	0.0053	0.0048	0.0051	0.0053	0.0050	0.0062	0.0060	0.0052	0.0064	1969	0.0658
1970	0.0060	0.0062	0.0057	0.0050	0.0053	0.0058	0.0052	0.0053	0.0054	0.0046	0.0046	0.0042	1970	0.0652
1971	0.0038	0.0033	0.0030	0.0028	0.0029	0.0037	0.0040	0.0047	0.0037	0.0037	0.0037	0.0037	1971	0.0439
1972	0.0029	0.0025	0.0027	0.0029	0.0030	0.0029	0.0031	0.0029	0.0034	0.0040	0.0037	0.0037	1972	0.0384
1973	0.0044	0.0042	0.0046	0.0052	0.0051	0.0051	0.0064	0.0070	0.0068	0.0065	0.0056	0.0064	1973	0.0693
1974	0.0063	0.0058	0.0056	0.0075	0.0075	0.0060	0.0070	0.0060	0.0081	0.0051	0.0054	0.0070	1974	0.0800
1975	0.0058	0.0043	0.0041	0.0044	0.0044	0.0041	0.0048	0.0048	0.0053	0.0056	0.0041	0.0048	1975	0.0580
1976	0.0047	0.0034	0.0040	0.0042	0.0037	0.0043	0.0047	0.0042	0.0044	0.0041	0.0040	0.0040	1976	0.0508
1977	0.0036	0.0035	0.0038	0.0038	0.0037	0.0040	0.0042	0.0044	0.0043	0.0049	0.0050	0.0049	1977	0.0512
1978	0.0049	0.0046	0.0053	0.0054	0.0051	0.0054	0.0056	0.0056	0.0062	0.0068	0.0070	0.0078	1978	0.0718
1979	0.0077	0.0073	0.0081	0.0080	0.0082	0.0081	0.0077	0.0077	0.0083	0.0087	0.0099	0.0095	1979	0.1038
1980	0.0080	0.0089	0.0121	0.0126	0.0081	0.0061	0.0053	0.0064	0.0075	0.0095	0.0096	0.0131	1980	0.1124
1981	0.0104	0.0107	0.0121	0.0108	0.0115	0.0135	0.0124	0.0128	0.0124	0.0121	0.0107	0.0087	1981	0.1471
1982	0.0080	0.0092	0.0098	0.0113	0.0106	0.0096	0.0105	0.0076	0.0051	0.0059	0.0063	0.0067	1982	0.1054
1983	0.0069	0.0062	0.0063	0.0071	0.0069	0.0067	0.0074	0.0076	0.0076	0.0076	0.0070	0.0073	1983	0.0880
1984	0.0076	0.0071	0.0073	0.0081	0.0078	0.0075	0.0082	0.0083	0.0086	0.0100	0.0073	0.0064	1984	0.0985
1985	0.0065	0.0058	0.0062	0.0072	0.0066	0.0055	0.0062	0.0055	0.0060	0.0065	0.0061	0.0065	1985	0.0772

*Compound annual return

Appendix A-14

U.S. Treasury Bills: Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1986	0.0056	0.0053	0.0060	0.0052	0.0049	0.0052	0.0052	0.0046	0.0045	0.0046	0.0039	0.0049	1986	0.0616
1987	0.0042	0.0043	0.0047	0.0044	0.0038	0.0048	0.0046	0.0047	0.0045	0.0060	0.0035	0.0039	1987	0.0547
1988	0.0029	0.0046	0.0044	0.0046	0.0051	0.0049	0.0051	0.0059	0.0062	0.0061	0.0057	0.0063	1988	0.0635
1989	0.0055	0.0061	0.0067	0.0067	0.0079	0.0071	0.0070	0.0074	0.0065	0.0068	0.0069	0.0061	1989	0.0837
1990	0.0057	0.0057	0.0064	0.0069	0.0068	0.0063	0.0068	0.0066	0.0060	0.0068	0.0057	0.0060	1990	0.0781
1991	0.0052	0.0048	0.0044	0.0053	0.0047	0.0042	0.0049	0.0046	0.0046	0.0042	0.0039	0.0038	1991	0.0560
1992	0.0034	0.0028	0.0034	0.0032	0.0028	0.0032	0.0031	0.0026	0.0026	0.0023	0.0023	0.0028	1992	0.0351
1993	0.0023	0.0022	0.0025	0.0024	0.0022	0.0025	0.0024	0.0025	0.0026	0.0022	0.0025	0.0023	1993	0.0290
1994	0.0025	0.0021	0.0027	0.0027	0.0032	0.0031	0.0028	0.0037	0.0037	0.0038	0.0037	0.0044	1994	0.0390
1995	0.0042	0.0040	0.0046	0.0045	0.0054	0.0047	0.0045	0.0047	0.0043	0.0047	0.0042	0.0049	1995	0.0560
1996	0.0043	0.0039	0.0039	0.0046	0.0042	0.0040	0.0045	0.0041	0.0044	0.0042	0.0041	0.0046	1996	0.0521
1997	0.0045	0.0039	0.0043	0.0043	0.0049	0.0037	0.0043	0.0041	0.0044	0.0042	0.0039	0.0048	1997	0.0526
1998	0.0043	0.0039	0.0039	0.0043	0.0040	0.0041	0.0040	0.0043	0.0046	0.0032	0.0031	0.0038	1998	0.0486
1999	0.0035	0.0035	0.0043	0.0037	0.0034	0.0040	0.0038	0.0039	0.0039	0.0039	0.0036	0.0044	1999	0.0468
2000	0.0041	0.0043	0.0047	0.0046	0.0050	0.0040	0.0048	0.0050	0.0051	0.0056	0.0051	0.0050	2000	0.0589
2001	0.0054	0.0038	0.0042	0.0039	0.0032	0.0028	0.0030	0.0031	0.0028	0.0022	0.0017	0.0015	2001	0.0383
2002	0.0014	0.0013	0.0013	0.0015	0.0014	0.0013	0.0015	0.0014	0.0014	0.0014	0.0012	0.0011	2002	0.0165
2003	0.0010	0.0009	0.0010	0.0010	0.0009	0.0010	0.0007	0.0007	0.0008	0.0007	0.0007	0.0008	2003	0.0102
2004	0.0007	0.0006	0.0009	0.0008	0.0006	0.0008	0.0010	0.0011	0.0011	0.0011	0.0015	0.0016	2004	0.0120
2005	0.0016	0.0016	0.0021	0.0021	0.0024	0.0023	0.0024	0.0030	0.0029	0.0027	0.0031	0.0032	2005	0.0298
2006	0.0035	0.0034	0.0037	0.0036	0.0043	0.0040	0.0040	0.0042	0.0041	0.0041	0.0042	0.0040	2006	0.0480
2007	0.0044	0.0038	0.0043	0.0044	0.0041	0.0040	0.0040	0.0042	0.0032	0.0032	0.0034	0.0027	2007	0.0466
2008	0.0021	0.0013	0.0017	0.0018	0.0018	0.0017	0.0015	0.0013	0.0015	0.0008	0.0003	0.0000	2008	0.0160
2009	0.0000	0.0001	0.0002	0.0001	0.0000	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0001	2009	0.0010
2010	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	2010	0.0012
2011	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	2011	0.0004
2012	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	2012	0.0006
2013	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	2013	0.0002
2014	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	2014	0.0002
2015	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	2015	0.0002
2016	0.0001	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0003	2016	0.0020

*Compound annual return

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Inflation

From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1926	0.0000	-0.0037	-0.0056	0.0094	-0.0056	-0.0075	-0.0094	-0.0057	0.0057	0.0038	0.0038	0.0000	1926	-0.0149
1927	-0.0076	-0.0076	-0.0058	0.0000	0.0077	0.0096	-0.0190	-0.0058	0.0058	0.0058	-0.0019	-0.0019	1927	-0.0208
1928	-0.0019	-0.0097	0.0000	0.0020	0.0058	-0.0078	0.0000	0.0020	0.0078	-0.0019	-0.0019	-0.0039	1928	-0.0097
1929	-0.0019	-0.0020	-0.0039	-0.0039	0.0059	0.0039	0.0098	0.0039	-0.0019	0.0000	-0.0019	-0.0058	1929	0.0020
1930	-0.0039	-0.0039	-0.0059	0.0059	-0.0059	-0.0059	-0.0139	-0.0060	0.0061	-0.0060	-0.0081	-0.0143	1930	-0.0603
1931	-0.0145	-0.0147	-0.0064	-0.0064	-0.0108	-0.0109	-0.0022	-0.0022	-0.0044	-0.0067	-0.0112	-0.0091	1931	-0.0952
1932	-0.0206	-0.0140	-0.0047	-0.0071	-0.0144	-0.0073	0.0000	-0.0123	-0.0050	-0.0075	-0.0050	-0.0101	1932	-0.1030
1933	-0.0153	-0.0155	-0.0079	-0.0027	0.0027	0.0106	0.0289	0.0102	0.0000	0.0000	0.0000	-0.0051	1933	0.0051
1934	0.0051	0.0076	0.0000	-0.0025	0.0025	0.0025	0.0000	0.0025	0.0150	-0.0074	-0.0025	-0.0025	1934	0.0203
1935	0.0149	0.0074	-0.0024	0.0098	-0.0048	-0.0024	-0.0049	0.0000	0.0049	0.0000	0.0049	0.0024	1935	0.0299
1936	0.0000	-0.0048	-0.0049	0.0000	0.0000	0.0098	0.0048	0.0072	0.0024	-0.0024	0.0000	0.0000	1936	0.0121
1937	0.0072	0.0024	0.0071	0.0047	0.0047	0.0023	0.0046	0.0023	0.0092	-0.0046	-0.0069	-0.0023	1937	0.0310
1938	-0.0139	-0.0094	0.0000	0.0047	-0.0047	0.0000	0.0024	-0.0024	0.0000	-0.0047	-0.0024	0.0024	1938	-0.0278
1939	-0.0048	-0.0048	-0.0024	-0.0024	0.0000	0.0000	0.0000	0.0000	0.0193	-0.0047	0.0000	-0.0048	1939	-0.0048
1940	-0.0024	0.0072	-0.0024	0.0000	0.0024	0.0024	-0.0024	-0.0024	0.0024	0.0000	0.0000	0.0048	1940	0.0096
1941	0.0000	0.0000	0.0047	0.0094	0.0070	0.0186	0.0046	0.0091	0.0180	0.0110	0.0087	0.0022	1941	0.0972
1942	0.0130	0.0085	0.0127	0.0063	0.0104	0.0021	0.0041	0.0061	0.0020	0.0101	0.0060	0.0080	1942	0.0929
1943	0.0000	0.0020	0.0158	0.0117	0.0077	-0.0019	-0.0076	-0.0038	0.0039	0.0038	-0.0019	0.0019	1943	0.0316
1944	-0.0019	-0.0019	0.0000	0.0058	0.0038	0.0019	0.0057	0.0038	0.0000	0.0000	0.0000	0.0038	1944	0.0211
1945	0.0000	-0.0019	0.0000	0.0019	0.0075	0.0093	0.0018	0.0000	-0.0037	0.0000	0.0037	0.0037	1945	0.0225
1946	0.0000	-0.0037	0.0074	0.0055	0.0055	0.0109	0.0590	0.0220	0.0116	0.0196	0.0240	0.0078	1946	0.1816
1947	0.0000	-0.0016	0.0218	0.0000	-0.0030	0.0076	0.0091	0.0105	0.0238	0.0000	0.0058	0.0130	1947	0.0901
1948	0.0114	-0.0085	-0.0028	0.0142	0.0070	0.0070	0.0125	0.0041	0.0000	-0.0041	-0.0068	-0.0069	1948	0.0271
1949	-0.0014	-0.0111	0.0028	0.0014	-0.0014	0.0014	-0.0070	0.0028	0.0042	-0.0056	0.0014	-0.0056	1949	-0.0180
1950	-0.0042	-0.0028	0.0043	0.0014	0.0042	0.0056	0.0098	0.0083	0.0069	0.0055	0.0041	0.0135	1950	0.0579
1951	0.0160	0.0118	0.0039	0.0013	0.0039	-0.0013	0.0013	0.0000	0.0064	0.0051	0.0051	0.0038	1951	0.0587
1952	0.0000	-0.0063	0.0000	0.0038	0.0013	0.0025	0.0076	0.0013	-0.0012	0.0013	0.0000	-0.0012	1952	0.0088
1953	-0.0025	-0.0050	0.0025	0.0013	0.0025	0.0038	0.0025	0.0025	0.0012	0.0025	-0.0037	-0.0012	1953	0.0062
1954	0.0025	-0.0012	-0.0012	-0.0025	0.0037	0.0012	0.0000	-0.0012	-0.0025	-0.0025	0.0012	-0.0025	1954	-0.0050
1955	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0037	-0.0025	0.0037	0.0000	0.0012	-0.0025	1955	0.0037

*Compound annual return

Appendix A-15

Inflation

From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1956	-0.0012	0.0000	0.0012	0.0012	0.0050	0.0062	0.0074	-0.0012	0.0012	0.0061	0.0000	0.0024	1956	0.0286
1957	0.0012	0.0036	0.0024	0.0036	0.0024	0.0060	0.0047	0.0012	0.0012	0.0000	0.0035	0.0000	1957	0.0302
1958	0.0059	0.0012	0.0070	0.0023	0.0000	0.0012	0.0012	-0.0012	0.0000	0.0000	0.0012	-0.0012	1958	0.0176
1959	0.0012	-0.0012	0.0000	0.0012	0.0012	0.0046	0.0023	-0.0011	0.0034	0.0034	0.0000	0.0000	1959	0.0150
1960	-0.0011	0.0011	0.0000	0.0057	0.0000	0.0023	0.0000	0.0000	0.0011	0.0045	0.0011	0.0000	1960	0.0148
1961	0.0000	0.0000	0.0000	0.0000	0.0000	0.0011	0.0045	-0.0011	0.0022	0.0000	0.0000	0.0000	1961	0.0067
1962	0.0000	0.0022	0.0022	0.0022	0.0000	0.0000	0.0022	0.0000	0.0055	-0.0011	0.0000	-0.0011	1962	0.0122
1963	0.0011	0.0011	0.0011	0.0000	0.0000	0.0044	0.0044	0.0000	0.0000	0.0011	0.0011	0.0022	1963	0.0165
1964	0.0011	-0.0011	0.0011	0.0011	0.0000	0.0022	0.0022	-0.0011	0.0022	0.0011	0.0021	0.0011	1964	0.0119
1965	0.0000	0.0000	0.0011	0.0032	0.0021	0.0053	0.0011	-0.0021	0.0021	0.0011	0.0021	0.0032	1965	0.0192
1966	0.0000	0.0063	0.0031	0.0042	0.0010	0.0031	0.0031	0.0051	0.0020	0.0041	0.0000	0.0010	1966	0.0335
1967	0.0000	0.0010	0.0020	0.0020	0.0030	0.0030	0.0050	0.0030	0.0020	0.0030	0.0030	0.0030	1967	0.0304
1968	0.0039	0.0029	0.0049	0.0029	0.0029	0.0058	0.0048	0.0029	0.0029	0.0057	0.0038	0.0028	1968	0.0472
1969	0.0028	0.0037	0.0084	0.0065	0.0028	0.0064	0.0046	0.0045	0.0045	0.0036	0.0054	0.0062	1969	0.0611
1970	0.0035	0.0053	0.0053	0.0061	0.0043	0.0052	0.0034	0.0017	0.0051	0.0051	0.0034	0.0051	1970	0.0549
1971	0.0008	0.0017	0.0034	0.0033	0.0050	0.0058	0.0025	0.0025	0.0008	0.0016	0.0016	0.0041	1971	0.0336
1972	0.0008	0.0049	0.0016	0.0024	0.0032	0.0024	0.0040	0.0016	0.0040	0.0032	0.0024	0.0032	1972	0.0341
1973	0.0031	0.0070	0.0093	0.0069	0.0061	0.0068	0.0023	0.0181	0.0030	0.0081	0.0073	0.0065	1973	0.0880
1974	0.0087	0.0129	0.0113	0.0056	0.0111	0.0096	0.0075	0.0128	0.0120	0.0086	0.0085	0.0071	1974	0.1220
1975	0.0045	0.0070	0.0038	0.0051	0.0044	0.0082	0.0106	0.0031	0.0049	0.0061	0.0061	0.0042	1975	0.0701
1976	0.0024	0.0024	0.0024	0.0042	0.0059	0.0053	0.0059	0.0047	0.0041	0.0041	0.0029	0.0029	1976	0.0481
1977	0.0057	0.0103	0.0062	0.0079	0.0056	0.0066	0.0044	0.0038	0.0038	0.0027	0.0049	0.0038	1977	0.0677
1978	0.0054	0.0069	0.0069	0.0090	0.0099	0.0103	0.0072	0.0051	0.0071	0.0080	0.0055	0.0055	1978	0.0903
1979	0.0089	0.0117	0.0097	0.0115	0.0123	0.0093	0.0130	0.0101	0.0104	0.0090	0.0093	0.0105	1979	0.1331
1980	0.0144	0.0137	0.0144	0.0113	0.0099	0.0110	0.0008	0.0065	0.0092	0.0087	0.0091	0.0086	1980	0.1240
1981	0.0081	0.0104	0.0072	0.0064	0.0082	0.0086	0.0114	0.0077	0.0101	0.0021	0.0029	0.0029	1981	0.0894
1982	0.0036	0.0032	-0.0011	0.0042	0.0098	0.0122	0.0055	0.0021	0.0017	0.0027	-0.0017	-0.0041	1982	0.0387
1983	0.0024	0.0003	0.0007	0.0072	0.0054	0.0034	0.0040	0.0033	0.0050	0.0027	0.0017	0.0013	1983	0.0380
1984	0.0056	0.0046	0.0023	0.0049	0.0029	0.0032	0.0032	0.0042	0.0048	0.0025	0.0000	0.0006	1984	0.0395
1985	0.0019	0.0041	0.0044	0.0041	0.0037	0.0031	0.0016	0.0022	0.0031	0.0031	0.0034	0.0025	1985	0.0377

*Compound annual return

Appendix A-15

Inflation

From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1986	0.0031	-0.0027	-0.0046	-0.0021	0.0031	0.0049	0.0003	0.0018	0.0049	0.0009	0.0009	0.0009	1986	0.0113
1987	0.0060	0.0039	0.0045	0.0054	0.0030	0.0041	0.0021	0.0056	0.0050	0.0026	0.0014	-0.0003	1987	0.0441
1988	0.0026	0.0026	0.0043	0.0052	0.0034	0.0043	0.0042	0.0042	0.0067	0.0033	0.0008	0.0017	1988	0.0442
1989	0.0050	0.0041	0.0058	0.0065	0.0057	0.0024	0.0024	0.0016	0.0032	0.0048	0.0024	0.0016	1989	0.0465
1990	0.0103	0.0047	0.0055	0.0016	0.0023	0.0054	0.0038	0.0092	0.0084	0.0060	0.0022	0.0000	1990	0.0611
1991	0.0060	0.0015	0.0015	0.0015	0.0030	0.0029	0.0015	0.0029	0.0044	0.0015	0.0029	0.0007	1991	0.0306
1992	0.0015	0.0036	0.0051	0.0014	0.0014	0.0036	0.0021	0.0028	0.0028	0.0035	0.0014	-0.0007	1992	0.0290
1993	0.0049	0.0035	0.0035	0.0028	0.0014	0.0014	0.0000	0.0028	0.0021	0.0041	0.0007	0.0000	1993	0.0275
1994	0.0027	0.0034	0.0034	0.0014	0.0007	0.0034	0.0027	0.0040	0.0027	0.0007	0.0013	0.0000	1994	0.0267
1995	0.0040	0.0040	0.0033	0.0033	0.0020	0.0020	0.0000	0.0026	0.0020	0.0033	-0.0007	-0.0007	1995	0.0254
1996	0.0059	0.0032	0.0052	0.0039	0.0019	0.0006	0.0019	0.0019	0.0032	0.0032	0.0019	0.0000	1996	0.0332
1997	0.0032	0.0031	0.0025	0.0013	-0.0006	0.0012	0.0012	0.0019	0.0025	0.0025	-0.0006	-0.0012	1997	0.0170
1998	0.0019	0.0019	0.0019	0.0018	0.0018	0.0012	0.0012	0.0012	0.0012	0.0024	0.0000	-0.0006	1998	0.0161
1999	0.0024	0.0012	0.0030	0.0073	0.0000	0.0000	0.0030	0.0024	0.0048	0.0018	0.0006	0.0000	1999	0.0268
2000	0.0030	0.0059	0.0082	0.0006	0.0012	0.0052	0.0023	0.0000	0.0052	0.0017	0.0006	-0.0006	2000	0.0339
2001	0.0063	0.0040	0.0023	0.0040	0.0045	0.0017	-0.0028	0.0000	0.0045	-0.0034	-0.0017	-0.0039	2001	0.0155
2002	0.0023	0.0040	0.0056	0.0056	0.0000	0.0006	0.0011	0.0033	0.0017	0.0017	0.0000	-0.0022	2002	0.0238
2003	0.0044	0.0077	0.0060	-0.0022	-0.0016	0.0011	0.0011	0.0038	0.0033	-0.0011	-0.0027	-0.0011	2003	0.0188
2004	0.0049	0.0054	0.0064	0.0032	0.0059	0.0032	-0.0016	0.0005	0.0021	0.0053	0.0005	-0.0037	2004	0.0326
2005	0.0021	0.0058	0.0078	0.0067	-0.0010	0.0005	0.0046	0.0051	0.0122	0.0020	-0.0080	-0.0040	2005	0.0342
2006	0.0076	0.0020	0.0055	0.0085	0.0050	0.0020	0.0030	0.0020	-0.0049	-0.0054	-0.0015	0.0015	2006	0.0254
2007	0.0031	0.0054	0.0091	0.0065	0.0061	0.0019	-0.0003	-0.0018	0.0028	0.0021	0.0059	-0.0007	2007	0.0408
2008	0.0050	0.0029	0.0087	0.0061	0.0084	0.0101	0.0053	-0.0040	-0.0014	-0.0101	-0.0192	-0.0103	2008	0.0009
2009	0.0044	0.0050	0.0024	0.0025	0.0029	0.0086	-0.0016	0.0022	0.0006	0.0010	0.0007	-0.0018	2009	0.0272
2010	0.0034	0.0002	0.0041	0.0017	0.0008	-0.0010	0.0002	0.0014	0.0006	0.0012	0.0004	0.0017	2010	0.0150
2011	0.0048	0.0049	0.0098	0.0064	0.0047	-0.0011	0.0009	0.0028	0.0015	-0.0021	-0.0008	-0.0025	2011	0.0296
2012	0.0044	0.0044	0.0076	0.0030	-0.0012	-0.0015	-0.0016	0.0056	0.0045	-0.0004	-0.0047	-0.0027	2012	0.0174
2013	0.0030	0.0082	0.0026	-0.0010	0.0018	0.0024	0.0004	0.0012	0.0012	-0.0026	-0.0020	-0.0001	2013	0.0151
2014	0.0037	0.0037	0.0064	0.0033	0.0035	0.0019	-0.0004	-0.0017	0.0008	-0.0025	-0.0054	-0.0057	2014	0.0076
2015	-0.0047	0.0043	0.0060	0.0020	0.0051	0.0035	0.0001	-0.0014	-0.0016	-0.0004	-0.0021	-0.0034	2015	0.0073
2016	0.0017	0.0008	0.0043	0.0047	0.0040	0.0033	-0.0016	0.0009	0.0024	0.0012	-0.0016	0.0003	2016	0.0207

*Compound annual return

Appendix A-16

U.S. Treasury Bills: Inflation-Adjusted Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1926	0.0034	0.0064	0.0086	-0.0059	0.0057	0.0110	0.0118	0.0083	-0.0035	-0.0006	-0.0007	0.0028	1926	0.0483
1927	0.0101	0.0103	0.0088	0.0025	-0.0047	-0.0069	0.0224	0.0086	-0.0037	-0.0033	0.0040	0.0042	1927	0.0531
1928	0.0045	0.0131	0.0029	0.0003	-0.0026	0.0110	0.0032	0.0013	-0.0051	0.0060	0.0058	0.0045	1928	0.0457
1929	0.0054	0.0055	0.0074	0.0075	-0.0015	0.0013	-0.0064	0.0002	0.0055	0.0046	0.0057	0.0095	1929	0.0454
1930	0.0053	0.0069	0.0094	-0.0038	0.0085	0.0087	0.0161	0.0070	-0.0039	0.0069	0.0095	0.0159	1930	0.0898
1931	0.0162	0.0153	0.0077	0.0072	0.0118	0.0118	0.0028	0.0026	0.0047	0.0078	0.0130	0.0104	1931	0.1171
1932	0.0234	0.0166	0.0064	0.0083	0.0152	0.0076	0.0003	0.0127	0.0053	0.0077	0.0052	0.0103	1932	0.1255
1933	0.0157	0.0155	0.0084	0.0036	-0.0022	-0.0103	-0.0279	-0.0098	0.0002	0.0001	0.0002	0.0053	1933	-0.0021
1934	-0.0046	-0.0073	0.0002	0.0026	-0.0024	-0.0024	0.0001	-0.0024	-0.0147	0.0075	0.0026	0.0026	1934	-0.0183
1935	-0.0146	-0.0071	0.0026	-0.0095	0.0050	0.0026	0.0050	0.0001	-0.0047	0.0001	-0.0046	-0.0023	1935	-0.0273
1936	0.0001	0.0050	0.0051	0.0002	0.0002	-0.0094	-0.0047	-0.0070	-0.0023	0.0026	0.0001	0.0000	1936	-0.0102
1937	-0.0070	-0.0022	-0.0069	-0.0043	-0.0040	-0.0020	-0.0043	-0.0021	-0.0088	0.0048	0.0071	0.0024	1937	-0.0271
1938	0.0141	0.0095	-0.0001	-0.0046	0.0048	0.0000	-0.0024	0.0024	0.0002	0.0049	0.0018	-0.0024	1938	0.0284
1939	0.0047	0.0049	0.0023	0.0024	0.0001	0.0001	0.0000	-0.0001	-0.0189	0.0048	0.0000	0.0048	1939	0.0050
1940	0.0024	-0.0071	0.0024	0.0000	-0.0025	-0.0023	0.0025	0.0023	-0.0024	0.0000	0.0000	-0.0047	1940	-0.0094
1941	-0.0001	-0.0001	-0.0046	-0.0094	-0.0069	-0.0182	-0.0042	-0.0089	-0.0176	-0.0109	-0.0086	-0.0021	1941	-0.0880
1942	-0.0126	-0.0083	-0.0124	-0.0062	-0.0100	-0.0018	-0.0038	-0.0058	-0.0017	-0.0097	-0.0057	-0.0076	1942	-0.0825
1943	0.0003	-0.0017	-0.0152	-0.0112	-0.0074	0.0022	0.0080	0.0042	-0.0036	-0.0035	0.0022	-0.0016	1943	-0.0273
1944	0.0022	0.0022	0.0002	-0.0055	-0.0036	-0.0016	-0.0054	-0.0035	0.0002	0.0003	0.0003	-0.0035	1944	-0.0174
1945	0.0003	0.0021	0.0002	-0.0016	-0.0072	-0.0090	-0.0015	0.0003	0.0040	0.0003	-0.0034	-0.0034	1945	-0.0188
1946	0.0003	0.0040	-0.0070	-0.0052	-0.0051	-0.0105	-0.0554	-0.0212	-0.0111	-0.0189	-0.0232	-0.0075	1946	-0.1507
1947	0.0003	0.0018	-0.0210	0.0003	0.0033	-0.0073	-0.0087	-0.0101	-0.0226	0.0006	-0.0052	-0.0120	1947	-0.0780
1948	-0.0105	0.0093	0.0037	-0.0132	-0.0062	-0.0060	-0.0115	-0.0032	0.0004	0.0045	0.0073	0.0074	1948	-0.0185
1949	0.0023	0.0121	-0.0018	-0.0005	0.0024	-0.0004	0.0079	-0.0019	-0.0033	0.0065	-0.0006	0.0065	1949	0.0296
1950	0.0052	0.0037	-0.0033	-0.0006	-0.0032	-0.0046	-0.0087	-0.0073	-0.0058	-0.0043	-0.0030	-0.0123	1950	-0.0434
1951	-0.0145	-0.0107	-0.0028	0.0000	-0.0026	0.0025	0.0001	0.0013	-0.0052	-0.0035	-0.0040	-0.0026	1951	-0.0414
1952	0.0015	0.0075	0.0011	-0.0026	0.0000	-0.0010	-0.0060	0.0002	0.0029	0.0001	0.0010	0.0029	1952	0.0077
1953	0.0041	0.0064	-0.0007	0.0004	-0.0008	-0.0019	-0.0010	-0.0008	0.0004	-0.0012	0.0045	0.0025	1953	0.0119
1954	-0.0014	0.0019	0.0020	0.0034	-0.0032	-0.0007	0.0005	0.0017	0.0034	0.0032	-0.0006	0.0033	1954	0.0137
1955	0.0008	0.0009	0.0010	0.0010	0.0014	0.0010	-0.0027	0.0041	-0.0021	0.0018	0.0005	0.0043	1955	0.0119

*Compound annual return

Appendix A-16

U.S. Treasury Bills: Inflation-Adjusted Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1956	0.0035	0.0019	0.0003	0.0006	-0.0027	-0.0042	-0.0052	0.0029	0.0006	-0.0036	0.0020	0.0000	1956	-0.0039
1957	0.0015	-0.0012	-0.0001	-0.0011	0.0002	-0.0035	-0.0018	0.0013	0.0014	0.0029	-0.0008	0.0024	1957	0.0011
1958	-0.0031	0.0000	-0.0060	-0.0015	0.0011	-0.0009	-0.0005	0.0016	0.0019	0.0018	-0.0001	0.0034	1958	-0.0022
1959	0.0009	0.0030	0.0022	0.0008	0.0010	-0.0021	0.0002	0.0030	-0.0003	-0.0004	0.0026	0.0034	1959	0.0143
1960	0.0045	0.0017	0.0035	-0.0037	0.0027	0.0001	0.0013	0.0017	0.0005	-0.0023	0.0002	0.0016	1960	0.0117
1961	0.0019	0.0014	0.0020	0.0017	0.0018	0.0009	-0.0026	0.0025	-0.0006	0.0019	0.0015	0.0019	1961	0.0144
1962	0.0024	-0.0002	-0.0002	0.0000	0.0024	0.0020	0.0005	0.0023	-0.0034	0.0037	0.0020	0.0034	1962	0.0149
1963	0.0014	0.0012	0.0012	0.0025	0.0024	-0.0021	-0.0017	0.0025	0.0027	0.0018	0.0016	0.0008	1963	0.0144
1964	0.0019	0.0037	0.0020	0.0018	0.0026	0.0009	0.0008	0.0039	0.0006	0.0019	0.0008	0.0020	1964	0.0232
1965	0.0028	0.0030	0.0025	-0.0001	0.0010	-0.0018	0.0020	0.0054	0.0010	0.0021	0.0014	0.0002	1965	0.0197
1966	0.0038	-0.0028	0.0007	-0.0007	0.0031	0.0007	0.0005	-0.0010	0.0020	0.0005	0.0040	0.0030	1966	0.0136
1967	0.0043	0.0026	0.0019	0.0012	0.0003	-0.0004	-0.0019	0.0001	0.0012	0.0010	0.0006	0.0004	1967	0.0113
1968	0.0001	0.0009	-0.0011	0.0014	0.0015	-0.0015	0.0000	0.0013	0.0014	-0.0013	0.0005	0.0014	1968	0.0046
1969	0.0024	0.0009	-0.0037	-0.0011	0.0021	-0.0013	0.0008	0.0005	0.0017	0.0024	-0.0002	0.0002	1969	0.0045
1970	0.0025	0.0009	0.0004	-0.0011	0.0009	0.0006	0.0018	0.0036	0.0002	-0.0005	0.0012	-0.0008	1970	0.0098
1971	0.0030	0.0016	-0.0004	-0.0006	-0.0020	-0.0020	0.0015	0.0022	0.0029	0.0020	0.0021	-0.0004	1971	0.0099
1972	0.0021	-0.0024	0.0011	0.0005	-0.0002	0.0005	-0.0009	0.0013	-0.0006	0.0008	0.0013	0.0006	1972	0.0041
1973	0.0012	-0.0029	-0.0047	-0.0017	-0.0010	-0.0017	0.0041	-0.0109	0.0038	-0.0016	-0.0017	-0.0002	1973	-0.0172
1974	-0.0024	-0.0070	-0.0057	0.0019	-0.0035	-0.0036	-0.0004	-0.0068	-0.0039	-0.0035	-0.0031	-0.0002	1974	-0.0374
1975	0.0013	-0.0027	0.0003	-0.0007	-0.0001	-0.0040	-0.0057	0.0017	0.0004	-0.0006	-0.0020	0.0006	1975	-0.0113
1976	0.0023	0.0010	0.0016	0.0000	-0.0022	-0.0010	-0.0012	-0.0005	0.0003	0.0000	0.0011	0.0012	1976	0.0026
1977	-0.0021	-0.0067	-0.0024	-0.0041	-0.0018	-0.0026	-0.0002	0.0006	0.0005	0.0022	0.0001	0.0011	1977	-0.0155
1978	-0.0005	-0.0023	-0.0016	-0.0036	-0.0048	-0.0049	-0.0016	0.0005	-0.0009	-0.0012	0.0015	0.0024	1978	-0.0169
1979	-0.0011	-0.0043	-0.0015	-0.0035	-0.0041	-0.0012	-0.0052	-0.0024	-0.0021	-0.0002	0.0005	-0.0010	1979	-0.0259
1980	-0.0063	-0.0048	-0.0023	0.0013	-0.0018	-0.0049	0.0045	-0.0001	-0.0017	0.0008	0.0005	0.0044	1980	-0.0103
1981	0.0022	0.0003	0.0048	0.0043	0.0033	0.0049	0.0010	0.0051	0.0023	0.0099	0.0078	0.0059	1981	0.0530
1982	0.0044	0.0060	0.0109	0.0070	0.0007	-0.0026	0.0050	0.0056	0.0034	0.0032	0.0081	0.0109	1982	0.0642
1983	0.0045	0.0058	0.0056	0.0000	0.0015	0.0033	0.0034	0.0043	0.0026	0.0049	0.0054	0.0059	1983	0.0482
1984	0.0020	0.0025	0.0050	0.0032	0.0049	0.0043	0.0050	0.0041	0.0038	0.0074	0.0073	0.0058	1984	0.0567
1985	0.0046	0.0017	0.0017	0.0031	0.0029	0.0024	0.0047	0.0033	0.0029	0.0034	0.0027	0.0040	1985	0.0381

*Compound annual return

Appendix A-16

U.S. Treasury Bills: Inflation-Adjusted Total Returns
From 1926 to 2016

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year	Jan–Dec*
1986	0.0025	0.0081	0.0106	0.0074	0.0019	0.0003	0.0049	0.0028	-0.0004	0.0037	0.0030	0.0040	1986	0.0498
1987	-0.0019	0.0004	0.0002	-0.0009	0.0008	0.0007	0.0025	-0.0009	-0.0004	0.0034	0.0020	0.0042	1987	0.0101
1988	0.0003	0.0020	0.0001	-0.0005	0.0016	0.0006	0.0008	0.0017	-0.0006	0.0028	0.0048	0.0047	1988	0.0185
1989	0.0005	0.0020	0.0009	0.0002	0.0022	0.0047	0.0045	0.0058	0.0033	0.0020	0.0045	0.0045	1989	0.0356
1990	-0.0046	0.0010	0.0010	0.0053	0.0044	0.0008	0.0029	-0.0026	-0.0024	0.0008	0.0034	0.0060	1990	0.0161
1991	-0.0008	0.0033	0.0029	0.0038	0.0018	0.0012	0.0034	0.0017	0.0002	0.0028	0.0010	0.0031	1991	0.0246
1992	0.0019	-0.0008	-0.0017	0.0018	0.0013	-0.0004	0.0009	-0.0002	-0.0003	-0.0012	0.0009	0.0035	1992	0.0059
1993	-0.0026	-0.0013	-0.0010	-0.0004	0.0008	0.0011	0.0024	-0.0003	0.0005	-0.0019	0.0018	0.0023	1993	0.0014
1994	-0.0002	-0.0013	-0.0007	0.0014	0.0025	-0.0003	0.0000	-0.0004	0.0010	0.0032	0.0023	0.0044	1994	0.0120
1995	0.0001	0.0000	0.0013	0.0011	0.0034	0.0027	0.0045	0.0020	0.0023	0.0014	0.0049	0.0055	1995	0.0298
1996	-0.0016	0.0007	-0.0012	0.0007	0.0023	0.0034	0.0026	0.0022	0.0012	0.0011	0.0022	0.0046	1996	0.0182
1997	0.0013	0.0007	0.0018	0.0031	0.0056	0.0024	0.0030	0.0022	0.0019	0.0017	0.0045	0.0060	1997	0.0349
1998	0.0024	0.0020	0.0021	0.0024	0.0022	0.0029	0.0028	0.0031	0.0033	0.0008	0.0031	0.0044	1998	0.0319
1999	0.0011	0.0023	0.0012	-0.0035	0.0034	0.0040	0.0008	0.0015	-0.0009	0.0021	0.0030	0.0044	1999	0.0195
2000	0.0012	-0.0016	-0.0035	0.0040	0.0039	-0.0013	0.0025	0.0050	-0.0001	0.0039	0.0045	0.0056	2000	0.0242
2001	-0.0009	-0.0002	0.0019	0.0000	-0.0013	0.0011	0.0058	0.0031	-0.0017	0.0056	0.0034	0.0054	2001	0.0224
2002	-0.0009	-0.0026	-0.0043	-0.0040	0.0014	0.0007	0.0004	-0.0019	-0.0002	-0.0003	0.0012	0.0033	2002	-0.0071
2003	-0.0034	-0.0068	-0.0050	0.0032	0.0025	-0.0001	-0.0004	-0.0031	-0.0024	0.0018	0.0034	0.0019	2003	-0.0084
2004	-0.0042	-0.0048	-0.0055	-0.0024	-0.0052	-0.0023	0.0026	0.0006	-0.0010	-0.0041	0.0010	0.0053	2004	-0.0199
2005	-0.0005	-0.0041	-0.0057	-0.0046	0.0034	0.0018	-0.0022	-0.0021	-0.0093	0.0007	0.0113	0.0072	2005	-0.0042
2006	-0.0041	0.0013	-0.0019	-0.0049	-0.0006	0.0020	0.0010	0.0023	0.0090	0.0095	0.0057	0.0025	2006	0.0220
2007	0.0014	-0.0015	-0.0048	-0.0021	-0.0020	0.0020	0.0042	0.0060	0.0005	0.0011	-0.0025	0.0034	2007	0.0056
2008	-0.0028	-0.0016	-0.0069	-0.0043	-0.0066	-0.0083	-0.0037	0.0053	0.0029	0.0110	0.0198	0.0105	2008	0.0151
2009	-0.0043	-0.0048	-0.0023	-0.0024	-0.0029	-0.0084	0.0017	-0.0021	-0.0005	-0.0009	-0.0007	0.0018	2009	-0.0256
2010	-0.0034	-0.0002	-0.0040	-0.0016	-0.0007	0.0011	-0.0001	-0.0013	-0.0005	-0.0011	-0.0003	-0.0016	2010	-0.0135
2011	-0.0047	-0.0048	-0.0096	-0.0064	-0.0047	0.0011	-0.0009	-0.0027	-0.0015	0.0021	0.0008	0.0025	2011	-0.0284
2012	-0.0044	-0.0044	-0.0075	-0.0030	0.0012	0.0015	0.0017	-0.0055	-0.0044	0.0005	0.0048	0.0028	2012	-0.0165
2013	-0.0029	-0.0081	-0.0026	0.0011	-0.0018	-0.0024	-0.0004	-0.0012	-0.0012	0.0026	0.0020	0.0001	2013	-0.0146
2014	-0.0037	-0.0036	-0.0064	-0.0033	-0.0035	-0.0018	0.0004	0.0017	-0.0007	0.0025	0.0054	0.0057	2014	-0.0073
2015	0.0047	-0.0043	-0.0059	-0.0020	-0.0051	-0.0035	-0.0001	0.0015	0.0016	0.0004	0.0021	0.0035	2015	-0.0071
2016	-0.0016	-0.0006	-0.0041	-0.0046	-0.0039	-0.0031	0.0018	-0.0007	-0.0022	-0.0011	0.0017	-0.0001	2016	-0.0184

*Compound annual return

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appropriate price adjustments are made to account for stock splits and dividends. The return on a portfolio for one month is calculated as the weighted average of the returns for its individual stocks. Annual portfolio returns are calculated by compounding the monthly portfolio returns.[7.7]

Size of the Deciles

Exhibit 7.1 provides an overview of the CRSP deciles and size groupings in terms of relative size (by aggregate market capitalization) and number of companies as of December 31, 2016.

Decile 1 has 191 companies in it, and accounts for nearly two-thirds of aggregate market cap (66.12%). Decile 10 has 790 companies in it, and accounts for less than 1% of aggregate market cap (0.40%).

Exhibit 7.1: Aggregate Market Capitalization and Company Counts of the CRSP (NYSE/NYSE MKT/NASDAQ) Deciles and Size Groupings
December 31, 2016

Decile	Historic Average Percentage of Total Capitalization	Recent Number of Companies	Recent Decile Market Capitalization (in $thousands)	Recent Percentage of Total Capitalization
1-Largest	63.13%	191	15,290,475,300	66.12%
2	13.95%	200	3,010,671,018	13.02%
3	7.55%	202	1,609,575,618	6.96%
4	4.73%	221	1,010,851,810	4.37%
5	3.26%	227	677,120,067	2.93%
6	2.41%	259	541,037,999	2.34%
7	1.79%	283	384,129,198	1.66%
8	1.33%	361	297,164,943	1.28%
9	1.03%	487	212,609,644	0.92%
10-Smallest	0.82%	790	92,882,169	0.40%
Mid-Cap 3-5	15.54%	650	3,297,547,494	14.26%
Low-Cap 6-8	5.53%	903	1,222,332,139	5.29%
Micro-Cap 9-10	1.85%	1,277	305,491,813	1.32%

Source of underlying data: Calculated (or derived) based on data from CRSP ©2017 Center for Research in Security Prices (CRSP®), The University of Chicago Booth School of Business (2017). Calculations by Duff & Phelps, LLC.

[7.7] According to CRSP, in 2016 CRSP "performed a comprehensive check and found changes to index levels back to 1977. Almost all of the changes are due to CRSP adding factor[s] to adjust price values for distribution codes 5663 & 5773. These edits were made in the 201612 iteration...". These edits are detailed in the CRSP document "STOCK & INDEX RELEASE NOTES, December 2016 Annual UPDATE" (available at: http://www.crsp.com/files/images/release_notes/mdaz_201612_annual.pdf). This review of the database caused small changes in the returns over the 1926–2015 period (calculated using the December 31, 2015 data cut) compared to the returns over the 1926–2015 period (calculated using the December 31, 2016 data cut). These changes were not material: the largest/smallest change to the geometric mean return of CRSP standard market-cap-weighted deciles 1–10 over this period was 0.0044%/-0.0146%; the average/median change was -0.0007%/0.0003%.

Exhibit 7.8: Size-Decile Portfolios of the NYSE/NYSE MKT/NASDAQ Long-Term Returns in Excess of CAPM
1926–2016

Size Grouping	OLS Beta	Arithmetic Mean	Return in Excess of Risk-free Rate (actual)	Return in Excess of Risk-free Rate (as predicted by CAPM)	Size Premium
Mid-Cap (3–5)	1.12	13.82%	8.80%	7.79%	1.02%
Low-Cap (6–8)	1.22	15.26%	10.24%	8.49%	1.75%
Micro-Cap (9–10)	1.35	18.04%	13.02%	9.35%	3.67%
Breakdown of Deciles 1–10					
1-Largest	0.92	11.05%	6.04%	6.38%	-0.35%
2	1.04	12.82%	7.81%	7.19%	0.61%
3	1.11	13.57%	8.55%	7.66%	0.89%
4	1.13	13.80%	8.78%	7.80%	0.98%
5	1.17	14.62%	9.60%	8.09%	1.51%
6	1.17	14.81%	9.79%	8.14%	1.66%
7	1.25	15.41%	10.39%	8.67%	1.72%
8	1.30	16.14%	11.12%	9.04%	2.08%
9	1.34	16.97%	11.96%	9.28%	2.68%
10-Smallest	1.39	20.27%	15.25%	9.66%	5.59%

Betas are estimated from monthly returns in excess of the 30-day U.S. Treasury bill total return, January 1926–December 2016. Historical riskless rate measured by the 91-year arithmetic mean income return component of 20-year government bonds (5.02%). Calculated in the context of the CAPM by multiplying the equity risk premium by beta. The equity risk premium is estimated by the arithmetic mean total return of the S&P 500 (11.95%) minus the arithmetic mean income return component of 20-year government bonds (5.02%) from 1926–2016. Source: Morningstar *Direct* and CRSP. Calculated based on data from CRSP US Stock Database and CRSP US Indices Database ©2017 Center for Research. Used with permission. All calculations performed by Duff & Phelps, LLC.

The equity risk premium is calculated by subtracting the arithmetic mean of the government bond income return from the arithmetic mean of the stock market total return. Exhibit 10.9 demonstrates this calculation for the long-horizon equity risk premium.

Exhibit 10.9: Long-Horizon Equity Risk Premium Calculation (%)
1926–2016

	Arithmetic Mean			
	Market Total Return	Risk-free Rate		Equity Risk Premium
S&P 500	11.95	5.02	=	6.94
CRSP NYSE/NYSE MKT/NASDAQ Deciles 1-10	11.77	5.02	=	6.75
CRSP NYSE/NYSE MKT/NASDAQ Deciles 1-2	11.31	5.02	=	6.30*

* difference due to rounding

Source of underlying data in both Exhibit 10.8 and 10.9: (i) "IA SBBI US Large Stock TR USD Ext" series retrieved from the Morningstar *Direct* database. The "IA SBBI US Large Stock TR USD Ext" return series is essentially the S&P 500 index. The long-term, intermediate-term, and short-term risk-free series used are the "IA SBBI US LT Govt IR USD" series, the "IA SBBI US IT Govt IR USD" series, and the "IA SBBI US 30 Day TBill TR USD" series, respectively. All rights reserved. Used with permission. (ii) CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2017 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. Used with permission. All rights reserved. Calculations performed by Duff & Phelps, LLC.

The Market Benchmark and Firm Size

Although not restricted to the 500 largest companies, the S&P 500 is considered a large-cap index. The returns of the S&P 500 are cap-weighted. The larger companies in the index therefore receive the majority of the weight. The use of the "NYSE/NYSE MKT/NASDAQ Deciles 1–2" series results in an even purer large-cap index. However, if using a large-cap index to calculate the equity risk premium, an adjustment is usually needed to account for the different risk and return characteristics of small stocks. This was discussed further in Chapter 7 on the size premium.

The Risk-Free Asset

The equity risk premium can be calculated for a variety of time horizons when given the choice of risk-free asset to be used in the calculation. Chapter 3 provides equity risk premium calculations for short-, intermediate-, and long-term horizons. The short-, intermediate-, and long-horizon equity risk premiums are calculated using the income return from a 30-day Treasury bill, a 5-year Treasury bond, and a 20-year Treasury bond, respectively.

20-Year vs. 30-Year Treasuries

Our methodology for estimating the long-horizon equity risk premium makes use of the income return on a 20-year Treasury bond; however, the Treasury stopped issuing 20-year bonds in 1986. The 30-year bond that the Treasury returned to issuing in 2006 is theoretically more correct when dealing with the long-term nature of business valuation, yet Ibbotson Associates instead creates a series of returns using bonds on the market with approximately 20 years to maturity. The reason for the use of a 20-year maturity bond is that 30-year Treasury securities have only been issued over the relatively recent past, starting in February of 1977, and were suspended from 2002 to 2006.

The same reason applies to why we do not use the 10-year Treasury bond – a long history of market data is not available for 10-year bonds. We have persisted in using a 20-year bond to keep the basis of the time series consistent.

Income Return

Another point to keep in mind when calculating the equity risk premium is that the income return on the appropriate-horizon Treasury security, rather than the total return, is used in the calculation.

The total return comprises three return components: the income return, the capital appreciation return, and the reinvestment return. The income return is defined as the portion of the total return that results from a periodic cash flow or, in this case, the bond coupon payment. The capital appreciation return results from the price change of a bond over a specific period. Bond prices generally change in reaction to unexpected fluctuations in yields. Reinvestment return is the return on a given month's investment income when reinvested into the same asset class in the subsequent months of the year. The income return is thus used in the estimation of the equity risk premium because it represents the truly riskless portion of the return.

Arithmetic vs. Geometric Mean

The equity risk premium data presented in this book are arithmetic average risk premiums as opposed to geometric average risk premiums. The arithmetic average equity risk premium can be demonstrated to be most appropriate when discounting future cash flows. For use as the expected equity risk premium in either the CAPM or the building-block approach, the arithmetic mean or the simple difference of the arithmetic means of stock market returns and riskless rates is the relevant number. This is because both the CAPM and the building-block approach are additive models, in which the cost of capital is the sum of its parts. The geometric average is more appropriate for reporting past performance because it represents the compound average return.

Appropriate Historical Period

The equity risk premium can be estimated using any historical time period. For the U.S., market data exist at least as far back as the late 1800s. Therefore, it is possible to estimate the equity risk premium using data that covers roughly the past 125 years.

Our equity risk premium covers 1926 to the present. The original data source for the time series comprising the equity risk premium is the Center for Research in Security Prices. CRSP chose to begin its analysis of market returns with 1926 for two main reasons. CRSP determined that 1926 was approximately when quality financial data became available. They also made a conscious effort to include the period of extreme market volatility from the late 1920s and early 1930s; 1926 was chosen because it includes one full business cycle of data before the market crash of 1929.

Implicit in using history to forecast the future is the assumption that investors' expectations for future outcomes conform to past results. This method assumes that the price of taking on risk changes only slowly, if at all, over time. This "future equals the past" assumption is most applicable to a random time-series variable. A time-series variable is random if its value in one period is independent of its value in other periods.

Choosing an Appropriate Historical Period

The estimate of the equity risk premium depends on the length of the data series studied. A proper estimate of the equity risk premium requires a data series long enough to give a reliable average without being unduly influenced by very good and very poor short-term returns. When calculated using a long data series, the historical equity risk premium is relatively stable. Furthermore, because an average of the realized equity risk premium is quite volatile when calculated using a short history, using a long series makes it less likely that the analyst can justify any number he or she wants. The magnitude of how shorter periods can affect the result will be explored later in this chapter.

Some analysts estimate the expected equity risk premium using a shorter, more recent period on the basis that recent events are more likely to be repeated in the near future; furthermore, they believe that the 1920s, 1930s, and 1940s contain too many unusual events. This view is suspect because all periods contain unusual events. Some of the most unusual events of the last 100 years took place quite recently, including the inflation of the late 1970s and early 1980s, the October 1987 stock market crash, the collapse of the high-yield bond market, the major contraction and consolidation of the thrift industry, the collapse of the Soviet Union, the development of the European Economic Community, the attacks of Sept. 11, 2001, and the more recent global financial crisis of 2008–2009.

It is even difficult for economists to predict the economic environment of the future. For example, if one were analyzing the stock market in 1987 before the crash, it would be statistically improbable to predict the impending short-term volatility without considering the stock market crash and market volatility of the 1929–1931 period.

Without an appreciation of the 1920s and 1930s, no one would believe that such events could happen. The 91-year period starting with 1926 represents what can happen: It includes high and low returns, volatile and quiet markets, war and peace, inflation and deflation, and prosperity and depression. Restricting attention to a shorter historical period underestimates the amount of change that could occur in a long future period. Finally, because historical event-types (not specific events) tend to repeat themselves, long-run capital market return studies can reveal a great deal about the future. Investors probably expect unusual events to occur from time to time, and their return expectations reflect this.

A Look at the Historical Results

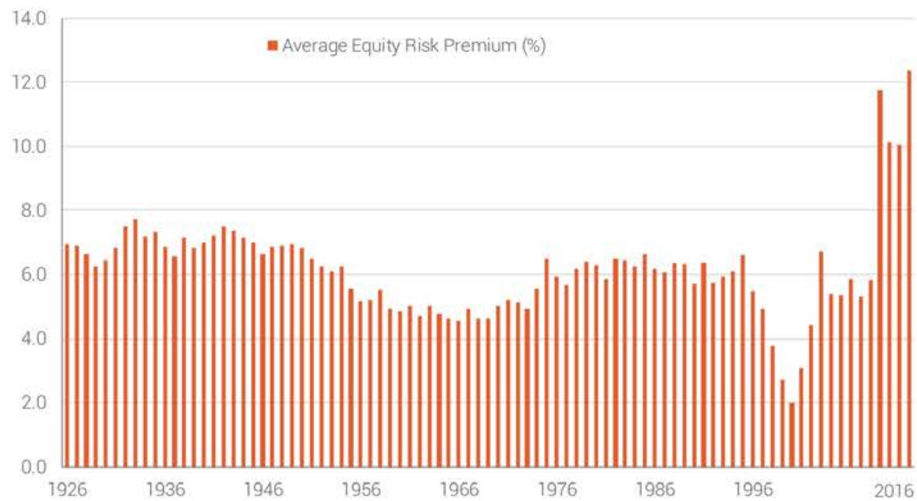
It is interesting to look at the realized returns and realized equity risk premium in the context of the above discussion. Exhibit 10.10 shows the average stock market return and the average (arithmetic mean) realized long-horizon equity risk premium over various historical periods. The exhibit shows that using a longer historical period provides a more stable estimate of the equity risk premium. The reason is that any unique period will not be weighted heavily in an average covering a longer historical period. It better represents the probability of these unique events occurring over a long period of time.

Exhibit 10.10: Stock Market Return and Equity Risk Premium Over Time (%)

Length	Period Dates	Large-Cap Arithmetic Mean (%)	Long-horizon Equity Risk Premium (%)
91	1926–2016	11.95	6.94
90	1927–2016	11.96	6.92
80	1937–2016	11.82	6.56
70	1947–2016	12.53	6.86
60	1957–2016	11.38	5.19
50	1967–2016	11.56	4.92
40	1977–2016	12.36	5.68
30	1987–2016	11.61	6.07
20	1997–2016	9.39	4.92
15	2002–2016	8.38	4.44
10	2007–2016	8.76	5.32
5	2012–2016	15.08	12.38

Looking carefully at Exhibit 10.11 will clarify this point. The graph shows the realized equity risk premium for a series of periods through 2016, starting with 1926. In other words, the first value on the graph represents the average realized equity risk premium over the period 1926–2016. The next value on the graph represents the average realized equity risk premium over the period 1927–2016, and so on, with the last value representing the average over the most recent five years, 2012–2016.

Exhibit 10.11: Equity Risk Premium Using Different Starting Dates
Average Equity Risk Premium (%)
1926–2016



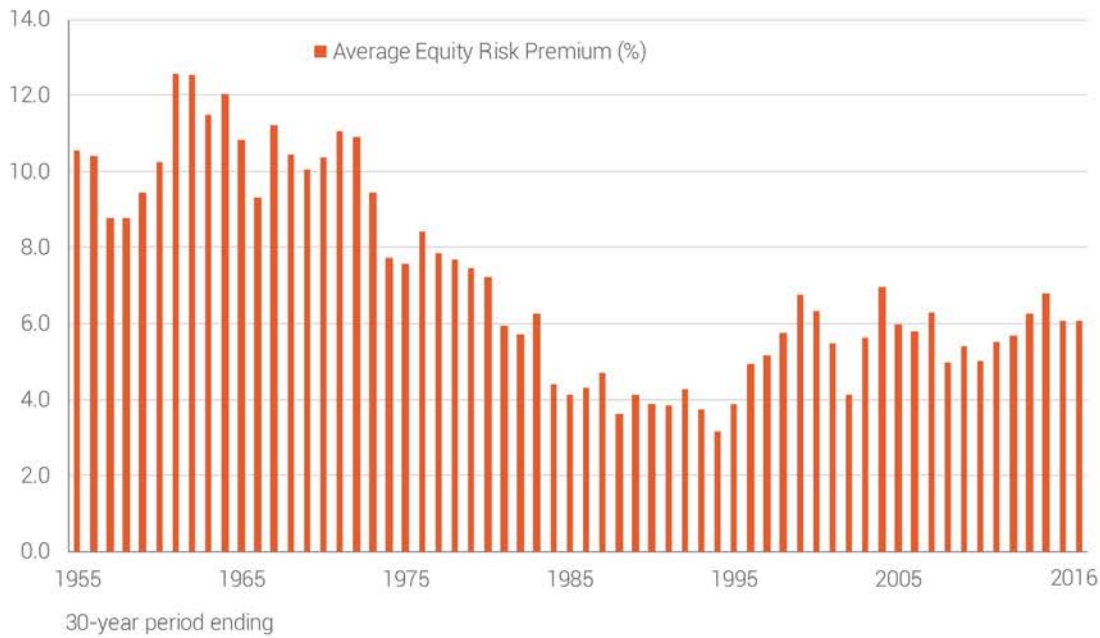
Concentrating on the left side of Exhibit 10.11, one notices that the realized equity risk premium, when measured over long periods, is relatively stable. In viewing the graph from left to right, moving from longer to shorter historical periods, one sees that the value of the realized equity risk premium begins to decline significantly. Why does this occur? The reason is that the severe bear market of 1973–1974 is receiving proportionately more weight in the shorter, more recent average. If you continue to follow the line to the right, however, you will also notice that when 1973 and 1974 fall out of the recent average, the realized equity risk premium jumps up by nearly 1.2 percentage points.

Additionally, use of recent historical periods for estimation purposes can lead to illogical conclusions. As seen in Exhibit 10.10, the bear market in the early 2000s and in 2008 has caused the realized equity risk premium in the shorter historical periods to be lower than the long-term average.

The impact of adding one additional year of data to a historical average is lessened the greater the initial period of measurement. Short-term averages can be affected considerably by one or more unique observations. On the other hand, long-term averages produce more stable results.

Some practitioners argue for a shorter historical period, such as 30 years, as a basis for the equity risk premium estimation. The logic for the use of a shorter period is that historical events and economic scenarios present before this time are unlikely to be repeated. Exhibit 10.12 shows the equity risk premium measured over rolling 30-year periods, and it appears from the graph that the premium has been trending downwards. The 30-year equity risk premium remained close to 4 percentage points for several years in the 1980s and 1990s. However, it has fallen and then risen in the most recent 30-year periods.

Exhibit 10.12: Equity Risk Premium Over Rolling 30-year Periods
Average Equity Risk Premium (%)
1926–2016



The key to understanding this result lies again in the years 1973 and 1974. The oil embargo during this period had a tremendous effect on the market. The equity risk premium for these years alone was -21% and -34%, respectively. Periods that include the years 1973 and 1974 result in average equity risk premiums as low as 3.2 percentage points. The 2000s have also had an enormous effect on the equity risk premium.

It is difficult to justify such a large divergence in estimates of return over such a short period. This does not suggest, however, that the years 1973 and 1974 should be excluded from any estimate of the equity risk premium; rather, it emphasizes the importance of using a long historical period when measuring the equity risk premium in order to obtain a reliable average that is not overly influenced by short-term returns. The same holds true when analyzing the poor performance of the early 2000s and 2008.

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THE COST OF CAPITAL, CORPORATION FINANCE AND THE THEORY OF INVESTMENT

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What is the "cost of capital" to a firm in a world in which funds are used to acquire assets whose yields are uncertain; and in which capital can be obtained by many different media, ranging from pure debt instruments, representing money-fixed claims, to pure equity issues, giving holders only the right to a pro-rata share in the uncertain venture? This question has vexed at least three classes of economists: (1) the corporation finance specialist concerned with the techniques of financing firms so as to ensure their survival and growth; (2) the managerial economist concerned with capital budgeting; and (3) the economic theorist concerned with explaining investment behavior at both the micro and macro levels.[1]

In much of his formal analysis, the economic theorist at least has tended to side-step the essence of this cost-of-capital problem by proceeding as though physical assets—like bonds—could be regarded as yielding known, sure streams. Given this assumption, the theorist has concluded that the cost of capital to the owners of a firm is simply the rate of interest on bonds; and has derived the familiar proposition that the firm, acting rationally, will tend to push investment to the point

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[1] The literature bearing on the cost-of-capital problem is far too extensive for listing here. Numerous references to it will be found throughout the paper though we make no claim to completeness. One phase of the problem which we do not consider explicitly, but which has a considerable literature of its own is the relation between the cost of capital and public utility rates. For a recent summary of the "cost-of-capital theory" of rate regulation and a brief discussion of some of its implications, the reader may refer to H. M. Somers [20].

where the marginal yield on physical assets is equal to the market rate of interest.[2] This proposition can be shown to follow from either of two criteria of rational decision-making which are equivalent under certainty, namely (1) the maximization of profits and (2) the maximization of market value.

According to the first criterion, a physical asset is worth acquiring if it will increase the net profit of the owners of the firm. But net profit will increase only if the expected rate of return, or yield, of the asset exceeds the rate of interest. According to the second criterion, an asset is worth acquiring if it increases the value of the owners' equity, *i.e.*, if it adds more to the market value of the firm than the costs of acquisition. But what the asset adds is given by capitalizing the stream it generates at the market rate of interest, and this capitalized value will exceed its cost if and only if the yield of the asset exceeds the rate of interest. Note that, under either formulation, the cost of capital is equal to the rate of interest on bonds, regardless of whether the funds are acquired through debt instruments or through new issues of common stock. Indeed, in a world of sure returns, the distinction between debt and equity funds reduces largely to one of terminology.

It must be acknowledged that some attempt is usually made in this type of analysis to allow for the existence of uncertainty. This attempt typically takes the form of superimposing on the results of the certainty analysis the notion of a "risk discount" to be subtracted from the expected yield (or a "risk premium" to be added to the market rate of interest). Investment decisions are then supposed to be based on a comparison of this "risk adjusted" or "certainty equivalent" yield with the market rate of interest.[3] No satisfactory explanation has yet been provided, however, as to what determines the size of the risk discount and how it varies in response to changes in other variables.

Considered as a convenient approximation, the model of the firm constructed via this certainty—or certainty-equivalent—approach has admittedly been useful in dealing with some of the grosser aspects of the processes of capital accumulation and economic fluctuations. Such a model underlies, for example, the familiar Keynesian aggregate investment function in which aggregate investment is written as a function of the rate of interest—the same riskless rate of interest which appears later in the system in the liquidity-preference equation. Yet few would maintain that this approximation is adequate. At the macroeconomic level there are ample grounds for doubting that the rate of interest has

[2] Or, more accurately, to the marginal cost of borrowed funds since it is customary, at least in advanced analysis, to draw the supply curve of borrowed funds to the firm as a rising one. For an advanced treatment of the certainty case, see F. and V. Lutz [13].

[3] The classic examples of the certainty-equivalent approach are found in J. R. Hicks [8] and O. Lange [11].

as large and as direct an influence on the rate of investment as this analysis would lead us to believe. At the microeconomic level the certainty model has little descriptive value and provides no real guidance to the finance specialist or managerial economist whose main problems cannot be treated in a framework which deals so cavalierly with uncertainty and ignores all forms of financing other than debt issues.[4]

Only recently have economists begun to face up seriously to the problem of the cost of capital *cum* risk. In the process they have found their interests and endeavors merging with those of the finance specialist and the managerial economist who have lived with the problem longer and more intimately. In this joint search to establish the principles which govern rational investment and financial policy in a world of uncertainty two main lines of attack can be discerned. These lines represent, in effect, attempts to extrapolate to the world of uncertainty each of the two criteria—profit maximization and market value maximization—which were seen to have equivalent implications in the special case of certainty. With the recognition of uncertainty this equivalence vanishes. In fact, the profit maximization criterion is no longer even well defined. Under uncertainty there corresponds to each decision of the firm not a unique profit outcome, but a plurality of mutually exclusive outcomes which can at best be described by a subjective probability distribution. The profit outcome, in short, has become a random variable and as such its maximization no longer has an operational meaning. Nor can this difficulty generally be disposed of by using the mathematical expectation of profits as the variable to be maximized. For decisions which affect the expected value will also tend to affect the dispersion and other characteristics of the distribution of outcomes. In particular, the use of debt rather than equity funds to finance a given venture may well increase the expected return to the owners, but only at the cost of increased dispersion of the outcomes.

Under these conditions the profit outcomes of alternative investment and financing decisions can be compared and ranked only in terms of a *subjective* "utility function" of the owners which weighs the expected yield against other characteristics of the distribution. Accordingly, the extrapolation of the profit maximization criterion of the certainty model has tended to evolve into utility maximization, sometimes explicitly, more frequently in a qualitative and heuristic form.[5]

The utility approach undoubtedly represents an advance over the certainty or certainty-equivalent approach. It does at least permit us

[4] Those who have taken a "case-method" course in finance in recent years will recall in this connection the famous Liquigas case of Hunt and Williams, [9, pp. 193–96] a case which is often used to introduce the student to the cost-of-capital problem and to poke a bit of fun at the economist's certainty-model.

[5] For an attempt at a rigorous explicit development of this line of attack, see F. Modigliani and M. Zeman [14].

to explore (within limits) some of the implications of different financing arrangements, and it does give some meaning to the "cost" of different types of funds. However, because the cost of capital has become an essentially subjective concept, the utility approach has serious drawbacks for normative as well as analytical purposes. How, for example, is management to ascertain the risk preferences of its stockholders and to compromise among their tastes? And how can the economist build a meaningful investment function in the face of the fact that any given investment opportunity might or might not be worth exploiting depending on precisely who happen to be the owners of the firm at the moment?

Fortunately, these questions do not have to be answered; for the alternative approach, based on market value maximization, can provide the basis for an operational definition of the cost of capital and a workable theory of investment. Under this approach any investment project and its concomitant financing plan must pass only the following test: Will the project, as financed, raise the market value of the firm's shares? If so, it is worth undertaking; if not, its return is less than the marginal cost of capital to the firm. Note that such a test is entirely independent of the tastes of the current owners, since market prices will reflect not only their preferences but those of all potential owners as well. If any current stockholder disagrees with management and the market over the valuation of the project, he is free to sell out and reinvest elsewhere, but will still benefit from the capital appreciation resulting from management's decision.

The potential advantages of the market-value approach have long been appreciated; yet analytical results have been meager. What appears to be keeping this line of development from achieving its promise is largely the lack of an adequate theory of the effect of financial structure on market valuations, and of how these effects can be inferred from objective market data. It is with the development of such a theory and of its implications for the cost-of-capital problem that we shall be concerned in this paper.

Our procedure will be to develop in Section I the basic theory itself and to give some brief account of its empirical relevance. In Section II, we show how the theory can be used to answer the cost-of-capital question and how it permits us to develop a theory of investment of the firm under conditions of uncertainty. Throughout these sections the approach is essentially a partial-equilibrium one focusing on the firm and "industry." Accordingly, the "prices" of certain income streams will be treated as constant and given from outside the model, just as in the standard Marshallian analysis of the firm and industry the prices of all inputs and of all other products are taken as given. We have chosen to focus at this level rather than on the economy as a whole because it is at the level of the firm and the industry that the interests of the various specialists concerned with the cost-of-capital problem come most closely together. Although the emphasis has thus been placed on partial-equilibrium analysis, the results obtained also provide the essential building blocks for a general equilibrium model which shows how those prices which are here taken as given, are themselves determined. For reasons of space, however, and because the material is of interest in its own right, the presentation of the general equilibrium model which rounds out the analysis must be deferred to a subsequent paper.

I. *The Valuation of Securities, Leverage, and the Cost of Capital*

A. *The Capitalization Rate for Uncertain Streams*

As a starting point, consider an economy in which all physical assets are owned by corporations. For the moment, assume that these corporations can finance their assets by issuing common stock only; the introduction of bond issues, or their equivalent, as a source of corporate funds is postponed until the next part of this section.

The physical assets held by each firm will yield to the owners of the firm—its stockholders—a stream of "profits" over time; but the elements of this series need not be constant and in any event are uncertain. This stream of income, and hence the stream accruing to any share of common stock, will be regarded as extending indefinitely into the future. We assume, however, that the mean value of the stream over time, or average profit per unit of time, is finite and represents a random variable subject to a (subjective) probability distribution. We shall refer to the average value over time of the stream accruing to a given share as the return of that share; and to the mathematical expectation of this average as the expected return of the share.[6] Although individual investors may have different views as to the shape of the probability distri-

[6] These propositions can be restated analytically as follows: The assets of the ith firm generate a stream:

$$X_i(1),\ X_i(2)\ \cdots\ X_i(T)$$

whose elements are random variables subject to the joint probability distribution:

$$\chi_i[X_i(1),\ X_i(2)\ \cdots\ X_i(t)].$$

The return to the ith firm is defined as:

$$X_i = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} X_i(t).$$

X_i is itself a random variable with a probability distribution $\Phi_i(X_i)$ whose form is determined uniquely by χ_i. The expected return $\bar{X}_i$ is defined as $\bar{X}_i = E(X_i) = \int_{X_i} X_i \Phi_i(X_i) dX_i$. If N_i is the number of shares outstanding, the return of the ith share is $x_i = (1/N)X_i$ with probability distribution $\phi_i(x_i)dx_i = \Phi_i(Nx_i)d(Nx_i)$ and expected value $\bar{x}_i = (1/N)\bar{X}_i$.

bution of the return of any share, we shall assume for simplicity that they are at least in agreement as to the expected return.[7]

This way of characterizing uncertain streams merits brief comment. Notice first that the stream is a stream of profits, not dividends. As will become clear later, as long as management is presumed to be acting in the best interests of the stockholders, retained earnings can be regarded as equivalent to a fully subscribed, pre-emptive issue of common stock. Hence, for present purposes, the division of the stream between cash dividends and retained earnings in any period is a mere detail. Notice also that the uncertainty attaches to the mean value over time of the stream of profits and should not be confused with variability over time of the successive elements of the stream. That variability and uncertainty are two totally different concepts should be clear from the fact that the elements of a stream can be variable even though known with certainty. It can be shown, furthermore, that whether the elements of a stream are sure or uncertain, the effect of variability per se on the valuation of the stream is at best a second-order one which can safely be neglected for our purposes (and indeed most others too).[8]

The next assumption plays a strategic role in the rest of the analysis. We shall assume that firms can be divided into "equivalent return" classes such that the return on the shares issued by any firm in any given class is proportional to (and hence perfectly correlated with) the return on the shares issued by any other firm in the same class. This assumption implies that the various shares within the same class differ, at most, by a "scale factor." Accordingly, if we adjust for the difference in scale, by taking the *ratio* of the return to the expected return, the probability distribution of that ratio is identical for all shares in the class. It follows that all relevant properties of a share are uniquely characterized by specifying (1) the class to which it belongs and (2) its expected return.

The significance of this assumption is that it permits us to classify firms into groups within which the shares of different firms are "homogeneous," that is, perfect substitutes for one another. We have, thus, an analogue to the familiar concept of the industry in which it is the commodity produced by the firms that is taken as homogeneous. To complete this analogy with Marshallian price theory, we shall assume in the

[7] To deal adequately with refinements such as differences among investors in estimates of expected returns would require extensive discussion of the theory of portfolio selection. Brief references to these and related topics will be made in the succeeding article on the general equilibrium model.

[8] The reader may convince himself of this by asking how much he would be willing to rebate to his employer for the privilege of receiving his annual salary in equal monthly installments rather than in irregular amounts over the year. See also J. M. Keynes [10, esp. pp. 53–54].

analysis to follow that the shares concerned are traded in perfect markets under conditions of atomistic competition.[9]

From our definition of homogeneous classes of stock it follows that in equilibrium in a perfect capital market the price per dollar's worth of expected return must be the same for all shares of any given class. Or, equivalently, in any given class the price of every share must be proportional to its expected return. Let us denote this factor of proportionality for any class, say the kth class, by $1/\rho_k$. Then if p_j denotes the price and $\bar{x}_j$ is the expected return per share of the jth firm in class k, we must have:

$$p_j = \frac{1}{\rho_k}\bar{x}_j; \tag{1}$$

or, equivalently,

$$\frac{\bar{x}_j}{p_j} = \rho_k \text{ a constant for all firms } j \text{ in class } k. \tag{2}$$

The constants ρ_k (one for each of the k classes) can be given several economic interpretations: (a) From (2) we see that each ρ_k is the expected rate of return of any share in class k. (b) From (1) $1/\rho_k$ is the price which an investor has to pay for a dollar's worth of expected return in the class k. (c) Again from (1), by analogy with the terminology for perpetual bonds, ρ_k can be regarded as the market rate of capitalization for the expected value of the uncertain streams of the kind generated by the kth class of firms.[10]

B. *Debt Financing and Its Effects on Security Prices*

Having developed an apparatus for dealing with uncertain streams we can now approach the heart of the cost-of-capital problem by dropping the assumption that firms cannot issue bonds. The introduction of debt-financing changes the market for shares in a very fundamental way. Because firms may have different proportions of debt in their capi-

[9] Just what our classes of stocks contain and how the different classes can be identified by outside observers are empirical questions to which we shall return later. For the present, it is sufficient to observe: (1) Our concept of a class, while not identical to that of the industry is at least closely related to it. Certainly the basic characteristics of the probability distributions of the returns on assets will depend to a significant extent on the product sold and the technology used. (2) What are the appropriate class boundaries will depend on the particular problem being studied. An economist concerned with general tendencies in the market, for example, might well be prepared to work with far wider classes than would be appropriate for an investor planning his portfolio, or a firm planning its financial strategy.

[10] We cannot, on the basis of the assumptions so far, make any statements about the relationship or spread between the various ρ's or capitalization rates. Before we could do so we would have to make further specific assumptions about the way investors believe the probability distributions vary from class to class, as well as assumptions about investors' preferences as between the characteristics of different distributions.

tal structure, shares of different companies, even in the same class, can give rise to different probability distributions of returns. In the language of finance, the shares will be subject to different degrees of financial risk or "leverage" and hence they will no longer be perfect substitutes for one another.

To exhibit the mechanism determining the relative prices of shares under these conditions, we make the following two assumptions about the nature of bonds and the bond market, though they are actually stronger than is necessary and will be relaxed later: (1) All bonds (including any debts issued by households for the purpose of carrying shares) are assumed to yield a constant income per unit of time, and this income is regarded as certain by all traders regardless of the issuer. (2) Bonds, like stocks, are traded in a perfect market, where the term perfect is to be taken in its usual sense as implying that any two commodities which are perfect substitutes for each other must sell, in equilibrium, at the same price. It follows from assumption (1) that all bonds are in fact perfect substitutes up to a scale factor. It follows from assumption (2) that they must all sell at the same price per dollar's worth of return, or what amounts to the same thing must yield the same rate of return. This rate of return will be denoted by r and referred to as the rate of interest or, equivalently, as the capitalization rate for sure streams. We now can derive the following two basic propositions with respect to the valuation of securities in companies with different capital structures:

Proposition I. Consider any company j and let $\overline{X}_j$ stand as before for the expected return on the assets owned by the company (that is, its expected profit before deduction of interest). Denote by D_j the market value of the debts of the company; by S_j the market value of its common shares; and by $V_j \equiv S_j + D_j$ the market value of all its securities or, as we shall say, the market value of the firm. Then, our Proposition I asserts that we must have in equilibrium:

$$(3) \qquad V_j \equiv (S_j + D_j) = \overline{X}_j / \rho_k, \text{ for any firm } j \text{ in class } k.$$

That is, the *market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate ρ_k appropriate to its class.*

This proposition can be stated in an equivalent way in terms of the firm's "average cost of capital," $\overline{X}_j / V_j$, which is the ratio of its expected return to the market value of all its securities. Our proposition then is:

$$(4) \qquad \frac{\overline{X}_j}{(S_j + D_j)} \equiv \frac{\overline{X}_j}{V_j} = \rho_k, \text{ for any firm } j, \text{ in class } k.$$

That is, *the average cost of capital to any firm is completely independent of* *its capital structure and is equal to the capitalization rate of a pure equity stream of its class.*

To establish Proposition I we will show that as long as the relations (3) or (4) do not hold between any pair of firms in a class, arbitrage will take place and restore the stated equalities. We use the term arbitrage advisedly. For if Proposition I did not hold, an investor could buy and sell stocks and bonds in such a way as to exchange one income stream for another stream, identical in all relevant respects but selling at a lower price. The exchange would therefore be advantageous to the investor quite independently of his attitudes toward risk.[11] As investors exploit these arbitrage opportunities, the value of the overpriced shares will fall and that of the underpriced shares will rise, thereby tending to eliminate the discrepancy between the market values of the firms.

By way of proof, consider two firms in the same class and assume for simplicity only, that the expected return, X, is the same for both firms. Let company 1 be financed entirely with common stock while company 2 has some debt in its capital structure. Suppose first the value of the levered firm, V_2, to be larger than that of the unlevered one, V_1. Consider an investor holding s_2 dollars' worth of the shares of company 2, representing a fraction α of the total outstanding stock, S_2. The return from this portfolio, denoted by Y_2, will be a fraction α of the income available for the stockholders of company 2, which is equal to the total return X_2 less the interest charge, rD_2. Since under our assumption of homogeneity, the anticipated total return of company 2, X_2, is, under all circumstances, the same as the anticipated total return to company 1, X_1, we can hereafter replace X_2 and X_1 by a common symbol X. Hence, the return from the initial portfolio can be written as:

$$Y_2 = \alpha(X - rD_2). \tag{5}$$

Now suppose the investor sold his αS_2 worth of company 2 shares and acquired instead an amount $s_1 = \alpha(S_2 + D_2)$ of the shares of company 1. He could do so by utilizing the amount αS_2 realized from the sale of his initial holding and borrowing an additional amount αD_2 on his own credit, pledging his new holdings in company 1 as a collateral. He would thus secure for himself a fraction $s_1/S_1 = \alpha(S_2 + D_2)/S_1$ of the shares and earnings of company 1. Making proper allowance for the interest payments on his personal debt αD_2, the return from the new portfolio, Y_1, is given by:

[11] In the language of the theory of choice, the exchanges are movements from inefficient points in the interior to efficient points on the boundary of the investor's opportunity set; and not movements between efficient points along the boundary. Hence for this part of the analysis nothing is involved in the way of specific assumptions about investor attitudes or behavior other than that investors behave consistently and prefer more income to less income, *ceteris paribus*.

$$Y_1 = \frac{\alpha(S_2 + D_2)}{S_1} X - r\alpha D_2 = \alpha \frac{V_2}{V_1} X - r\alpha D_2. \tag{6}$$

Comparing (5) with (6) we see that as long as $V_2 > V_1$ we must have $Y_1 > Y_2$, so that it pays owners of company 2's shares to sell their holdings, thereby depressing S_2 and hence V_2; and to acquire shares of company 1, thereby raising S_1 and thus V_1. We conclude therefore that levered companies cannot command a premium over unlevered companies because investors have the opportunity of putting the equivalent leverage into their portfolio directly by borrowing on personal account.

Consider now the other possibility, namely that the market value of the levered company V_2 is less than V_1. Suppose an investor holds initially an amount s_1 of shares of company 1, representing a fraction α of the total outstanding stock, S_1. His return from this holding is:

$$Y_1 = \frac{s_1}{S_1} X = \alpha X.$$

Suppose he were to exchange this initial holding for another portfolio, also worth s_1, but consisting of s_2 dollars of stock of company 2 and of d dollars of bonds, where s_2 and d are given by:

$$s_2 = \frac{S_2}{V_2} s_1, \qquad d = \frac{D_2}{V_2} s_1. \tag{7}$$

In other words the new portfolio is to consist of stock of company 2 and of bonds in the proportions S_2/V_2 and D_2/V_2, respectively. The return from the stock in the new portfolio will be a fraction s_2/S_2 of the total return to stockholders of company 2, which is $(X - rD_2)$, and the return from the bonds will be rd. Making use of (7), the total return from the portfolio, Y_2, can be expressed as follows:

$$Y_2 = \frac{s_2}{S_2}(X - rD_2) + rd = \frac{s_1}{V_2}(X - rD_2) + r\frac{D_2}{V_2}s_1 = \frac{s_1}{V_2}X = \alpha\frac{S_1}{V_2}X$$

(since $s_1 = \alpha S_1$). Comparing Y_2 with Y_1 we see that, if $V_2 < S_1 \equiv V_1$, then Y_2 will exceed Y_1. Hence it pays the holders of company 1's shares to sell these holdings and replace them with a mixed portfolio containing an appropriate fraction of the shares of company 2.

The acquisition of a mixed portfolio of stock of a levered company j and of bonds in the proportion S_j/V_j and D_j/V_j respectively, may be regarded as an operation which "undoes" the leverage, giving access to an appropriate fraction of the unlevered return X_j. It is this possibility of undoing leverage which prevents the value of levered firms from being consistently less than those of unlevered firms, or more generally prevents the average cost of capital $\bar{X}_j/V_j$ from being systematically higher for levered than for nonlevered companies in the same class.

Since we have already shown that arbitrage will also prevent V_2 from being larger than V_1, we can conclude that in equilibrium we must have $V_2 = V_1$, as stated in Proposition I.

Proposition II. From Proposition I we can derive the following proposition concerning the rate of return on common stock in companies whose capital structure includes some debt: the expected rate of return or yield, i, on the stock of any company j belonging to the kth class is a linear function of leverage as follows:

$$i_j = \rho_k + (\rho_k - r)D_j/S_j. \tag{8}$$

That is, *the expected yield of a share of stock is equal to the appropriate capitalization rate ρ_k for a pure equity stream in the class, plus a premium related to financial risk equal to the debt-to-equity ratio times the spread between ρ_k and r.* Or equivalently, the market price of any share of stock is given by capitalizing its expected return at the continuously variable rate i_j of (8).[12]

A number of writers have stated close equivalents of our Proposition I although by appealing to intuition rather than by attempting a proof and only to insist immediately that the results were not applicable to the actual capital markets.[13] Proposition II, however, so far as we have been able to discover is new.[14] To establish it we first note that, by definition, the expected rate of return, i, is given by:

$$i_j \equiv \frac{\bar{X}_j - rD_j}{S_j}. \tag{9}$$

From Proposition I, equation (3), we know that:

$$\bar{X}_j = \rho_k(S_j + D_j).$$

Substituting in (9) and simplifying, we obtain equation (8).

[12] To illustrate, suppose $\bar{X} = 1000$, $D = 4000$, $r = 5$ per cent and $\rho_k = 10$ per cent. These values imply that $V = 10{,}000$ and $S = 6000$ by virtue of Proposition I. The expected yield or rate of return per share is then:

$$i = \frac{1000 - 200}{6000} = .1 + (.1 - .05)\frac{4000}{6000} = 13\tfrac{1}{3} \text{ per cent.}$$

[13] See, for example, J. B. Williams [21, esp. pp. 72–73]; David Durand [3]; and W. A. Morton [15]. None of these writers describe in any detail the mechanism which is supposed to keep the average cost of capital constant under changes in capital structure. They seem, however, to be visualizing the equilibrating mechanism in terms of switches by investors between stocks and bonds as the yields of each get out of line with their "riskiness." This is an argument quite different from the pure arbitrage mechanism underlying our proof, and the difference is crucial. Regarding Proposition I as resting on investors' attitudes toward risk leads inevitably to a misunderstanding of many factors influencing relative yields such as, for example, limitations on the portfolio composition of financial institutions. See below, esp. Section I.D.

[14] Morton does make reference to a linear yield function but only " . . . for the sake of simplicity and because the particular function used makes no essential difference in my conclusions" [15, p. 443, note 2].

C. *Some Qualifications and Extensions of the Basic Propositions*

The methods and results developed so far can be extended in a number of useful directions, of which we shall consider here only three: (1) allowing for a corporate profits tax under which interest payments are deductible; (2) recognizing the existence of a multiplicity of bonds and interest rates; and (3) acknowledging the presence of market imperfections which might interfere with the process of arbitrage. The first two will be examined briefly in this section with some further attention given to the tax problem in Section II. Market imperfections will be discussed in Part D of this section in the course of a comparison of our results with those of received doctrines in the field of finance.

Effects of the Present Method of Taxing Corporations. The deduction of interest in computing taxable corporate profits will prevent the arbitrage process from making the value of all firms in a given class proportional to the expected returns generated by their physical assets. Instead, it can be shown (by the same type of proof used for the original version of Proposition I) that the market values of firms in each class must be proportional in equilibrium to their expected return net of taxes (that is, to the sum of the interest paid and expected net stockholder income). This means we must replace each $\bar{X}_j$ in the original versions of Propositions I and II with a new variable $\bar{X}_j^\tau$ representing the total income net of taxes generated by the firm:

$$\bar{X}_j^\tau \equiv (\bar{X}_j - rD_j)(1 - \tau) + rD_j \equiv \bar{\pi}_j^\tau + rD_j, \tag{10}$$

where $\bar{\pi}_j^\tau$ represents the expected net income accruing to the common stockholders and τ stands for the average rate of corporate income tax.[15]

After making these substitutions, the propositions, when adjusted for taxes, continue to have the same form as their originals. That is, Proposition I becomes:

$$\frac{\bar{X}_j^\tau}{V_j} = \rho_k^\tau, \text{ for any firm in class } k, \tag{11}$$

and Proposition II becomes

$$i_j \equiv \frac{\bar{\pi}_j^\tau}{S_j} = \rho_j^\tau + (\rho_k^\tau - r)D_j/S_j \tag{12}$$

where ρ_k^τ is the capitalization rate for income net of taxes in class k.

Although the form of the propositions is unaffected, certain interpretations must be changed. In particular, the after-tax capitalization rate

[15] For simplicity, we shall ignore throughout the tiny element of progression in our present corporate tax and treat τ as a constant independent of $(X_j - rD_j)$.

ρ_k^{τ} can no longer be identified with the "average cost of capital" which is $\rho_k = \overline{X}_j/V_j$. The difference between ρ_k^{τ} and the "true" average cost of capital, as we shall see, is a matter of some relevance in connection with investment planning within the firm (Section II). For the description of market behavior, however, which is our immediate concern here, the distinction is not essential. To simplify presentation, therefore, and to preserve continuity with the terminology in the standard literature we shall continue in this section to refer to ρ_k^{τ} as the average cost of capital, though strictly speaking this identification is correct only in the absence of taxes.

Effects of a Plurality of Bonds and Interest Rates. In existing capital markets we find not one, but a whole family of interest rates varying with maturity, with the technical provisions of the loan and, what is most relevant for present purposes, with the financial condition of the borrower.[16] Economic theory and market experience both suggest that the yields demanded by lenders tend to increase with the debt-equity ratio of the borrowing firm (or individual). If so, and if we can assume as a first approximation that this yield curve, $r = r\ (D/S)$, whatever its precise form, is the same for all borrowers, then we can readily extend our propositions to the case of a rising supply curve for borrowed funds.[17]

Proposition I is actually unaffected in form and interpretation by the fact that the rate of interest may rise with leverage; while the average cost of *borrowed* funds will tend to increase as debt rises, the average cost of funds from *all* sources will still be independent of leverage (apart from the tax effect). This conclusion follows directly from the ability of those who engage in arbitrage to undo the leverage in any financial structure by acquiring an appropriately mixed portfolio of bonds and stocks. Because of this ability, the ratio of earnings (*before* interest charges) to market value—*i.e.*, the average cost of capital from all

[16] We shall not consider here the extension of the analysis to encompass the time structure of interest rates. Although some of the problems posed by the time structure can be handled within our comparative statics framework, an adequate discussion would require a separate paper.

[17] We can also develop a theory of bond valuation along lines essentially parallel to those followed for the case of shares. We conjecture that the curve of bond yields as a function of leverage will turn out to be a nonlinear one in contrast to the linear function of leverage developed for common shares. However, we would also expect that the rate of increase in the yield on new issues would not be substantial in practice. This relatively slow rise would reflect the fact that interest rate increases by themselves can never be completely satisfactory to creditors as compensation for their increased risk. Such increases may simply serve to raise r so high relative to ρ that they become self-defeating by giving rise to a situation in which even normal fluctuations in earnings may force the company into bankruptcy. The difficulty of borrowing more, therefore, tends to show up in the usual case not so much in higher rates as in the form of increasingly stringent restrictions imposed on the company's management and finances by the creditors; and ultimately in a complete inability to obtain new borrowed funds, at least from the institutional investors who normally set the standards in the market for bonds.

sources—must be the same for all firms in a given class.[18] In other words, the increased cost of borrowed funds as leverage increases will tend to be offset by a corresponding reduction in the yield of common stock. This seemingly paradoxical result will be examined more closely below in connection with Proposition II.

A significant modification of Proposition I would be required only if the yield curve $r=r(D/S)$ were different for different borrowers, as might happen if creditors had marked preferences for the securities of a particular class of debtors. If, for example, corporations as a class were able to borrow at lower rates than individuals having equivalent personal leverage, then the average cost of capital to corporations might fall slightly, as leverage increased over some range, in reflection of this differential. In evaluating this possibility, however, remember that the relevant interest rate for our arbitrage operators is the rate on brokers' loans and, historically, that rate has not been noticeably higher than representative corporate rates.[19] The operations of holding companies and investment trusts which can borrow on terms comparable to operating companies represent still another force which could be expected to wipe out any marked or prolonged advantages from holding levered stocks.[20]

Although Proposition I remains unaffected as long as the yield curve is the same for all borrowers, the relation between common stock yields and leverage will no longer be the strictly linear one given by the original Proposition II. If r increases with leverage, the yield i will still tend to

[18] One normally minor qualification might be noted. Once we relax the assumption that all bonds have certain yields, our arbitrage operator faces the danger of something comparable to "gambler's ruin." That is, there is always the possibility that an otherwise sound concern—one whose long-run expected income is greater than its interest liability—might be forced into liquidation as a result of a run of temporary losses. Since reorganization generally involves costs, and because the operation of the firm may be hampered during the period of reorganization with lasting unfavorable effects on earnings prospects, we might perhaps expect heavily levered companies to sell at a slight discount relative to less heavily indebted companies of the same class.

[19] Under normal conditions, moreover, a substantial part of the arbitrage process could be expected to take the form, not of having the arbitrage operators go into debt on personal account to put the required leverage into their portfolios, but simply of having them reduce the amount of corporate bonds they already hold when they acquire underpriced unlevered stock. Margin requirements are also somewhat less of an obstacle to maintaining any desired degree of leverage in a portfolio than might be thought at first glance. Leverage could be largely restored in the face of higher margin requirements by switching to stocks having more leverage at the corporate level.

[20] An extreme form of inequality between borrowing and lending rates occurs, of course, in the case of preferred stocks, which can not be directly issued by individuals on personal account. Here again, however, we would expect that the operations of investment corporations plus the ability of arbitrage operators to sell off their holdings of preferred stocks would act to prevent the emergence of any substantial premiums (for this reason) on capital structures containing preferred stocks. Nor are preferred stocks so far removed from bonds as to make it impossible for arbitrage operators to approximate closely the risk and leverage of a corporate preferred stock by incurring a somewhat smaller debt on personal account.

rise as D/S increases, but at a decreasing rather than a constant rate. Beyond some high level of leverage, depending on the exact form of the interest function, the yield may even start to fall.[21] The relation between i and D/S could conceivably take the form indicated by the curve MD

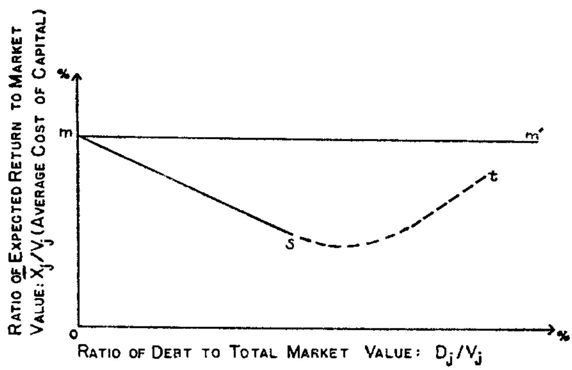

FIGURE 1

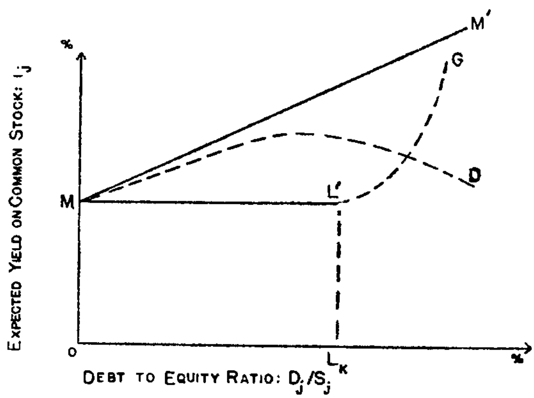

FIGURE 2

in Figure 2, although in practice the curvature would be much less pronounced. By contrast, with a constant rate of interest, the relation would be linear throughout as shown by line MM', Figure 2.

The downward sloping part of the curve MD perhaps requires some

[21] Since new lenders are unlikely to permit this much leverage (*cf.* note 17), this range of the curve is likely to be occupied by companies whose earnings prospects have fallen substantially since the time when their debts were issued.

comment since it may be hard to imagine why investors, other than those who like lotteries, would purchase stocks in this range. Remember, however, that the yield curve of Proposition II is a consequence of the more fundamental Proposition I. Should the demand by the risk-lovers prove insufficient to keep the market to the peculiar yield-curve *MD*, this demand would be reinforced by the action of arbitrage operators. The latter would find it profitable to own a pro-rata share of the firm as a whole by holding its stock *and* bonds, the lower yield of the shares being thus offset by the higher return on bonds.

D. *The Relation of Propositions I and II to Current Doctrines*

The propositions we have developed with respect to the valuation of firms and shares appear to be substantially at variance with current doctrines in the field of finance. The main differences between our view and the current view are summarized graphically in Figures 1 and 2. Our Proposition I [equation (4)] asserts that the average cost of capital, $\bar{X}_j^\tau/V_j$, is a constant for all firms j in class k, independently of their financial structure. This implies that, if we were to take a sample of firms in a given class, and if for each firm we were to plot the ratio of expected return to market value against some measure of leverage or financial structure, the points would tend to fall on a horizontal straight line with intercept ρ_k^τ, like the solid line mm' in Figure 1.[22] From Proposition I we derived Proposition II [equation (8)] which, taking the simplest version with r constant, asserts that, for all firms in a class, the relation between the yield on common stock and financial structure, measured by D_j/S_j, will approximate a straight line with slope $(\rho_k^\tau - r)$ and intercept ρ_k^τ. This relationship is shown as the solid line MM' in Figure 2, to which reference has been made earlier.[23]

By contrast, the conventional view among finance specialists appears to start from the proposition that, other things equal, the earnings-price ratio (or its reciprocal, the times-earnings multiplier) of a firm's common stock will normally be only slightly affected by "moderate" amounts of debt in the firm's capital structure.[24] Translated into our no-

[22] In Figure 1 the measure of leverage used is D_j/V_j (the ratio of debt to market value) rather than D_j/S_j (the ratio of debt to equity), the concept used in the analytical development. The D_j/V_j measure is introduced at this point because it simplifies comparison and contrast of our view with the traditional position.

[23] The line MM' in Figure 2 has been drawn with a positive slope on the assumption that $\rho_k^\tau > r$, a condition which will normally obtain. Our Proposition II as given in equation (8) would continue to be valid, of course, even in the unlikely event that $\rho_k^\tau < r$, but the slope of MM' would be negative.

[24] See, *e.g.*, Graham and Dodd [6, pp. 464–66]. Without doing violence to this position, we can bring out its implications more sharply by ignoring the qualification and treating the yield as a virtual constant over the relevant range. See in this connection the discussion in Durand [3, esp. pp. 225–37] of what he calls the "net income method" of valuation.

tation, it asserts that for any firm j in the class k,

$$\frac{\overline{X}_j^\tau - rD_j}{S_j} \equiv \frac{\bar{\pi}_j^\tau}{S_j} = i_k^*, \text{ a constant for } \frac{D_j}{S_j} \le L_k \tag{13}$$

or, equivalently,

$$S_j = \bar{\pi}_j^\tau / i_k^*. \tag{14}$$

Here i_k^* represents the capitalization rate or earnings-price ratio on the common stock and L_k denotes some amount of leverage regarded as the maximum "reasonable" amount for firms of the class k. This assumed relationship between yield and leverage is the horizontal solid line ML' of Figure 2. Beyond L', the yield will presumably rise sharply as the market discounts "excessive" trading on the equity. This possibility of a rising range for high leverages is indicated by the broken-line segment $L'G$ in the figure.[25]

If the value of shares were really given by (14) then the over-all market value of the firm must be:

$$V_j \equiv S_j + D_j = \frac{\overline{X}_j^\tau - rD_j}{i_k^*} + D_j = \frac{\overline{X}_j^\tau}{i_k^*} + \frac{(i_k^* - r)D_j}{i_k^*}. \tag{16}$$

That is, for any given level of expected total returns after taxes ($\overline{X}_j^\tau$) and assuming, as seems natural, that $i_k^* > r$, the value of the firm must tend to *rise* with debt;[26] whereas our Proposition I asserts that the value of the firm is completely independent of the capital structure. Another way of contrasting our position with the traditional one is in terms of the cost of capital. Solving (16) for $\overline{X}_j^\tau / V_j$ yields:

$$\overline{X}_j^\tau / V_j = i_k^* - (i_k^* - r) D_j / V_j. \tag{17}$$

According to this equation, the average cost of capital is not independent of capital structure as we have argued, but should tend to *fall* with increasing leverage, at least within the relevant range of moderate debt ratios, as shown by the line *ms* in Figure 1. Or to put it in more familiar terms, debt-financing should be "cheaper" than equity-financing if not carried too far.

When we also allow for the possibility of a rising range of stock yields for large values of leverage, we obtain a U-shaped curve like *mst* in

[25] To make it easier to see some of the implications of this hypothesis as well as to prepare the ground for later statistical testing, it will be helpful to assume that the notion of a critical limit on leverage beyond which yields rise rapidly, can be epitomized by a quadratic relation of the form:

$$\bar{\pi}_j^\tau / S_j = i_k^* + \beta(D_j/S_j) + \alpha(D_j/S_j)^2, \qquad \alpha > 0. \tag{15}$$

[26] For a typical discussion of how a promoter can, supposedly, increase the market value of a firm by recourse to debt issues, see W. J. Eiteman [4, esp. pp. 11–13].

Figure 1.[27] That a yield-curve for stocks of the form *ML'G* in Figure 2 implies a U-shaped cost-of-capital curve has, of course, been recognized by many writers. A natural further step has been to suggest that the capital structure corresponding to the trough of the U is an "optimal capital structure" towards which management ought to strive in the best interests of the stockholders.[28] According to our model, by contrast, no such optimal structure exists—all structures being equivalent from the point of view of the cost of capital.

Although the falling, or at least U-shaped, cost-of-capital function is in one form or another the dominant view in the literature, the ultimate rationale of that view is by no means clear. The crucial element in the position—that the expected earnings-price ratio of the stock is largely unaffected by leverage up to some conventional limit—is rarely even regarded as something which requires explanation. It is usually simply taken for granted or it is merely asserted that this is the way the market behaves.[29] To the extent that the constant earnings-price ratio has a rationale at all we suspect that it reflects in most cases the feeling that moderate amounts of debt in "sound" corporations do not really add very much to the "riskiness" of the stock. Since the extra risk is slight, it seems natural to suppose that firms will not have to pay noticeably higher yields in order to induce investors to hold the stock.[30]

A more sophisticated line of argument has been advanced by David Durand [3, pp. 231-33]. He suggests that because insurance companies and certain other important institutional investors are restricted to debt securities, nonfinancial corporations are able to borrow from them at interest rates which are lower than would be required to compensate

[27] The U-shaped nature of the cost-of-capital curve can be exhibited explicitly if the yield curve for shares as a function of leverage can be approximated by equation (15) of footnote 25. From that equation, multiplying both sides by S_j we obtain: $\bar{\pi}_j^\tau = \bar{X}_j^\tau - rD_j = i_k^* S_j + \beta D_j + \alpha D_j^2 / S_j$ or, adding and subtracting $i_k^* D_k$ from the right-hand side and collecting terms,

$$\bar{X}_j^\tau = i_k^*(S_j + D_j) + (\beta + r - i_k^*)D_j + \alpha D^2{}_j/S_j. \tag{18}$$

Dividing (18) by V_j gives an expression for the cost of capital:

$$\begin{aligned}\bar{X}_j^\tau/V_j = i_k^* - (i_k^* - r - \beta)D_j/V_j + \alpha D_j^2/S_jV_j = i_k^* - (i_k^* - r - \beta)D_j/V_j \\ + \alpha(D_j/V_j)^2/(1 - D_j/V_j)\end{aligned} \tag{19}$$

which is clearly U-shaped since α is supposed to be positive.

[28] For a typical statement see S. M. Robbins [16, p. 307]. See also Graham and Dodd [6, pp. 468–74].

[29] See *e.g.*, Graham and Dodd [6, p. 466].

[30] A typical statement is the following by Guthmann and Dougall [7, p. 245]: "Theoretically it might be argued that the increased hazard from using bonds and preferred stocks would counterbalance this additional income and so prevent the common stock from being more attractive than when it had a lower return but fewer prior obligations. In practice, the extra earnings from 'trading on the equity' are often regarded by investors as more than sufficient to serve as a 'premium for risk' when the proportions of the several securities are judiciously mixed."

creditors in a free market. Thus, while he would presumably agree with our conclusions that stockholders could not gain from leverage in an unconstrained market, he concludes that they can gain under present institutional arrangements. This gain would arise by virtue of the "safety superpremium" which lenders are willing to pay corporations for the privilege of lending.[31]

The defective link in both the traditional and the Durand version of the argument lies in the confusion between investors' subjective risk preferences and their objective market opportunities. Our Propositions I and II, as noted earlier, do not depend for their validity on any assumption about individual risk preferences. Nor do they involve any assertion as to what is an adequate compensation to investors for assuming a given degree of risk. They rely merely on the fact that a given commodity cannot consistently sell at more than one price in the market; or more precisely that the price of a commodity representing a "bundle" of two other commodities cannot be consistently different from the weighted average of the prices of the two components (the weights being equal to the proportion of the two commodities in the bundle).

An analogy may he helpful at this point. The relations between $1/\rho_k$, the price per dollar of an unlevered stream in class k; $1/r$, the price per dollar of a sure stream, and $1/i_j$, the price per dollar of a levered stream j, in the kth class, are essentially the same as those between, respectively, the price of whole milk, the price of butter fat, and the price of milk which has been thinned out by skimming off some of the butter fat. Our Proposition I states that a firm cannot reduce the cost of capital—*i.e.*, increase the market value of the stream it generates—by securing part of its capital through the sale of bonds, even though debt money appears to be cheaper. This assertion is equivalent to the proposition that, under perfect markets, a dairy farmer cannot in general earn more for the milk he produces by skimming some of the butter fat and selling it separately, even though butter fat per unit weight, sells for more than whole milk. The advantage from skimming the milk rather than selling whole milk would be purely illusory; for what would be gained from selling the high-priced butter fat would be lost in selling the low-priced residue of thinned milk. Similarly our Proposition II—that the price per dollar of a levered stream falls as leverage increases—is an ex-

[31] Like Durand, Morton [15] contends "that the actual market deviates from [Proposition I] by giving a changing over-all cost of money at different points of the [leverage] scale" (p. 443, note 2, inserts ours), but the basis for this contention is nowhere clearly stated. Judging by the great emphasis given to the lack of mobility of investment funds between stocks and bonds and to the psychological and institutional pressures toward debt portfolios (see pp. 444–51 and especially his discussion of the optimal capital structure on p. 453) he would seem to be taking a position very similar to that of Durand above.

act analogue of the statement that the price per gallon of thinned milk falls continuously as more butter fat is skimmed off.[32]

It is clear that this last assertion is true as long as butter fat is worth more per unit weight than whole milk, and it holds even if, for many consumers, taking a little cream out of the milk (adding a little leverage to the stock) does not detract noticeably from the taste (does not add noticeably to the risk). Furthermore the argument remains valid even in the face of instituional limitations of the type envisaged by Durand. For suppose that a large fraction of the population habitually dines in restaurants which are required by law to serve only cream in lieu of milk (entrust their savings to institutional investors who can only buy bonds). To be sure the price of butter fat will then tend to be higher in relation to that of skimmed milk than in the absence such restrictions (the rate of interest will tend to be lower), and this will benefit people who eat at home and who like skim milk (who manage their own portfolio and are able and willing to take risk). But it will still be the case that a farmer cannot gain by skimming some of the butter fat and selling it separately (firm cannot reduce the cost of capital by recourse to borrowed funds).[33]

Our propositions can be regarded as the extension of the classical theory of markets to the particular case of the capital markets. Those who hold the current view—whether they realize it or not—must as-

[32] Let M denote the quantity of whole milk, B/M the proportion of butter fat in the whole milk, and let p_M, p_B and p_α denote, respectively, the price per unit weight of whole milk, butter fat and thinned milk from which a fraction α of the butter fat has been skimmed off. We then have the fundamental perfect market relation:

$$\text{(a)} \qquad p_\alpha(M - \alpha B) + p_B \alpha B = p_M M, \qquad 0 \leq \alpha \leq 1,$$

stating that total receipts will be the same amount $p_M M$, independently of the amount αB of butter fat that may have been sold separately. Since p_M corresponds to $1/\rho$, p_B to $1/r$, p_α to $1/i$, M to $\bar{X}$ and αB to rD, (a) is equivalent to Proposition I, $S+D=\bar{X}/\rho$. From (a) we derive:

$$\text{(b)} \qquad p_\alpha = p_M \frac{M}{M - \alpha B} - p_B \frac{\alpha B}{M - \alpha B}$$

which gives the price of thinned milk as an explicit function of the proportion of butter fat skimmed off; the function decreasing as long as $p_B > p_M$. From (a) also follows:

$$\text{(c)} \qquad 1/p_\alpha = 1/p_M + (1/p_M - 1/p_B) \frac{p_B \alpha B}{p_\alpha(M - \alpha B)}$$

which is the exact analogue of Proposition II, as given by (8).

[33] The reader who likes parables will find that the analogy with interrelated commodity markets can be pushed a good deal farther than we have done in the text. For instance, the effect of changes in the market rate of interest on the over-all cost of capital is the same as the effect of a change in the price of butter on the price of whole milk. Similarly, just as the relation between the prices of skim milk and butter fat influences the kind of cows that will be reared, so the relation between i and r influences the kind of ventures that will be undertaken. If people like butter we shall have Guernseys; if they are willing to pay a high price for safety, this will encourage ventures which promise smaller but less uncertain streams per dollar of physical assets.

sume not merely that there are lags and frictions in the equilibrating process—a feeling we certainly share,[34] claiming for our propositions only that they describe the central tendency around which observations will scatter—but also that there are large and *systematic* imperfections in the market which permanently bias the outcome. This is an assumption that economists, at any rate, will instinctively eye with some skepticism.

In any event, whether such prolonged, systematic departures from equilibrium really exist or whether our propositions are better descriptions of long-run market behavior can be settled only by empirical research. Before going on to the theory of investment it may be helpful, therefore, to look at the evidence.

E. *Some Preliminary Evidence on the Basic Propositions*

Unfortunately the evidence which has been assembled so far is amazingly skimpy. Indeed, we have been able to locate only two recent studies—and these of rather limited scope—which were designed to throw light on the issue. Pending the results of more comprehensive tests which we hope will soon be available, we shall review briefly such evidence as is provided by the two studies in question: (1) an analysis of the relation between security yields and financial structure for some 43 large electric utilities by F. B. Allen [1], and (2) a parallel (unpublished) study by Robert Smith [19], for 42 oil companies designed to test whether Allen's rather striking results would be found in an industry with very different characteristics.[35] The Allen study is based on average figures for the years 1947 and 1948, while the Smith study relates to the single year 1953.

The Effect of Leverage on the Cost of Capital. According to the received view, as shown in equation (17) the average cost of capital, $\overline{X}^{\tau}/V$, should decline linearly with leverage as measured by the ratio D/V, at least through most of the relevant range.[36] According to Proposition I, the average cost of capital within a given class k should tend to have the same value ρ_k^{τ} independently of the degree of leverage. A simple test

[34] Several specific examples of the failure of the arbitrage mechanism can be found in Graham and Dodd [6, *e.g.*, pp. 646–48]. The price discrepancy described on pp. 646–47 is particularly curious since it persists even today despite the fact that a whole generation of security analysts has been brought up on this book!

[35] We wish to express our thanks to both writers for making available to us some of their original worksheets. In addition to these recent studies there is a frequently cited (but apparently seldom read) study by the Federal Communications Commission in 1938 [22] which purports to show the existence of an optimal capital structure or range of structures (in the sense defined above) for public utilities in the 1930's. By current standards for statistical investigations, however, this study cannot be regarded as having any real evidential value for the problem at hand.

[36] We shall simplify our notation in this section by dropping the subscript j used to denote a particular firm wherever this will not lead to confusion.

of the merits of the two alternative hypotheses can thus be carried out by correlating $\bar{X}^{\tau}/V$ with D/V. If the traditional view is correct, the correlation should be significantly negative; if our view represents a better approximation to reality, then the correlation should not be significantly different from zero.

Both studies provide information about the average value of D—the market value of bonds and preferred stock—and of V—the market value of all securities.[37] From these data we can readily compute the ratio D/V and this ratio (expressed as a percentage) is represented by the symbol d in the regression equations below. The measurement of the variable $\bar{X}^{\tau}/V$, however, presents serious difficulties. Strictly speaking, the numerator should measure the expected returns net of taxes, but this is a variable on which no direct information is available. As an approximation, we have followed both authors and used (1) the average value of actual net returns in 1947 and 1948 for Allen's utilities; and (2) actual net returns in 1953 for Smith's oil companies. Net return is defined in both cases as the sum of interest, preferred dividends and stockholders' income net of corporate income taxes. Although this approximation to expected returns is undoubtedly very crude, there is no reason to believe that it will systematically bias the test in so far as the sign of the regression coefficient is concerned. The roughness of the approximation, however, will tend to make for a wide scatter. Also contributing to the scatter is the crudeness of the industrial classification, since especially within the sample of oil companies, the assumption that all the firms belong to the same class in our sense, is at best only approximately valid.

Denoting by x our approximation to $\bar{X}^{\tau}/V$ (expressed, like d, as a percentage), the results of the tests are as follows:

$$\text{Electric Utilities} \quad x = 5.3 + \underset{(\pm .008)}{.006d} \qquad r = .12$$

$$\text{Oil Companies} \quad x = 8.5 + \underset{(\pm .024)}{.006d} \qquad r = .04.$$

The data underlying these equations are also shown in scatter diagram form in Figures 3 and 4.

The results of these tests are clearly favorable to our hypothesis.

[37] Note that for purposes of this test preferred stocks, since they represent an *expected* fixed obligation, are properly classified with bonds even though the tax status of preferred dividends is different from that of interest payments and even though preferred dividends are really fixed only as to their maximum in any year. Some difficulty of classification does arise in the case of convertible preferred stocks (and convertible bonds) selling at a substantial premium, but fortunately very few such issues were involved for the companies included in the two studies. Smith included bank loans and certain other short-term obligations (at book values) in his data on oil company debts and this treatment is perhaps open to some question. However, the amounts involved were relatively small and check computations showed that their elimination would lead to only minor differences in the test results.

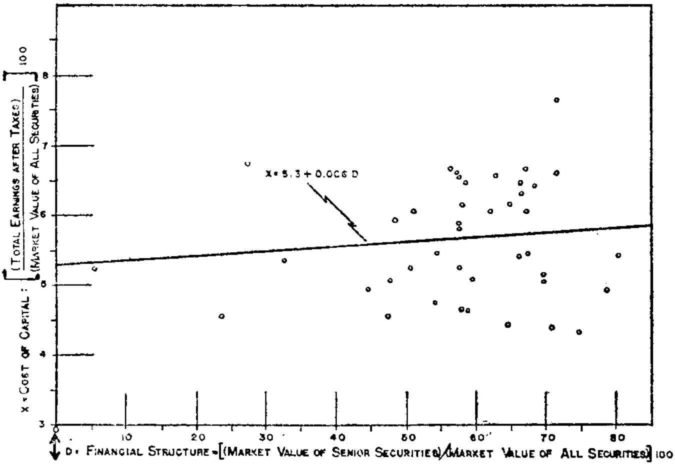

FIGURE 3. COST OF CAPITAL IN RELATION TO FINANCIAL STRUCTURE FOR 43 ELECTRIC UTILITIES, 1947–48

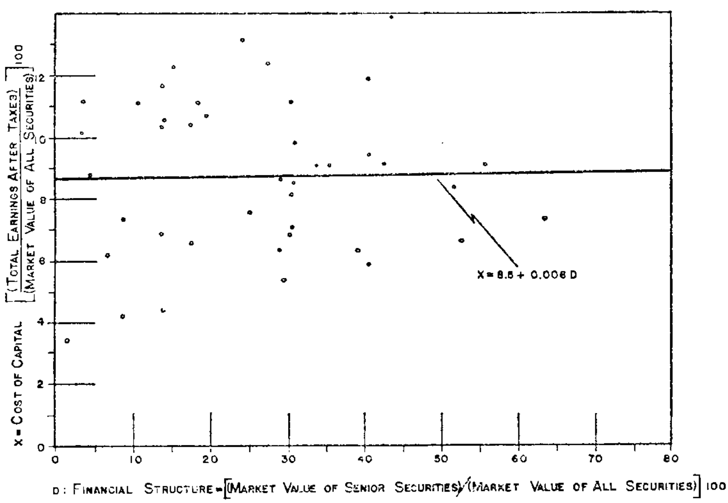

FIGURE 4. COST OF CAPITAL IN RELATION TO FINANCIAL STRUCTURE FOR 42 OIL COMPANIES, 1953

Both correlation coefficients are very close to zero and not statistically significant. Furthermore, the implications of the traditional view fail to be supported even with respect to the sign of the correlation. The data in short provide no evidence of any tendency for the cost of capital to fall as the debt ratio increases.[38]

It should also be apparent from the scatter diagrams that there is no hint of a curvilinear, U-shaped, relation of the kind which is widely believed to hold between the cost of capital and leverage. This graphical impression was confirmed by statistical tests which showed that for both industries the curvature was not significantly different from zero, its sign actually being opposite to that hypothesized.[39]

Note also that according to our model, the constant terms of the regression equations are measures of $\rho_k{}^\tau$, the capitalization rates for unlevered streams and hence the average cost of capital in the classes in question. The estimates of 8.5 per cent for the oil companies as against 5.3 per cent for electric utilities appear to accord well with a priori expectations, both in absolute value and relative spread.

The Effect of Leverage on Common Stock Yields. According to our Proposition II—see equation 12 and Figure 2—the expected yield on common stock, $\bar{\pi}^\tau/S$, in any given class, should tend to increase with leverage as measured by the ratio D/S. The relation should tend to be linear and with positive slope through most of the relevant range (as in the curve MM' of Figure 2), though it might tend to flatten out if we move

[38] It may be argued that a test of the kind used is biased against the traditional view. The fact that both sides of the regression equation are divided by the variable V which may be subject to random variation might tend to impart a positive bias to the correlation. As a check on the results presented in the text, we have, therefore, carried out a supplementary test based on equation (16). This equation shows that, if the traditional view is correct, the market value of a company should, for given $\bar{X}^\tau$, increase with debt through most of the relevant range; according to our model the market value should be uncorrelated with D, given $\bar{X}^\tau$. Because of wide variations in the size of the firms included in our samples, all variables must be divided by a suitable scale factor in order to avoid spurious results in carrying out a test of equation (16). The factor we have used is the book value of the firm denoted by A. The hypothesis tested thus takes the specific form:

$$V/A = a + b(\bar{X}^\tau/A) + c(D/A)$$

and the numerator of the ratio X^τ/A is again approximated by actual net returns. The partial correlation between V/A and D/A should now be positive according to the traditional view and zero according to our model. Although division by A should, if anything, bias the results in favor of the traditional hypothesis, the partial correlation turns out to be only .03 for the oil companies and $-.28$ for the electric utilities. Neither of these coefficients is significantly different from zero and the larger one even has the wrong sign.

[39] The tests consisted of fitting to the data the equation (19) of footnote 27. As shown there, it follows from the U-shaped hypothesis that the coefficient α of the variable $(D/V)^2/(1-D/V)$, denoted hereafter by d^*, should be significant and positive. The following regression equations and partials were obtained:

Electric Utilities $x = 5.0 + .017d - .003d^*;\ r_{xd^*.d} = -.15$

Oil Companies $x = 8.0 + .05d - .03d^*;\ r_{xd^*.d} = -.14.$

far enough to the right (as in the curve MD'), to the extent that high leverage tends to drive up the cost of senior capital. According to the conventional view, the yield curve as a function of leverage should be a horizontal straight line (like ML') through most of the relevant range; far enough to the right, the yield may tend to rise at an increasing rate. Here again, a straight-forward correlation—in this case between $\bar{\pi}^{\tau}/S$ and D/S—can provide a test of the two positions. If our view is correct, the correlation should be significantly positive; if the traditional view is correct, the correlation should be negligible.

Subject to the same qualifications noted above in connection with $\bar{X}^{\tau}$, we can approximate $\bar{\pi}^{\tau}$ by actual stockholder net income.[40] Letting z denote in each case the approximation to $\bar{\pi}^{\tau}/S$ (expressed as a percentage) and letting h denote the ratio D/S (also in percentage terms) the following results are obtained:

$$\text{Electric Utilities}\quad z = 6.6 + \underset{(+.004)}{.017h} \qquad r = .53$$

$$\text{Oil Companies}\quad z = 8.9 + \underset{(\pm .012)}{.051h} \qquad r = .53.$$

These results are shown in scatter diagram form in Figures 5 and 6.

Here again the implications of our analysis seem to be borne out by the data. Both correlation coefficients are positive and highly significant when account is taken of the substantial sample size. Furthermore, the estimates of the coefficients of the equations seem to accord reasonably well with our hypothesis. According to equation (12) the constant term should be the value of $\rho_k{}^{\tau}$ for the given class while the slope should be $(\rho_k{}^{\tau}-r)$. From the test of Proposition I we have seen that for the oil companies the mean value of $\rho_k{}^{\tau}$ could be estimated at around 8.7. Since the average yield of senior capital during the period covered was in the order of $3\frac{1}{2}$ per cent, we should expect a constant term of about 8.7 per cent and a slope of just over 5 per cent. These values closely approximate the regression estimates of 8.9 per cent and 5.1 per cent respectively. For the electric utilities, the yield of senior capital was also on the order of $3\frac{1}{2}$ per cent during the test years, but since the estimate of the mean value of $\rho_k{}^{\tau}$ from the test of Proposition I was 5.6 per cent,

[40] As indicated earlier, Smith's data were for the single year 1953. Since the use of a single year's profits as a measure of expected profits might be open to objection we collected profit data for 1952 for the same companies and based the computation of $\bar{\pi}^{\tau}/S$ on the average of the two years. The value of $\bar{\pi}^{\tau}/S$ was obtained from the formula:

$$\left(\text{net earnings in 1952}\cdot\frac{\text{assets in '53}}{\text{assets in '52}} + \text{net earnings in '1953}\right)\frac{1}{2}$$
$$\div \text{ (average market value of common stock in '53).}$$

The asset adjustment was introduced as rough allowance for the effects of possible growth in the size of the firm. It might be added that the correlation computed with $\bar{\pi}^{\tau}/S$ based on net profits in 1953 alone was found to be only slightly smaller, namely .50.

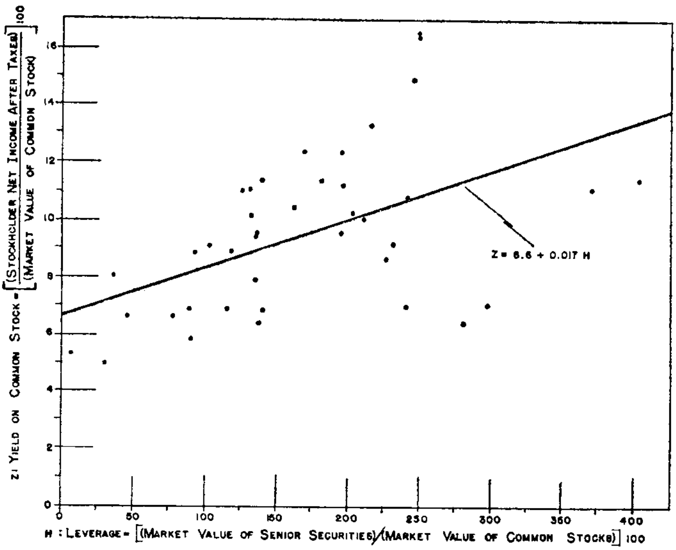

FIGURE 5. YIELD ON COMMON STOCK IN RELATION TO LEVERAGE FOR 43 ELECTRIC UTILITIES, 1947–48

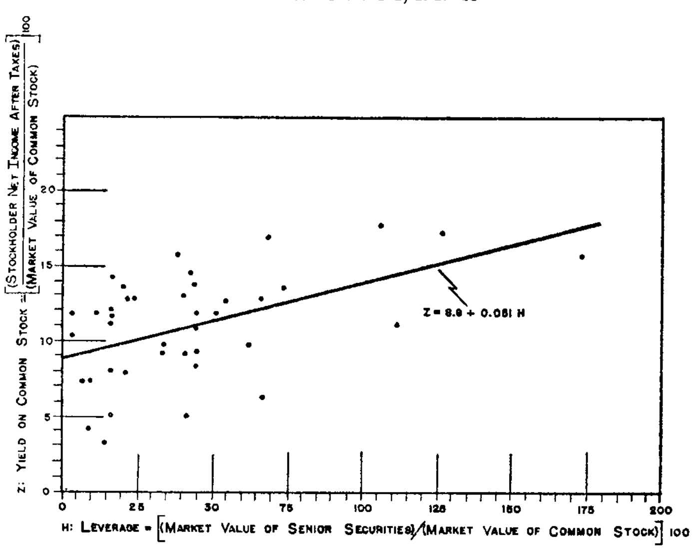

FIGURE 6. YIELD ON COMMON STOCK IN RELATION TO LEVERAGE FOR 42 OIL COMPANIES, 1952–53

the slope should be just above 2 per cent. The actual regression estimate for the slope of 1.7 per cent is thus somewhat low, but still within one standard error of its theoretical value. Because of this underestimate of the slope and because of the large mean value of leverage ($\bar{h}=160$ per cent) the regression estimate of the constant term, 6.6 per cent, is somewhat high, although not significantly different from the value of 5.6 per cent obtained in the test of Proposition I.

When we add a square term to the above equations to test for the presence and direction of curvature we obtain the following estimates:

$$\text{Electric Utilities} \quad z = 4.6 + .004h - .007h^2$$

$$\text{Oil Companies} \quad z = 8.5 + .072h - .016h^2.$$

For both cases the curvature is negative. In fact, for the electric utilities, where the observations cover a wider range of leverage ratios, the negative coefficient of the square term is actually significant at the 5 per cent level. Negative curvature, as we have seen, runs directly counter to the traditional hypothesis, whereas it can be readily accounted for by our model in terms of rising cost of borrowed funds.[41]

In summary, the empirical evidence we have reviewed seems to be broadly consistent with our model and largely inconsistent with traditional views. Needless to say much more extensive testing will be required before we can firmly conclude that our theory describes market behavior. Caution is indicated especially with regard to our test of Proposition II, partly because of possible statistical pitfalls[42] and partly because not all the factors that might have a systematic effect on stock yields have been considered. In particular, no attempt was made to test the possible influence of the dividend pay-out ratio whose role has tended to receive a great deal of attention in current research and thinking. There are two reasons for this omission. First, our main objective has been to assess the prima facie tenability of *our* model, and in this model, based as it is on rational behavior by investors, dividends per se play no role. Second, in a world in which the policy of dividend stabilization is widespread, there is no simple way of disentangling the true effect of dividend payments on stock prices from their apparent effect,

[41] That the yield of senior capital tended to rise for utilities as leverage increased is clearly shown in several of the scatter diagrams presented in the published version of Allen's study. This significant negative curvature between stock yields and leverage for utilities may be partly responsible for the fact, previously noted, that the constant in the linear regression is somewhat higher and the slope somewhat lower than implied by equation (12). Note also in connection with the estimate of ρ_k^τ that the introduction of the quadratic term reduces the constant considerably, pushing it in fact below the a priori expectation of 5.6, though the difference is again not statistically significant.

[42] In our test, *e.g.*, the two variables z and h are both ratios with S appearing in the denominator, which may tend to impart a positive bias to the correlation (*cf.* note 38). Attempts were made to develop alternative tests, but although various possibilities were explored, we have so far been unable to find satisfactory alternatives.

the latter reflecting only the role of dividends as a proxy measure of long-term earning anticipations.[43] The difficulties just mentioned are further compounded by possible interrelations between dividend policy and leverage.[44]

II. *Implications of the Analysis for the Theory of Investment*

A. *Capital Structure and Investment Policy*

On the basis of our propositions with respect to cost of capital and financial structure (and for the moment neglecting taxes), we can derive the following simple rule for optimal investment policy by the firm:

Proposition III. If a firm in class k is acting in the best interest of the stockholders at the time of the decision, it will exploit an investment opportunity if and only if the rate of return on the investment, say ρ^*, is as large as or larger than ρ_k. That is, *the cut-off point for investment in the firm will in all cases be ρ_k and will be completely unaffected by the type of security used to finance the investment.* Equivalently, we may say that regardless of the financing used, the marginal cost of capital to a firm is equal to the average cost of capital, which is in turn equal to the capitalization rate for an unlevered stream in the class to which the firm belongs.[45]

To establish this result we will consider the three major financing alternatives open to the firm—bonds, retained earnings, and common stock issues—and show that in each case an investment is worth undertaking if, and only if, $\rho^* \geqq \rho_k$.[46]

Consider first the case of an investment financed by the sale of bonds. We know from Proposition I that the market value of the firm before the investment was undertaken was:[47]

$$V_0 = \overline{X}_0/\rho_k \tag{20}$$

[43] We suggest that failure to appreciate this difficulty is responsible for many fallacious, or at least unwarranted, conclusions about the role of dividends.

[44] In the sample of electric utilities, there is a substantial negative correlation between yields and pay-out ratios, but also between pay-out ratios and leverage, suggesting that either the association of yields and leverage or of yields and pay-out ratios may be (at least partly) spurious. These difficulties however do not arise in the case of the oil industry sample. A preliminary analysis indicates that there is here no significant relation between leverage and pay-out ratios and also no significant correlation (either gross or partial) between yields and pay-out ratios.

[45] The analysis developed in this paper is essentially a comparative-statics, not a dynamic analysis. This note of caution applies with special force to Proposition III. Such problems as those posed by expected changes in r and in ρ_k over time will not be treated here. Although they are in principle amenable to analysis within the general framework we have laid out, such an undertaking is sufficiently complex to deserve separate treatment. *Cf.* note 17.

[46] The extension of the proof to other types of financing, such as the sale of preferred stock or the issuance of stock rights is straightforward.

[47] Since no confusion is likely to arise, we have again, for simplicity, eliminated the subscripts identifying the firm in the equations to follow. Except for ρ_k, the subscripts now refer to time periods.

and that the value of the common stock was:

$$S_0 = V_0 - D_0. \tag{21}$$

If now the firm borrows I dollars to finance an investment yielding ρ^* its market value will become:

$$V_1 = \frac{\overline{X}_0 + \rho^* I}{\rho_k} = V_0 + \frac{\rho^* I}{\rho_k} \tag{22}$$

and the value of its common stock will be:

$$S_1 = V_1 - (D_0 + I) = V_0 + \frac{\rho^* I}{\rho_k} - D_0 - I \tag{23}$$

or using equation 21,

$$S_1 = S_0 + \frac{\rho^* I}{\rho_k} - I. \tag{24}$$

Hence $S_1 \gtreqless S_0$ as $\rho^* \gtreqless \rho_k$.[48]

To illustrate, suppose the capitalization rate for uncertain streams in the kth class is 10 per cent and the rate of interest is 4 per cent. Then if a given company had an expected income of 1,000 and if it were financed entirely by common stock we know from Proposition I that the market value of its stock would be 10,000. Assume now that the managers of the firm discover an investment opportunity which will require an outlay of 100 and which is expected to yield 8 per cent. At first sight this might appear to be a profitable opportunity since the expected return is double the interest cost. If, however, the management borrows the necessary 100 at 4 per cent, the total expected income of the company rises to 1,008 and the market value of the firm to 10,080. But the firm now will have 100 of bonds in its capital structure so that, paradoxically, the market value of the stock must actually be reduced from 10,000 to 9,980 as a consequence of this apparently profitable investment. Or, to put it another way, the gains from being able to tap cheap, borrowed funds are more than offset for the stockholders by the market's discounting of the stock for the added leverage assumed.

Consider next the case of retained earnings. Suppose that in the course of its operations the firm acquired I dollars of cash (without impairing

[48] In the case of bond-financing the rate of interest on bonds does not enter explicitly into the decision (assuming the firm borrows at the market rate of interest). This is true, moreover, given the conditions outlined in Section I.C, even though interest rates may be an increasing function of debt outstanding. To the extent that the firm borrowed at a rate other than the market rate the two I's in equation (24) would no longer be identical and an additional gain or loss, as the case might be, would accrue to the shareholders. It might also be noted in passing that permitting the two I's in (24) to take on different values provides a simple method for introducing underwriting expenses into the analysis.

the earning power of its assets). If the cash is distributed as a dividend to the stockholders their wealth W_0, after the distribution will be:

$$W_0 = S_0 + I = \frac{\overline{X}_0}{\rho_k} - D_0 + I \tag{25}$$

where $\overline{X}_0$ represents the expected return from the assets exclusive of the amount I in question. If however the funds are retained by the company and used to finance new assets whose expected rate of return is ρ^*, then the stockholders' wealth would become:

$$W_1 = S_1 = \frac{\overline{X}_0 + \rho^* I}{\rho_k} - D_0 = S_0 + \frac{\rho^* I}{\rho_k} \cdot \tag{26}$$

Clearly $W_1 \gtreqless W_0$ as $\rho^* \gtreqless \rho_k$ so that an investment financed by retained earnings raises the net worth of the owners if and only if $\rho^* > \rho_k$.[49]

Consider finally, the case of common-stock financing. Let P_0 denote the current market price per share of stock and assume, for simplicity, that this price reflects currently expected earnings only, that is, it does not reflect any future increase in earnings as a result of the investment under consideration.[50] Then if N is the original number of shares, the price per share is:

$$P_0 = S_0/N \tag{27}$$

and the number of new shares, M, needed to finance an investment of I dollars is given by:

$$M = \frac{I}{P_0} \cdot \tag{28}$$

As a result of the investment the market value of the stock becomes:

$$S_1 = \frac{\overline{X}_0 + \rho^* I}{\rho_k} - D_0 = S_0 + \frac{\rho^* I}{\rho_k} = NP_0 + \frac{\rho^* I}{\rho_k}$$

and the price per share:

$$P_1 = \frac{S_1}{N + M} = \frac{1}{N + M}\left[NP_0 + \frac{\rho^* I}{\rho_k}\right]. \tag{29}$$

[49] The conclusion that ρ_k is the cut-off point for investments financed from internal funds applies not only to undistributed net profits, but to depreciation allowances (and even to the funds represented by the current sale value of any asset or collection of assets). Since the owners can earn ρ_k by investing funds elsewhere in the class, partial or total liquidating distributions should be made whenever the firm cannot achieve a marginal internal rate of return equal to ρ_k.

[50] If we assumed that the market price of the stock did reflect the expected higher future earnings (as would be the case if our original set of assumptions above were strictly followed) the analysis would differ slightly in detail, but not in essentials. The cut-off point for new investment would still be ρ_k, but where $\rho^* > \rho_k$ the gain to the original owners would be larger than if the stock price were based on the pre-investment expectations only.

Since by equation (28), $I = MP_0$, we can add MP_0 and subtract I from the quantity in bracket, obtaining:

$$(30)\qquad \begin{aligned} P_1 &= \frac{1}{N+M}\left[(N+M)P_0 + \frac{\rho^* - \rho_k}{\rho_k} I\right] \\ &= P_0 + \frac{1}{N+M}\,\frac{\rho^* - \rho_k}{\rho_k} I > P_0 \text{ if,} \end{aligned}$$

and only if, $\rho^* > \rho_k$.

Thus an investment financed by common stock is advantageous to the current stockholders if and only if its yield exceeds the capitalization rate ρ_k.

Once again a numerical example may help to illustrate the result and make it clear why the relevant cut-off rate is ρ_k and not the current yield on common stock, i. Suppose that ρ_k is 10 per cent, r is 4 per cent, that the original expected income of our company is 1,000 and that management has the opportunity of investing 100 having an expected yield of 12 per cent. If the original capital structure is 50 per cent debt and 50 per cent equity, and 1,000 shares of stock are initially outstanding, then, by Proposition I, the market value of the common stock must be 5,000 or 5 per share. Furthermore, since the interest bill is $.04 \times 5{,}000 = 200$, the yield on common stock is $800/5{,}000 = 16$ per cent. It may then appear that financing the additional investment of 100 by issuing 20 shares to outsiders at 5 per share would dilute the equity of the original owners since the 100 promises to yield 12 per cent whereas the common stock is currently yielding 16 per cent. Actually, however, the income of the company would rise to 1,012; the value of the firm to 10,120; and the value of the common stock to 5,120. Since there are now 1,020 shares, each would be worth 5.02 and the wealth of the original stockholders would thus have been increased. What has happened is that the dilution in expected earnings per share (from .80 to .796) has been more than offset, in its effect upon the market price of the shares, by the decrease in leverage.

Our conclusion is, once again, at variance with conventional views,[51] so much so as to be easily misinterpreted. Read hastily, Proposition III seems to imply that the capital structure of a firm is a matter of indifference; and that, consequently, one of the core problems of corporate finance—the problem of the optimal capital structure for a firm—is no problem at all. It may be helpful, therefore, to clear up such possible misundertandings.

[51] In the matter of investment policy under uncertainty there is no single position which represents "accepted" doctrine. For a sample of current formulations, all very different from ours, see Joel Dean [2, esp. Ch. 3], M. Gordon and E. Shapiro [5], and Harry Roberts [17].

B. *Proposition III and Financial Planning by Firms*

Misinterpretation of the scope of Proposition III can be avoided by remembering that this Proposition tells us only that the type of instrument used to finance an investment is irrelevant to the question of whether or not the investment is worth while. This does not mean that the owners (or the managers) have no grounds whatever for preferring one financing plan to another; or that there are no other policy or technical issues in finance at the level of the firm.

That grounds for preferring one type of financial structure to another will still exist within the framework of our model can readily be seen for the case of common-stock financing. In general, except for something like a widely publicized oil-strike, we would expect the market to place very heavy weight on current and recent past earnings in forming expectations as to future returns. Hence, if the owners of a firm discovered a major investment opportunity which they felt would yield much more than ρ_k, they might well prefer not to finance it via common stock at the then ruling price, because this price may fail to capitalize the new venture. A better course would be a pre-emptive issue of stock (and in this connection it should be remembered that stockholders are free to borrow and buy). Another possibility would be to finance the project initially with debt. Once the project had reflected itself in increased actual earnings, the debt could be retired either with an equity issue at much better prices or through retained earnings. Still another possibility along the same lines might be to combine the two steps by means of a convertible debenture or preferred stock, perhaps with a progressively declining conversion rate. Even such a double-stage financing plan may possibly be regarded as yielding too large a share to outsiders since the new stockholders are, in effect, being given an interest in any similar opportunities the firm may discover in the future. If there is a reasonable prospect that even larger opportunities may arise in the near future and if there is some danger that borrowing now would preclude more borrowing later, the owners might find their interests best protected by splitting off the current opportunity into a separate subsidiary with independent financing. Clearly the problems involved in making the crucial estimates and in planning the optimal financial strategy are by no means trivial, even though they should have no bearing on the basic decision to invest (as long as $\rho^* \geqq \rho_k$).[52]

Another reason why the alternatives in financial plans may not be a matter of indifference arises from the fact that managers are concerned

[52] Nor can we rule out the possibility that the existing owners, if unable to use a financing plan which protects their interest, may actually prefer to pass up an otherwise profitable venture rather than give outsiders an "excessive" share of the business. It is presumably in situations of this kind that we could justifiably speak of a shortage of "equity capital," though this kind of market imperfection is likely to be of significance only for small or new firms.

with more than simply furthering the interest of the owners. Such other objectives of the management—which need not be necessarily in conflict with those of the owners—are much more likely to be served by some types of financing arrangements than others. In many forms of borrowing agreements, for example, creditors are able to stipulate terms which the current management may regard as infringing on its prerogatives or restricting its freedom to maneuver. The creditors might even be able to insist on having a direct voice in the formation of policy.[53] To the extent, therefore, that financial policies have these implications for the management of the firm, something like the utility approach described in the introductory section becomes relevant to financial (as opposed to investment) decision-making. It is, however, the utility functions of the managers per se and not of the owners that are now involved.[54]

In summary, many of the specific considerations which bulk so large in traditional discussions of corporate finance can readily be superimposed on our simple framework without forcing any drastic (and certainly no systematic) alteration of the conclusion which is our principal concern, namely that for investment decisions, the marginal cost of capital is ρ_k.

C. *The Effect of the Corporate Income Tax on Investment Decisions*

In Section I it was shown that when an unintegrated corporate income tax is introduced, the original version of our Proposition I,

$$\overline{X}/V = \rho_k = \text{a constant}$$

must be rewritten as:

$$(11) \qquad \frac{(\overline{X} - rD)(1 - \tau) + rD}{V} \equiv \frac{\overline{X}^\tau}{V} = \rho_k^\tau = \text{a constant.}$$

Throughout Section I we found it convenient to refer to $\overline{X}^\tau/V$ as the cost of capital. The appropriate measure of the cost of capital relevant

[53] Similar considerations are involved in the matter of dividend policy. Even though the stockholders may be indifferent as to payout policy as long as investment policy is optimal, the management need not be so. Retained earnings involve far fewer threats to control than any of the alternative sources of funds and, of course, involve no underwriting expense or risk. But against these advantages management must balance the fact that sharp changes in dividend rates, which heavy reliance on retained earnings might imply, may give the impression that a firm's finances are being poorly managed, with consequent threats to the control and professional standing of the management.

[54] In principle, at least, this introduction of management's risk preferences with respect to financing methods would do much to reconcile the apparent conflict between Proposition III and such empirical findings as those of Modigliani and Zeman [14] on the close relation between interest rates and the ratio of new debt to new equity issues; or of John Lintner [12] on the considerable stability in target and actual dividend-payout ratios.

to investment decisions, however, is the ratio of the expected return *before* taxes to the market value, *i.e.*, $\bar{X}/V$. From (11) above we find:

$$\frac{\bar{X}}{V} = \frac{\rho_k^{\tau} - \tau_r(D/V)}{1-\tau} = \frac{\rho_k^{\tau}}{1-\tau}\left[1 - \frac{\tau r D}{\rho_k^{\tau} V}\right], \tag{31}$$

which shows that the cost of capital now depends on the debt ratio, decreasing, as D/V rises, at the constant rate $\tau r/(1-\tau)$.[55] Thus, with a corporate income tax under which interest is a deductible expense, gains can accrue to stockholders from having debt in the capital structure, even when capital markets are perfect. The gains however are small, as can be seen from (31), and as will be shown more explicitly below.

From (31) we can develop the tax-adjusted counterpart of Proposition III by interpreting the term D/V in that equation as the proportion of debt used in any additional financing of V dollars. For example, in the case where the financing is entirely by new common stock, $D=0$ and the required rate of return ρ_k^S on a venture so financed becomes:

$$\rho_k^S = \frac{\rho_k^{\tau}}{1-\tau}. \tag{32}$$

For the other extreme of pure debt financing $D=V$ and the required rate of return, ρ_k^D, becomes:

$$\rho_k^D = \frac{\rho_k^{\tau}}{1-\tau}\left[1 - \tau\frac{r}{\rho_k^{\tau}}\right] = \rho_k^S\left[1 - \tau\frac{r}{\rho_k^{\tau}}\right] = \rho_k^S - \frac{\tau}{1-\tau}r.^{56} \tag{33}$$

For investments financed out of retained earnings, the problem of defining the required rate of return is more difficult since it involves a comparison of the tax consequences to the individual stockholder of receiving a dividend versus having a capital gain. Depending on the time of realization, a capital gain produced by retained earnings may be taxed either at ordinary income tax rates, 50 per cent of these rates, 25 per

[55] Equation (31) is amenable, in principle, to statistical tests similar to those described in Section I.E. However we have not made any systematic attempt to carry out such tests so far, because neither the Allen nor the Smith study provides the required information. Actually, Smith's data included a very crude estimate of tax liability, and, using this estimate, we did in fact obtain a negative relation between $\bar{X}/V$ and D/V. However, the correlation (—.28) turned out to be significant only at about the 10 per cent level. While this result is not conclusive, it should be remembered that, according to our theory, the slope of the regression equation should be in any event quite small. In fact, with a value of τ in the order of .5, and values of ρ_k^{τ} and r in the order of 8.5 and 3.5 per cent respectively (*cf.* Section I.E) an increase in D/V from 0 to 60 per cent (which is, approximately, the range of variation of this variable in the sample) should tend to reduce the average cost of capital only from about 17 to about 15 per cent.

[56] This conclusion does not extend to preferred stocks even though they have been classed with debt issues previously. Since preferred dividends except for a portion of those of public utilities are not in general deductible from the corporate tax, the cut-off point for new financing via preferred stock is exactly the same as that for common stock.

cent, or zero, if held till death. The rate on any dividends received in the event of a distribution will also be a variable depending on the amount of other income received by the stockholder, and with the added complications introduced by the current dividend-credit provisions. If we assume that the managers proceed on the basis of reasonable estimates as to the average values of the relevant tax rates for the owners, then the required return for retained earnings ρ_k^R can be shown to be:

$$\rho_k^R = \rho_k^\tau \frac{1}{1-\tau} \frac{1-\tau_d}{1-\tau_g} = \frac{1-\tau_d}{1-\tau_g} \rho_k^s \qquad (34)$$

where τ_d is the assumed rate of personal income tax on dividends and τ_g is the assumed rate of tax on capital gains.

A numerical illustration may perhaps be helpful in clarifying the relationship between these required rates of return. If we take the following round numbers as representative order-of-magnitude values under present conditions: an after-tax capitalization rate ρ_k^τ of 10 per cent, a rate of interest on bonds of 4 per cent, a corporate tax rate of 50 per cent, a marginal personal income tax rate on dividends of 40 per cent (corresponding to an income of about $25,000 on a joint return), and a capital gains rate of 20 per cent (one-half the marginal rate on dividends), then the required rates of return would be: (1) 20 per cent for investments financed entirely by issuance of new common shares; (2) 16 per cent for investments financed entirely by new debt; and (3) 15 per cent for investments financed wholly from internal funds.

These results would seem to have considerable significance for current discussions of the effect of the corporate income tax on financial policy and on investment. Although we cannot explore the implications of the results in any detail here, we should at least like to call attention to the remarkably small difference between the "cost" of equity funds and debt funds. With the numerical values assumed, equity money turned out to be only 25 per cent more expensive than debt money, rather than something on the order of 5 times as expensive as is commonly supposed to be the case.[57] The reason for the wide difference is that the traditional

[57] See *e.g.*, D. T. Smith [18]. It should also be pointed out that our tax system acts in other ways to reduce the gains from debt financing. Heavy reliance on debt in the capital structure, for example, commits a company to paying out a substantial proportion of its income in the form of interest payments taxable to the owners under the personal income tax. A debt-free company, by contrast, can reinvest in the business all of its (smaller) net income and to this extent subject the owners only to the low capital gains rate (or possibly no tax at all by virtue of the loophole at death). Thus, we should expect a high degree of leverage to be of value to the owners, even in the case of closely held corporations, primarily in cases where their firm was not expected to have much need for additional funds to expand assets and earnings in the future. To the extent that opportunities for growth were available, as they presumably would be for most successful corporations, the interest of the stockholders would tend to be better served by a structure which permitted maximum use of retained earnings.

view starts from the position that debt funds are several times cheaper than equity funds even in the absence of taxes, with taxes serving simply to magnify the cost ratio in proportion to the corporate rate. By contrast, in our model in which the repercussions of debt financing on the value of shares are taken into account, the *only* difference in cost is that due to the tax effect, and its magnitude is simply the tax on the "grossed up" interest payment. Not only is this magnitude likely to be small but our analysis yields the further paradoxical implication that the stockholders' gain from, and hence incentive to use, debt financing is actually smaller the lower the rate of interest. In the extreme case where the firm could borrow for practically nothing, the advantage of debt financing would also be practically nothing.

III. *Conclusion*

With the development of Proposition III the main objectives we outlined in our introductory discussion have been reached. We have in our Propositions I and II at least the foundations of a theory of the valuation of firms and shares in a world of uncertainty. We have shown, moreover, how this theory can lead to an operational definition of the cost of capital and how that concept can be used in turn as a basis for rational investment decision-making within the firm. Needless to say, however, much remains to be done before the cost of capital can be put away on the shelf among the solved problems. Our approach has been that of static, partial equilibrium analysis. It has assumed among other things a state of atomistic competition in the capital markets and an ease of access to those markets which only a relatively small (though important) group of firms even come close to possessing. These and other drastic simplifications have been necessary in order to come to grips with the problem at all. Having served their purpose they can now be relaxed in the direction of greater realism and relevance, a task in which we hope others interested in this area will wish to share.

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equanimity a writing-down of the value of their reserves, or unless one is prepared to forego the possibility of exchange-rate adjustment, any major extension of the gold exchange standard is dependent upon the introduction of guarantees. It is misleading to suggest that the multiple key-currency system is an alternative to a guarantee, as implied by Roosa [6, pp. 5-7 and 9-12].

IV. *Conclusion*

The most noteworthy conclusion to be drawn from this analysis is that the successful operation of a multiple key-currency system would require both exchange guarantees and continuing cooperation between central bankers of a type that would effectively limit their choice as to the form in which they hold their reserves. Yet these are two of the conditions whose undesirability has frequently been held to be an obstacle to implementation of the alternative proposal to create a world central bank. The multiple key-currency proposal represents an attempt to avoid the impracticality supposedly associated with a world central bank, but if both proposals in fact depend on the fulfillment of similar conditions, it is difficult to convince oneself that the sacrifice of the additional liquidity that an almost closed system would permit is worth while. Unless, of course, the object of the exercise is to reinforce discipline rather than to expand liquidity.

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Corporate Income Taxes and the Cost of Capital: A Correction

The purpose of this communication is to correct an error in our paper "The Cost of Capital, Corporation Finance and the Theory of Investment" (this *Review*, June 1958). In our discussion of the effects of the present method of taxing corporations on the valuation of firms, we said (p. 272):

> The deduction of interest in computing taxable corporate profits will prevent the arbitrage process from making the value of all firms in a given class proportional to the expected returns generated by their

physical assets. Instead, it can be shown (by the same type of proof used for the original version of Proposition I) that *the market values of firms in each class must be proportional in equilibrium to their expected returns net of taxes* (*that is, to the sum of the interest paid and expected net stockholder income*). (Italics added.)

The statement in italics, unfortunately, is wrong. For even though one firm may have an *expected* return after taxes (our $\bar{X}^\tau$) twice that of another firm in the same risk-equivalent class, it will not be the case that the *actual* return after taxes (our X^τ) of the first firm will always be twice that of the second, if the two firms have different degrees of leverage.[1] And since the distribution of returns after taxes of the two firms will not be proportional, there can be no "arbitrage" process which forces their values to be proportional to their expected after-tax returns.[2] In fact, it can be shown—and this time it really will be shown—that "arbitrage" will make values within any class a function not only of expected after-tax returns, but of the tax rate and the degree of leverage. This means, among other things, that the tax advantages of debt financing are somewhat greater than we originally suggested and, to this extent, the quantitative difference between the valuations implied by our position and by the traditional view is narrowed. It still remains true, however, that under our analysis the tax advantages of debt are the *only* permanent advantages so that the gulf between the two views in matters of interpretation and policy is as wide as ever.

I. *Taxes, Leverage, and the Probability Distribution of After-Tax Returns*

To see how the distribution of after-tax earnings is affected by leverage, let us again denote by the random variable X the (long-run average) earnings before interest and taxes generated by the currently owned assets of a given firm in some stated risk class, k.[3] From our definition of a risk class it follows that X can be expressed in the form $\bar{X}Z$, where $\bar{X}$ is the expected value of X, and the random variable $Z = X/\bar{X}$, having the same value for all firms in class k, is a drawing from a distribution, say $f_k(Z)$. Hence the

[1] With some exceptions, which will be noted when they occur, we shall preserve here both the notation and the terminology of the original paper. A working knowledge of both on the part of the reader will be presumed.

[2] Barring, of course, the trivial case of universal linear utility functions. Note that in deference to Professor Durand (see his Comment on our paper and our reply, this *Review*, Sept. 1959, *49*, 639–69) we here and throughout use quotation marks when referring to arbitrage.

[3] Thus our X corresponds essentially to the familiar EBIT concept of the finance literature. The use of EBIT and related "income" concepts as the basis of valuation is strictly valid only when the underlying real assets are assumed to have perpetual lives. In such a case, of course, EBIT and "cash flow" are one and the same. This was, in effect, the interpretation of X we used in the original paper and we shall retain it here both to preserve continuity and for the considerable simplification it permits in the exposition. We should point out, however, that the perpetuity interpretation is much less restrictive than might appear at first glance. Before-tax cash flow and EBIT can also safely be equated even where assets have finite lives as soon as these assets attain a steady state age distribution in which annual replacements equal annual depreciation. The subject of finite lives of assets will be further discussed in connection with the problem of the cut-off rate for investment decisions.

random variable X^τ, measuring the after-tax return, can be expressed as:

$$(1)\quad X^\tau = (1-\tau)(X-R) + R = (1-\tau)X + \tau R = (1-\tau)\overline{X}Z + \tau R$$

where τ is the marginal corporate income tax rate (assumed equal to the average), and R is the interest bill. Since $E(X^\tau) \equiv \overline{X}^\tau = (1-\tau)\overline{X} + \tau R$ we can substitute $\overline{X}^\tau - \tau R$ for $(1-\tau)\overline{X}$ in (1) to obtain:

$$(2)\qquad X^\tau = (\overline{X}^\tau - \tau R)Z + \tau R = \overline{X}^\tau\left(1 - \frac{\tau R}{\overline{X}^\tau}\right)Z + \tau R.$$

Thus, if the tax rate is other than zero, the shape of the distribution of X^τ will depend not only on the "scale" of the stream $\overline{X}^\tau$ and on the distribution of Z, but also on the tax rate and the degree of leverage (one measure of which is $R/\overline{X}^\tau$). For example, if Var $(Z) = \sigma^2$, we have:

$$\text{Var}\,(X^\tau) = \sigma^2(\overline{X}^\tau)^2\left(1 - \tau\frac{R}{\overline{X}^\tau}\right)^2$$

implying that for given $\overline{X}^\tau$ the variance of after-tax returns is smaller, the higher τ and the degree of leverage.[4]

II. *The Valuation of After-Tax Returns*

Note from equation (1) that, from the investor's point of view, the long-run average stream of after-tax returns appears as a sum of two components: (1) an uncertain stream $(1-\tau)\overline{X}Z$; and (2) a sure stream τR.[5] This suggests that the equilibrium market value of the combined stream can be found by capitalizing each component separately. More precisely, let ρ^τ be the rate at which the market capitalizes the expected returns net of tax of an unlevered company of size $\overline{X}$ in class k, i.e.,

$$\rho^\tau = \frac{(1-\tau)\overline{X}}{V_U} \quad \text{or} \quad V_U = \frac{(1-\tau)\overline{X}}{\rho^\tau};^6$$

[4] It may seem paradoxical at first to say that leverage *reduces* the variability of outcomes, but remember we are here discussing the variability of total returns, interest plus net profits. The variability of stockholder net profits will, of course, be greater in the presence than in the absence of leverage, though relatively less so than in an otherwise comparable world of no taxes. The reasons for this will become clearer after the discussion in the next section.

[5] The statement that τR—the tax saving per period on the interest payments—is a sure stream is subject to two qualifications. First, it must be the case that firms can always obtain the tax benefit of their interest deductions either by offsetting them directly against other taxable income in the year incurred; or, in the event no such income is available in any given year, by carrying them backward or forward against past or future taxable earnings; or, in the extreme case, by merger of the firm with (or its sale to) another firm that can utilize the deduction. Second, it must be assumed that the tax rate will remain the same. To the extent that neither of these conditions holds exactly then some uncertainty attaches even to the tax savings, though, of course, it is of a different kind and order from that attaching to the stream generated by the assets. For simplicity, however, we shall here ignore these possible elements of delay or of uncertainty in the tax saving; but it should be kept in mind that this neglect means that the subsequent valuation formulas overstate, if anything, the value of the tax saving for any given permanent level of debt.

[6] Note that here, as in our original paper, we neglect dividend policy and "growth" in the

and let r be the rate at which the market capitalizes the sure streams generated by debts. For simplicity, assume this rate of interest is a constant independent of the size of the debt so that

$$r = \frac{R}{D} \quad \text{or} \quad D = \frac{R}{r}.^{7}$$

Then we would expect the value of a levered firm of size $\overline{X}$, with a permanent level of debt D_L in its capital structure, to be given by:

$$V_L = \frac{(1-\tau)\overline{X}}{\rho\tau} + \frac{\tau R}{r} = V_U + \tau D_L.^{8} \tag{3}$$

In our original paper we asserted instead that, within a risk class, market value would be proportional to expected after-tax return $\overline{X}^{\tau}$ (cf. our original equation [11]), which would imply:

$$V_L = \frac{\overline{X}^{\tau}}{\rho^{\tau}} = \frac{(1-\tau)\overline{X}}{\rho^{\tau}} + \frac{\tau R}{\rho^{\tau}} = V_U + \frac{r}{\rho^{\tau}}\tau D_L. \tag{4}$$

We will now show that if (3) does not hold, investors can secure a more efficient portfolio by switching from relatively overvalued to relatively undervalued firms. Suppose first that unlevered firms are overvalued or that

$$V_L - \tau D_L < V_U.$$

An investor holding m dollars of stock in the unlevered company has a right to the fraction m/V_U of the eventual outcome, i.e., has the uncertain income

$$Y_U = \left(\frac{m}{V_U}\right)(1-\tau)\overline{X}Z.$$

Consider now an alternative portfolio obtained by investing m dollars as follows: (1) the portion,

$$m\left(\frac{S_L}{S_L + (1-\tau)D_L}\right),$$

is invested in the stock of the levered firm, S_L; and (2) the remaining portion,

$$m\left(\frac{(1-\tau)D_L}{S_L + (1-\tau)D_L}\right),$$

sense of opportunities to invest at a rate of return greater than the market rate of return. These subjects are treated extensively in our paper, "Dividend Policy, Growth and the Valuation of Shares," *Jour. Bus.*, Univ. Chicago, Oct. 1961, 411–33.

[7] Here and throughout, the corresponding formulas when the rate of interest rises with leverage can be obtained merely by substituting $r(L)$ for r, where L is some suitable measure of leverage.

[8] The assumption that the debt is permanent is not necessary for the analysis. It is employed here both to maintain continuity with the original model and because it gives an upper bound on the value of the tax saving. See in this connection footnote 5 and footnote 9.

is invested in its bonds. The stock component entitles the holder to a fraction,

$$\frac{m}{S_L + (1 - \tau) D_L},$$

of the net profits of the levered company or

$$\left(\frac{m}{S_L + (1 - \tau) D_L}\right) [(1 - \tau)(\overline{X}Z - R_L)].$$

The holding of bonds yields

$$\left(\frac{m}{S_L + (1 - \tau) D_L}\right) [(1 - \tau) R_L].$$

Hence the total outcome is

$$Y_L = \left(\frac{m}{(S_L + (1 - \tau) D_L)}\right) [(1 - \tau)\overline{X}Z]$$

and this will dominate the uncertain income Y_U if (and only if)

$$S_L + (1 - \tau) D_L \equiv S_L + D_L - \tau D_L \equiv V_L - \tau D_L < V_U.$$

Thus, in equilibrium, V_U cannot exceed $V_L - \tau D_L$, for if it did investors would have an incentive to sell shares in the unlevered company and purchase the shares (and bonds) of the levered company.

Suppose now that $V_L - \tau D_L > V_U$. An investment of m dollars in the stock of the levered firm entitles the holder to the outcome

$$\begin{aligned} Y_L &= (m/S_L)[(1 - \tau)(\overline{X}Z - R_L)] \\ &= (m/S_L)(1 - \tau)\overline{X}Z - (m/S_L)(1 - \tau)R_L. \end{aligned}$$

Consider the following alternative portfolio: (1) borrow an amount $(m/S_L)(1-\tau)D_L$ for which the interest cost will be $(m/S_L)(1-\tau)R_L$ (assuming, of course, that individuals and corporations can borrow at the same rate, r); and (2) invest m plus the amount borrowed, i.e.,

$$m + \frac{m(1 - \tau) D_L}{S_L} = m \frac{S_L + (1 - \tau) D_L}{S_L} = (m/S_L)[V_L - \tau D_L]$$

in the stock of the unlevered firm. The outcome so secured will be

$$(m/S_L)\left(\frac{V_L - \tau D_L}{V_U}\right)(1 - \tau)\overline{X}Z.$$

Subtracting the interest charges on the borrowed funds leaves an income of

$$Y_U = (m/S_L)\left(\frac{V_L - \tau D_L}{V_U}\right)(1 - \tau)\overline{X}Z - (m/S_L)(1 - \tau)R_L$$

which will dominate Y_L if (and only if) $V_L - \tau D_L > V_U$. Thus, in equilibrium, both $V_L - \tau D_L > V_U$ and $V_L - \tau D_L < V_U$ are ruled out and (3) must hold.

III. *Some Implications of Formula* (3)

To see what is involved in replacing (4) with (3) as the rule of valuation, note first that both expressions make the value of the firm a function of leverage and the tax rate. The difference between them is a matter of the size and source of the tax advantages of debt financing. Under our original formulation, values within a class were strictly proportional to expected earnings after taxes. Hence the tax advantage of debt was due solely to the fact that the deductibility of interest payments implied a higher level of after-tax income for any given level of before-tax earnings (i.e., higher by the amount τR since $\bar{X}^{\tau}=(1-\tau)\bar{X}+\tau R$). Under the corrected rule (3), however, there is an additional gain due to the fact that the extra after-tax earnings, τR, represent a sure income in contrast to the uncertain outcome $(1-\tau)\bar{X}$. Hence τR is capitalized at the more favorable certainty rate, $1/r$, rather than at the rate for uncertain streams, $1/\rho^{\tau}$.[9]

Since the difference between (3) and (4) is solely a matter of the rate at which the tax savings on interest payments are capitalized, the required changes in all formulas and expressions derived from (4) are reasonably straightforward. Consider, first, the before-tax earnings yield, i.e., the ratio of expected earnings before interest and taxes to the value of the firm.[10] Dividing both sides of (3) by V and by $(1-\tau)$ and simplifying we obtain:

$$\text{(31.c)} \qquad \frac{\bar{X}}{V} = \frac{\rho^{\tau}}{1-\tau}\left[1 - \tau\frac{D}{V}\right]$$

which replaces our original equation (31) (p. 294). The new relation differs from the old in that the coefficient of D/V in the original (31) was smaller by a factor of r/ρ^{τ}.

Consider next the after-tax earnings yield, i.e., the ratio of interest payments plus profits after taxes to total market value.[11] This concept was discussed extensively in our paper because it helps to bring out more clearly the differences between our position and the traditional view, and because it facilitates the construction of empirical tests of the two hypotheses about the valuation process. To see what the new equation (3) implies for this yield we need merely substitute $\bar{X}^{\tau}-\tau R$ for $(1-\tau)\bar{X}$ in (3) obtaining:

[9] Remember, however, that in one sense formula (3) gives only an upper bound on the value of the firm since $\tau R/r=\tau D$ is an exact measure of the value of the tax saving only where both the tax rate and the level of debt are assumed to be fixed forever (and where the firm is certain to be able to use its interest deduction to reduce taxable income either directly or via transfer of the loss to another firm). Alternative versions of (3) can readily be developed for cases in which the debt is not assumed to be permanent, but rather to be outstanding only for some specified finite length of time. For reasons of space, we shall not pursue this line of inquiry here beyond observing that the shorter the debt period considered,the closer does the valuation formula approach our original (4). Hence, the latter is perhaps still of some interest if only as a lower bound.

[10] Following usage common in the field of finance we referred to this yield as the "average cost of capital." We feel now, however, that the term "before-tax earnings yield" would be preferable both because it is more immediately descriptive and because it releases the term "cost of capital" for use in discussions of optimal investment policy (in accord with standard usage in the capital budgeting literature).

[11] We referred to this yield as the "after-tax cost of capital." Cf. the previous footnote.

$$V = \frac{\bar{X}^\tau - \tau R}{\rho^\tau} + \tau D = \frac{\bar{X}^\tau}{\rho^\tau} + \tau \frac{\rho^\tau - r}{\rho^\tau} D, \tag{5}$$

from which it follows that the after-tax earnings yield must be:

$$\frac{\bar{X}^\tau}{V} = \rho^\tau - \tau(\rho^\tau - r) D/V. \tag{11.c}$$

This replaces our original equation (11) (p. 272) in which we had simply $\bar{X}^\tau/V = \rho^\tau$. Thus, in contrast to our earlier result, the corrected version (11.c) implies that even the after-tax yield is affected by leverage. The predicted rate of decrease of $\bar{X}^\tau/V$ with D/V, however, is still considerably smaller than under the naive traditional view, which, as we showed, implied essentially $\bar{X}^\tau/V = \rho^\tau\text{-}(\rho^\tau - r)D/V$. See our equation (17) and the discussion immediately preceding it (p. 277).[12] And, of course, (11.c) implies that the effect of leverage on $\bar{X}^\tau/V$ is *solely* a matter of the deductibility of interest payments whereas, under the traditional view, going into debt would lower the cost of capital regardless of the method of taxing corporate earnings.

Finally, we have the matter of the after-tax yield on *equity* capital, i.e., the ratio of net profits after taxes to the value of the shares.[13] By subtracting D from both sides of (5) and breaking $\bar{X}^\tau$ into its two components—expected net profits after taxes, $\bar{\pi}^\tau$, and interest payments, $R = rD$—we obtain after simplifying:

$$S = V - D = \frac{\bar{\pi}^\tau}{\rho^\tau} - (1 - \tau)\left(\frac{\rho^\tau - r}{\rho^\tau}\right) D. \tag{6}$$

From (6) it follows that the after-tax yield on equity capital must be:

$$\frac{\bar{\pi}^\tau}{S} = \rho^\tau + (1 - \tau)[\rho^\tau - r] D/S \tag{12.c}$$

which replaces our original equation (12), $\bar{\pi}^\tau/S = \rho^\tau + (\rho^\tau - r)D/S$ (p. 272). The new (12.c) implies an increase in the after-tax yield on equity capital as leverage increases which is smaller than that of our original (12) by a factor of $(1-\tau)$. But again, the linear increasing relation of the corrected (12.c) is still fundamentally different from the naive traditional view which asserts the cost of equity capital to be completely independent of leverage (at least as long as leverage remains within "conventional" industry limits).

IV. *Taxes and the Cost of Capital*

From these corrected valuation formulas we can readily derive corrected measures of the cost of capital in the capital budgeting sense of the minimum prospective yield an investment project must offer to be just worth

[12] The i_k^* of (17) is the same as ρ^τ in the present context, each measuring the ratio of net profits to the value of the shares (and hence of the whole firm) in an unlevered company of the class.

[13] We referred to this yield as the "after-tax cost of equity capital." Cf. footnote 9.

undertaking from the standpoint of the present stockholders. If we interpret earnings streams as perpetuities, as we did in the original paper, then we actually have two equally good ways of defining this minimum yield: either by the required increase in before-tax earnings, $d\overline{X}$, or by the required increase in earnings net of taxes, $d\overline{X}(1-\tau)$.[14] To conserve space, however, as well as to maintain continuity with the original paper, we shall concentrate here on the before-tax case with only brief footnote references to the net-of-tax concept.

Analytically, the derivation of the cost of capital in the above sense amounts to finding the minimum value of $d\overline{X}/dI$ for which $dV=dI$, where I denotes the level of new investment.[15] By differentiating (3) we see that:

$$(7) \qquad \frac{dV}{dI} = \frac{1-\tau}{\rho^\tau}\frac{d\overline{X}}{dI} + \tau\frac{dD}{dI} \geq 1 \qquad \text{if } \frac{d\overline{X}}{dI} \geq \frac{1-\tau\frac{dD}{dI}}{1-\tau}\rho^\tau.$$

Hence the before tax required rate of return cannot be defined without reference to financial policy. In particular, for an investment considered as being financed entirely by new equity capital $dD/dI=0$ and the required rate of return or marginal cost of equity financing (neglecting flotation costs) would be:

$$\rho^S = \frac{\rho^\tau}{1-\tau}\,.$$

This result is the same as that in the original paper (see equation [32], p. 294) and is applicable to any other sources of financing where the remuneration to the suppliers of capital is not deductible for tax purposes. It applies, therefore, to preferred stock (except for certain partially deductible issues of public utilities) and would apply also to retained earnings were it not for the favorable tax treatment of capital gains under the personal income tax.

For investments considered as being financed entirely by new debt capital $dI=dD$ and we find from (7) that:

$$(33.c) \qquad \rho^D = \rho^\tau$$

which replaces our original equation (33) in which we had:

$$(33) \qquad \rho^D = \rho^S - \frac{\tau}{1-\tau}r.$$

[14] Note that we use the term "earnings net of taxes" rather than "earnings after taxes." We feel that to avoid confusion the latter term should be reserved to describe what will actually appear in the firm's accounting statements, namely the net cash flow including the tax savings on the interest (our $\overline{X}^\tau$). Since financing sources cannot in general be allocated to particular investments (see below), the after-tax or accounting concept is not useful for capital budgeting purposes, although it can be extremely useful for valuation equations as we saw in the previous section.

[15] Remember that when we speak of the minimum required yield on an investment we are referring in principle only to investments which increase the *scale* of the firm. That is, the new

Thus for borrowed funds (or any other tax-deductible source of capital) the marginal cost or before-tax required rate of return is simply the market rate of capitalization for net of tax unlevered streams and is thus independent of both the tax rate and the interest rate. This required rate is lower than that implied by our original (33), but still considerably higher than that implied by the traditional view (see esp. pp. 276–77 of our paper) under which the before-tax cost of borrowed funds is simply the interest rate, r.

Having derived the above expressions for the marginal costs of debt and equity financing it may be well to warn readers at this point that these expressions represent at best only the hypothetical extremes insofar as costs are concerned and that neither is directly usable as a cut-off criterion for investment planning. In particular, care must be taken to avoid falling into the famous "Liquigas" fallacy of concluding that if a firm intends to float a bond issue in some given year then its cut-off rate should be set that year at ρ^D; while, if the next issue is to be an equity one, the cut-off is ρ^S. The point is, of course, that no investment can meaningfully be regarded as 100 per cent equity financed if the firm makes any use of debt capital—and most firms do, not only for the tax savings, but for many other reasons having nothing to do with "cost" in the present static sense (cf. our original paper pp. 292–93). And no investment can meaningfully be regarded as 100 per cent debt financed when lenders impose strict limitations on the maximum amount a firm can borrow relative to its equity (and when most firms actually plan on normally borrowing less than this external maximum so as to leave themselves with an emergency reserve of unused borrowing power). Since the firm's long-run capital structure will thus contain both debt and equity capital, investment planning must recognize that, over the long pull, *all* of the firm's assets are really financed by a mixture of debt and equity capital even though only one kind of capital may be raised in any particular year. More precisely, if L^* denotes the firm's long-run "target" debt ratio (around which its actual debt ratio will fluctuate as it "alternately" floats debt issues and retires them with internal or external equity) then the firm can assume, to a first approximation at least, that for any particular investment $dD/dI = L^*$. Hence, the relevant marginal cost of capital for investment planning, which we shall here denote by ρ^*, is:

$$\rho^* = \frac{1 - \tau L^*}{1 - \tau} \rho^\tau = \rho^S - \frac{\tau}{1 - \tau} \rho^D L^* = \rho^S(1 - L^*) + \rho^D L^*.$$

That is, the appropriate cost of capital for (repetitive) investment decisions over time is, to a first approximation, a weighted average of the costs of debt and equity financing, the weights being the proportions of each in the "target" capital structure.[16]

assets must be in the same "class" as the old. See in this connection, J. Hirshleifer, "Risk, the Discount Rate and Investment Decisions," *Am. Econ. Rev.*, May 1961, *51*, 112–20 (especially pp. 119–20). See also footnote 16.

[16] From the formulas in the text one can readily derive corresponding expressions for the required net-of-tax yield, or net-of-tax cost of capital for any given financing policy. Specifi-

V. *Some Concluding Observations*

Such, then, are the major corrections that must be made to the various formulas and valuation expressions in our earlier paper. In general, we can say that the force of these corrections has been to increase somewhat the estimate of the tax advantages of debt financing under our model and consequently to reduce somewhat the quantitative difference between the estimates of the effects of leverage under our model and under the naive traditional view. It may be useful to remind readers once again that the existence of a tax advantage for debt financing—even the larger advantage of the corrected version—does not necessarily mean that corporations should at all times seek to use the maximum possible amount of debt in their capital structures. For one thing, other forms of financing, notably retained earnings, may in some circumstances be cheaper still when the tax status of investors under the personal income tax is taken into account. More important, there are, as we pointed out, limitations imposed by lenders (see pp. 292–93), as well as many other dimensions (and kinds of costs) in real-world problems of financial strategy which are not fully comprehended within the framework of static equilibrium models, either our own or those of the traditional variety. These additional considerations, which are typically grouped under the rubric of "the need for preserving flexibility," will normally imply the maintenance by the corporation of a substantial reserve of untapped borrowing power. The tax advantage of debt may well tend to lower the optimal size of that reserve, but it is hard to believe that advantages of the size contemplated under our model could justify any substantial reduction, let alone their complete elimination. Nor do the data

cally, let $\bar{\rho}(L)$ denote the required net-of-tax yield for investment financed with a proportion of debt $L=dD/dI$. (More generally L denotes the proportion financed with tax deductible sources of capital.) Then from (7) we find:

$$\bar{\rho}(L) = (1-\tau)\frac{d\bar{X}}{dI} = (1-L\tau)\rho^{\tau} \quad (8)$$

and the various costs can be found by substituting the appropriate value for L. In particular, if we substitute in this formula the "target" leverage ratio, L^*, we obtain:

$$\bar{\rho}^* \equiv \bar{\rho}(L^*) = (1-\tau L^*)\rho^{\tau}$$

and $\bar{\rho}^*$ measures the average net-of-tax cost of capital in the sense described above.

Although the before-tax and the net-of-tax approaches to the cost of capital provide equally good criteria for investment decisions when assets are assumed to generate perpetual (i.e., non-depreciating) streams, such is not the case when assets are assumed to have finite lives (even when it is also assumed that the firm's assets are in a steady state age distribution so that our X or EBIT is approximately the same as the net cash flow before taxes). See footnote 3 above. In the latter event, the correct method for determining the desirability of an investment would be, in principle, to discount the net-of-tax stream at the net-of-tax cost of capital. Only under this net-of-tax approach would it be possible to take into account the deductibility of depreciation (and also to choose the most advantageous depreciation policy for tax purposes). Note that we say that the net-of-tax approach is correct "in principle" because, strictly speaking, nothing in our analysis (or anyone else's, for that matter) has yet established that it is indeed legitimate to "discount" an uncertain stream. One can hope that subsequent research will show the analogy to discounting under the certainty case is a valid one; but, at the moment, this is still only a hope.

indicate that there has in fact been a substantial increase in the use of debt (except relative to preferred stock) by the corporate sector during the recent high tax years.[17]

As to the differences between our modified model and the traditional one, we feel that they are still large in quantitative terms and still very much worth trying to detect. It is not only a matter of the two views having different implications for corporate financial policy (or even for national tax policy). But since the two positions rest on fundamentally different views about investor behavior and the functioning of the capital markets, the results of tests between them may have an important bearing on issues ranging far beyond the immediate one of the effects of leverage on the cost of capital.

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[17] See, e.g., Merton H. Miller, "The Corporate Income Tax and Corporate Financial Policies," in *Staff Reports to the Commission on Money and Credit* (forthcoming).

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Consumption, Savings and Windfall Gains: Comment

In her recent article in this *Review* [3], Margaret Reid attempted to answer previous articles by Bodkin [1] and Jones [2] challenging the validity of the permanent income hypothesis. Bodkin and Jones used income and expenditure data for those consumer units who had received the soldiers' bonus (National Service Life Insurance dividends) during 1950, the year of the urban consumption survey [4]. These bonuses were regarded as windfall gains for the purposes of their analyses.

Professor Reid used data from the same survey, but her windfall gains were represented by "other money receipts." These are defined as "inheritances and occasional large gifts of money from persons outside the family . . . and net receipts from the settlement of fire and accident policies" [4, Vol. 1, p. xxix]. She assumed that the soldiers' bonus was included, and that it accounted for about one-half of other money receipts. Here she made an unfortunate mistake in interpreting the data for the main critical purpose of her article.

The soldiers' bonus is not part of "other money receipts" (O) but rather a part of "disposable money income" (Y). It is the main part of an item in the disposable money income category called "military pay, allotments, and pensions" [4, Vol. 11, p. xxix].

This would appear to alter completely the relationship of Professor Reid's main findings to the Bodkin results and to change the windfall interpretation of the O variable. Surely, fire and accident policy settlements are not windfall income, but rather a (partial) recovery of real assets previously lost. Likewise, inheritances are probably best considered as a long-anticipated increase in assets—not an increase in transitory income.

The discovery of this error probably does not affect whatever importance Professor Reid's secondary finding may have: ". . . the need, in any study of